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Article *in* Wireless Communications and Mobile Computing · October 2015

Impact Factor: 0.86 · DOI: 10.1002/wcm.2642

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Opportunistic Distributed Channel Access for a Dense Wireless Small-Cell Zone

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***Abstract*—This paper considers uplink channel access in a *zone* of closed-access small-cells that is deployed in a macrocell service area. All small-cell user equipments (SUEs) have access to a common orthogonal set of channels, leading to intercell interference. In addition, each channel forms a separate collision domain in each cell, thus can be successfully used only by one SUE of that cell. This paper proposes two noncooperative Bayesian games, G_1 and G_2, that are played among the SUEs. G_1 assumes the availability of channel state information (CSI) at the transmitters while G_2 assumes the availability of only the distribution of the CSI. Each SUE can choose to transmit over one of the channels or not to transmit. The emphasis of the paper is on the set of symmetric threshold strategies where the Nash equilibrium is fully determined by a single parameter. The existence and uniqueness of pure Bayesian-Nash symmetric equilibrium (BNSE) of G_1 in threshold strategies and mixed BNSE of G_2 in uniformly distributed threshold strategies are proven. Numerical results corroborate the theoretical findings and benchmark against another decentralized scheme.**

***Index Terms*—Small-cells, distributed uplink access, Bayesian games, stochastic coupling.**

I. Introduction

Small-cells (SCs) are introduced to improve the coverage and meet the increasing demand for throughput of indoor users. They are low-power, cost-effective, and short-range radio access networks. The throughput is improved due to the reduced link distance and improved channel between the indoor located user equipments (UEs) and the small-cell access point (SAP). SCs include microcells, picocells, and femtocells [1]. A UE connected to an SAP is called a small-cell user equipment (SUE). In an underlay deployment, SCs have access to the same spectrum as the existing macrocell. A wireless network with co-existing macrocells and SCs is called a heterogeneous small-cell network (HetSNet). In a HetSNet, intercell interference is a key limiting factor [2]. Centralized coordinated scheduling of SUEs among multiple SAPs is optimal for interference mitigation, but such a scheme requires extensive global channel state information (CSI) of all the UEs. Acquisition of time-varying global CSI involves a large signaling overhead, which renders centralized schemes less practical. Therefore, distributed interference management with low-signaling in HetSNets is a true challenge [3]. *Non-cooperative-game theory* is one of the tools employed in this domain [4].

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Interference mitigation schemes that use game theoretic tools are being developed in recent work. A potential-game-theoretic [4] solution for intracell interference mitigation in a cognitive radio network (CRN) through combined power allocation and base station association is considered in [5]. Another potential-game is introduced in [6] for the problem of uplink channel and power allocation in a multicell environment. In a potential game, the convergence time to equilibrium can be large and best response dynamics generally require the knowledge of current action profile at each player. For a detailed study of potential games in wireless networks the reader is referred to [7]. In [8], a concave game is proposed for a set of interfering orthogonal frequency division multiplex (OFDM) transmit-receive pairs. In a concave game, the action set of each player is compact and convex, the utility of a player is continuous in the action profile and is quasi-concave in its own action [9]. The strategy space of the transmitters is the rate assignment over the set of OFDM channels. Finding the Nash equilibrium (NE) in a concave game requires extensive information sharing between players.

Multi-user channel access games are also addressed in relation to medium access control (MAC) protocol design. This approach is taken in random access networks, such as ad-hoc, ALOHA, and carrier sense multiple access (CSMA). A game theory inspired MAC protocol for the interference channel is designed in [10]. In [11], an uplink channel access game is proposed for a set of co-located transmit-receive pairs over a single channel. They consider a collision-domain approach, instead of interference, and present a mixed-strategy nonsymmetric equilibrium. Likewise authors in [12] assume a collision-domain and model the access probability design as a continuous-action-space game. In the ALOHA setting, a noncooperative game is designed in [13], where the strategy space is the probability of transmission. In [14], a mobile ad-hoc network where nodes follow slotted ALOHA protocol is considered and they develop a game for the channel access probability at the symmetric NE.

A. Related Work

This paper explores orthogonal channel access in a dense cluster of SCs. Application of game theory for channel access in multichannel case appears in orthogonal frequency division multiple access (OFDMA) networks. In [15], subchannel allocation in a OFDMA based network is considered as a potential game. The utility of a player is a function of interference. An uplink power allocation game among UEs is analyzed in [16]. Interference is controlled through a quadratic cost function on transmitted power. In [17], uplink channel allocation in an OFDMA multicell system is formulated and solved for the correlated equilibrium (CE). In [18], a distributed cell selection and resource allocation scheme that is performed by UEs is presented. It is a two stage game. A UE first selects the cell and then selects the radio resource. In [19], resource allocation problem of the OFDMA downlink is addressed in the context of mechanism design. The authors demonstrate that the problem is NP-hard and provide an α-optimal solution. In a multiple femtocell scenario, an OFDMA based downlink power allocation with interference constraints is considered in [20], as a generalized NE problem. In [21], a threshold strategy based game in a multichannel environment is presented. A threshold strategy of a player is defined by a single parameter.

The above discussed research consider games that require complete CSI. Therefore, even if a NE exists they have limited practical applicability as an individual player in a wireless network is unlikely to possess full knowledge of the network. Also there is the question of multiple NEs. The incomplete CSI case is taken into consideration through Bayesian games in [22]. Therein, power allocation for transmit-receive pairs over a multichannel system with interference is considered. The authors prove that spreading power equally among the flat fading channels is a pure-strategy NE. The Bayesian symmetric games in [23]–[25] consider threshold strategies in a single channel wireless network. The channel access model in [23], [24] is a single collision domain and a single access point (AP). In [25], the work of [23] is extended to multiple APs with intercell interference.

A key limitation of applying game theory to design distributed solutions is the complexity in finding the NE. It has been proven, that even for a two player game finding the NE is PPAD (Polynomial Parity Arguments on Directed graphs) complete [26]. Therefore, we motivate a symmetric game which possesses NE that is computable individually by each player as the unique root of a function [23]. In a symmetric game the utility of a player, given the action profile of other players, is independent of the player [4]. This paper considers the orthogonal multichannel uplink transmission in a *dense zone* of SCs. The qualifier *dense*, in the context of this paper, means that the coverage areas of the SCs overlap with each other. That is, as opposed to sparse deployment where SCs are deployed far apart as to not interfere with each other [27]. The qualifier *zone*, in the context of this paper, means that the set of SCs in consideration is confined to a localized area that is small relative to the macrocell coverage area. Examples of dense zones of SCs are convention centres, hotel lobbies or shopping malls, where more than one telecommunication service providers maintain picocells, each for their own users. Thus it is important to this research that a *dense zone of SCs* is not understood as a large number of SCs, rather it is an overlapping localized deployment. Fig. 1 depicts a dense zone of SCs. In such a dense deployment, cochannel interference between SCs is a key limitation and opportunistic channel access has been proposed as a solution [28]. In opportunistic channel access, an SUE exploits the fading of the channels to judiciously access, while its own channel gain is relatively higher in comparison to the interference [29].

B. Contributions and Organization of the Paper

The objective of this paper is to propose low complexity, decentralized-opportunistic channel access schemes based on Bayesian games. To that end, the paper considers symmetric threshold strategies; those that are defined by a single parameter [21], [23]. Two uplink channel access games are discussed. The first game $\mathbf{G}_1$, considers the case where each SUE knows its CSI. This situation is identified in literature as channel state information available at the transmitter (CSIT). The second game $\mathbf{G}_2$, considers the case where each SUE has statistical knowledge of its CSI (statistical-CSIT). In this paper:

- we bring together in a game model; multiple channels, intracell per-channel collision domains, inter-SC and macrocell-SC interference, and random symbol availability at the SUEs which are well identified resource allocation constraints in an SC deployment.
- we prove the existence of a unique pure-strategy Bayesian-Nash symmetric equilibrium (BNSE) in threshold strategies for game $\mathbf{G}_1$ with CSIT.
- we prove the existence of a unique mixed-strategy BNSE in uniformly distributed threshold strategies for game $\mathbf{G}_2$ with statistical-CSIT.
- we corroborate the theoretical results through numerical simulations and compare against a locally optimal scheme where each SC schedules the SUE with highest channel gain.

The pure-strategy BNSE proved in $\mathbf{G}_1$ is an extension of the single channel result of [25] to a multichannel case. However, the extension is nontrivial and the proof method is novel, in that we employ stochastic coupling theory [30]. The mixed-strategy BNSE proved in $\mathbf{G}_2$ for statistical-CSIT is novel in SC research to the best of our knowledge. The advantage of BNSE in threshold strategies is that each player is independently able to find the equilibrium without message passing. However its applicability is limited to symmetric players, which is a fair approximation of a dense SC zone as justified in the following sections.

The remainder of this paper is organized as follows. The assumptions and system model are detailed in Section II. Development of game $\mathbf{G}_1$ for CSIT is presented in Section III, and Section IV solves it. Development of game $\mathbf{G}_2$ for statistical-CSIT and its solution is presented in Section V. Numerical results and a discussion on non-symmetric case is presented in Section VI and finally Section VII concludes the paper.

II. System Model

This paper considers the uplink access in a dense zone of SCs that is underlayed in a single macrocell coverage area. Each SAP forms a single SC and hence SAP and SC are synonymous. As was defined in Section I, a dense SC zone is a set of SAPs that is deployed in a confined area having overlapping coverage. For example a convention centre where more than one operator maintain picocells. Fig. 1 depicts an example dense zone of SCs. The set of SAPs is $\mathcal{M}$. Let $\mathcal{N}$ denote the set of SUEs and $\mathcal{K}$ the set of orthogonal channels whose cardinalities are given by N and K, respectively. The

Table I: Notation summary

Symbol(s)	Description
$\mathcal{M}$, $\mathcal{N}$, $\mathcal{K}$,	set of SAP, SUE, and channels with cardinality M, N, and K
$\mathcal{N}_m$, $\mathcal{N}_{-m}$	set of SUEs of SAP m and set of SUEs not of SAP m
$\mathcal{N}_{b_i k}$	set of SUEs except $i \in \mathcal{N}_{b_i}$ of SAP b_i that transmits on channel k
$\mathcal{N}_{-mk}$	set of SUEs not of SAP m that transmits on channel k
$\mathcal{N}'_{-mk}$	set of SUEs not of SAP m that does not transmit on channel k
$\mathcal{A}_i$, $\mathcal{A}_{-i}$,$\mathcal{A}$	action set of SUE i, product set $\prod_{j\in\mathcal{N}\setminus\{i\}}\mathcal{A}_j$, and $\prod_{j\in\mathcal{N}}\mathcal{A}_j$
Θ_i, Θ_{-i}, Θ	of game $\mathbf{G}_1$- type set of SUE i, product set $\prod_{j\in\mathcal{N}\setminus\{i\}}\Theta_j$, and $\prod_{j\in\mathcal{N}}\Theta_j$
Ω_i, Ω_{-i}, Ω	of game $\mathbf{G}_2$- type set of SUE i, product set $\prod_{j\in\mathcal{N}\setminus\{i\}}\Omega_j$, and $\prod_{j\in\mathcal{N}}\Omega_j$
α_i	random symbol availability indicator of SUE i
h_i^k	channel power gain of SUE i to home-SAP on channel k
g_{jm}^k	interference channel power gain of a UE j to SAP m on channel k
γ_{km}	sum of noise power of MUEs at SAP m on channel k
$f_{h_i^k}$, $f_{g_{im}^k}$, f_{α_i}, f_{γ}	probability densities of h_i^k, g_{im}^k, α_i, and γ_{km}
$\bar{f}_{\boldsymbol{\theta}_{-i}}$, $\bar{f}_{\boldsymbol{\omega}_{-i}}$	belief densities of SUE i over set Θ_{-i} and Ω_{-i}
u_i	utility of player i
s_i, r_i	pure and mixed strategy of SUE i
$\mathbf{s}_{-i}$, $\mathbf{r}_{-i}$	pure and mixed strategies of SUEs except i
$h_{i\text{th}}$, $\lambda_{i\text{th}}$	threshold of SUE i in game $\mathbf{G}_1$ and $\mathbf{G}_2$
q_1, q_2	probability that an SUE transmits on channel k in game $\mathbf{G}_1$ and $\mathbf{G}_2$
$\bar{\mathrm{p}}_l\left(\mathcal{N}_{b_i k}\right)$, $\bar{\mathrm{p}}_l\left(\mathcal{N}'_{-b_i k}\right)$	probability that SUEs $\mathcal{N}_{b_i k}$ and $\mathcal{N}_{-b_i k}$ do not transmit on channel k, of game $\mathbf{G}_l$ $(l \in \{1,2\})$
$\mathrm{p}_l\left(\mathcal{N}_{-b_i k}\right)$	probability that SUEs $\mathcal{N}_{-b_i k}$ transmit on channel k, of game $\mathbf{G}_l$ $(l \in \{1,2\})$
$\mathscr{B}_l\left(N, \mathrm{p}\right)$	of game $\mathbf{G}_l$ $(l \in \{1,2\})$ binomial distribution with N trials and success probability p
$\mathbb{E}$, $\Pr$, $\mathbb{P}$	expectation, probability, and power set

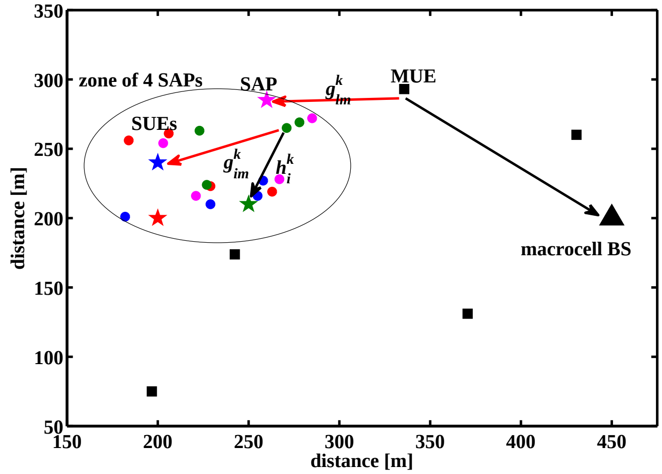


Figure 1: A scenario illustrating a dense zone of SCs where 4 operators serve a commercial building. A ⋆ denotes an SAP, ● denotes an SUE, ▲ denotes the macrocell base station (BS) ■ denotes an macrocell UE (MUE). The colors match the SUEs to home-SAPs.

set of macrocell user equipments (MUEs) is denoted by $\mathfrak{N}$. Each SAP $m \in \mathcal{M}$ operates in *closed-access* mode and hence is accessible only by SUEs that are in its access list and they are called the home-SUEs of that SAP [31]. We assume that an SUE i can only be in the access list of only one SAP, which is called its home-SAP and denoted by $b_i \in \mathcal{M}$. The set of home-SUEs of SAP m is denoted by $\mathcal{N}_m$ and its complement by $\mathcal{N}_{-m}$. Narrow-band single tap Rayleigh fading channels are assumed in this paper. Then the baseband equivalent received signal y_m^k, at SAP m on channel $k \in \mathcal{K}$ is,

$$y_m^k = \sum_{i\in\mathcal{N}_m} \bar{h}_i^k x_i^k + \sum_{j\in\mathcal{N}_{-m}} \bar{g}_{jm}^k x_j^k + \sum_{l\in\mathfrak{N}} \bar{g}_{lm}^k x_l^k + n_m^k, \quad (1)$$

where $\bar{h}_i^k$ is the complex channel gain from SUE i to its home-SAP m on channel k. The complex interference gain from a UE (can be an SUE or an MUE) $j \in \mathcal{N}_{-m} \cup \mathfrak{N}$ to SAP m on channel k is denoted by $\bar{g}_{jm}^k$ [29]. The corresponding channel power gains are denoted by $h_i^k := \left|\bar{h}_i^k\right|^2$, $g_{im}^k := \left|\bar{g}_{jm}^k\right|^2$ and they follow exponential distribution. The complex valued transmit symbols of a UE $j \in \mathcal{N} \cup \mathfrak{N}$ on channel k is denoted by x_j^k. The circular symmetric complex additive white Gaussian noise (AWGN) with zero mean and σ^2 variance is denoted by n_m^k. The second right hand term of (1) is the sum interference from the SUEs that belongs to SAPs other than m. The third term is the sum interference from the MUEs. Note that although we sum over all UEs, if a UE j does not transmit, the respective symbol x_j^k is 0. In order to enhance the readability we summarize key notations in Table I.

In the following development, the number of SUEs in each SC is set to a constant with probability one. However the results derived in the paper apply to independently and identically populated SC with finite number of SUEs as well. This extension is detailed in Section VI-B and an example is provided there. In practice, this corresponds to a situation where the SAP operators have equal market share. Unlike in macrocells, in SCs all home-SUEs lie sufficiently close to the home-SAP so that no power control is needed and therefore SUEs transmit at constant normalized unit power [32].

The load of the MUEs is assumed to be balanced in distribution over the set of channels $\mathcal{K}$, which is realistic as the channel gains are random and there is a large number of MUEs. From the above assumption and since the SAPs in the zone are close to each other with respect to the coverage area of the macrocell, the sum of MUE interference received at each SAP can be modeled by identically distributed random variables [33]. In other words, defining $\gamma_{km} := \sum_{l\in\mathfrak{N}} g_{lm}^k$ with the probability density $f_{\gamma_{km}}$, we have that $f_{\gamma_{km}} = f_{\gamma_{k'm'}} = f_\gamma$, $\forall k, k' \in \mathcal{K}\ \forall\ m, m' \in \mathcal{M}$. The distribution f_γ can be estimated through cognitive features in SCs and the errors associated with estimation are disregarded in this research [34]. The symbol availability at an SUE i is denoted by the Boolean random variable α_i. If a symbol is available then $\alpha_i = 1$ otherwise 0.

III. Design of $\mathbf{G}_1$: A Game with CSIT

This section defines the components of the Bayesian game with CSIT. The set of players are the SUEs. By the definition of CSIT, each SUE i possess perfect information of $\left(h_i^k, k \in \mathcal{K}\right)$ and it also knows the realization of α_i. The actions available for an SUE are: transmit on a channel k denoted by $\mathtt{T}_k$ and the action of *"do not transmit"* denoted by $\mathtt{X}$. Then the action set of SUE i is $\mathcal{A}_i := \{\mathtt{X}, \mathtt{T}_1, \ldots, \mathtt{T}_K\}$. We define the joint action spaces by $\mathcal{A} := \prod_{i\in\mathcal{N}} \mathcal{A}_i$ such that $\mathbf{a} \in \mathcal{A}$ and $\mathcal{A}_{-i} := \prod_{j\in\mathcal{N}\setminus\{i\}} \mathcal{A}_j$ such that $\mathbf{a}_{-i} \in \mathcal{A}_{-i}$. The MUEs do not take part in the game, but exogenously affect the outcome through interference.

A. Symmetric-Independent Types

Type of a player in a Bayesian games is the private information of that player [4]. In our system model, the private information available at SUE i is its channel power gains to the home-SAP and its symbol availability. We define a single private information vector $\boldsymbol{\theta}_i$, called the *type vector*, containing all private information, $\boldsymbol{\theta}_i := \left(\left(h_i^k\right)_{k\in K}, \left(g_{im}^k\right)_{k\in\mathcal{K}, m\in\mathcal{M}\setminus\{b_i\}}, \alpha_i\right)$. The *type set* is denoted by Θ_i such that $\boldsymbol{\theta}_i \in \Theta_i$. We also define the type set product $\Theta := \prod_{i\in\mathcal{N}} \Theta_i$ such that $\boldsymbol{\theta} \in \Theta$ and $\Theta_{-i} := \prod_{j\in\mathcal{N}\setminus\{i\}} \Theta_j$ such that $\boldsymbol{\theta}_{-i} \in \Theta_{-i}$.

Let the probability densities of h_i^k, g_{im}^k, and α_i, be $f_{h_i^k}$, $f_{g_{im}^k}$, and f_{α_i}, respectively. Then the belief that SUE i holds about the types of other players $\boldsymbol{\theta}_{-i}$ is given by the density function $f_{\boldsymbol{\theta}_{-i}} = \prod_{\substack{k\in\mathcal{K}\\ j\in\mathcal{N}\setminus\{i\}}} \left(f_{h_j^k} \cdot f_{g_{jm}^k}\right) \cdot \prod_{j\in\mathcal{N}\setminus\{i\}} f_{\alpha_j}$. The paper also consider i.i.d. symbol availability among SUEs, which implies that they have demands identical in distribution, hence $f_{\alpha_i} = f_\alpha$, $\forall\ i \in \mathcal{N}$. Recall from Section II that the SCs lie in a dense confined zone. Therefore, all the SUEs lie close to the SAPs and experience a similar scattering environment. As such, we assume that all players hold independent and identical beliefs about each others channels, which in Bayesian games is called *symmetric-independent types* [4]. From symmetric-independent types, $f_{h_i^k} = f_h$ and $f_{g_{im}^k} = f_g$ $\forall i \in \mathcal{N}$, $\forall m \in \mathcal{M}$, and $\forall k \in \mathcal{K}$.

B. Utility Design

The utility of player i is a function $u_i : \mathcal{A} \times \Theta \to \mathbb{R}$. When $\alpha_i = 0$, SUE i does not possess a symbol and hence the utility is zero. This paper models each channel on each SAP as a separate *collision domain*. This implies that if more than one home-SUE of a given SAP transmits simultaneously on the same channel, all those SUEs obtain a zero utility due to collision. It is important to emphasize that the SUEs from different SAPs may transmit simultaneously over the same channel. Home-SUEs of a given SAP may transmit simultaneously and not experience collisions as long as they employ different channels.

However, to obtain a positive rate, avoiding a collision is not sufficient. The signal-to-interference plus noise ratio (SINR) needs to be above a detectable threshold Γ_{th} as well [28]. Let $\mathcal{N}_{b_ik}$ denote the set of the home-SUEs of SAP b_i except SUE i , that transmits on channel k. Let $\mathcal{N}_{-b_ik}$ denote the set of SUEs of SAPs $\mathcal{M} \setminus \{b_i\}$ that transmits on channel k. Their cardinalities are denoted by N_{b_ik} and N_{-b_ik} respectively. According to the above discussion, we define the utility $u_i(\mathbf{a}, \boldsymbol{\theta})$, as follows. If SUE i does not transmit

$$u_i(\mathtt{X}, \mathbf{a}_{-i}, \boldsymbol{\theta}) = \begin{cases} \rho & \text{if}\, \alpha_i = 1, \\ 0 & \text{else}\, \alpha_i = 0, \end{cases} \tag{2}$$

where the modeling parameter $\rho \in \mathbb{R}$ is an incentive given to the player.

For the notational convenience, as the SAP in discussion is clear, subscript m is omitted and g_{jm}^k simplifies to g_j^k, while γ_{km} simplifies to γ_k. If SUE i has $\alpha_i = 1$, and transmits successfully on channel k, i.e., obtains SINR $\geq \Gamma_{\text{th}}$, and $N_{b_ik} = 0$, then

$$u_i(\mathtt{T}_k, \mathbf{a}_{-i}, \boldsymbol{\theta}) = \log\Big(1 + \frac{h_i^k}{\sum\limits_{j\in\mathcal{N}_{-b_ik}} g_j^k + \gamma_k + \sigma^2}\Big). \tag{3}$$

Otherwise, if SUE i has $\alpha_i = 1$, and transmits unsuccessfully on channel k, i.e., obtains SINR $< \Gamma_{\text{th}}$ or $N_{b_ik} \neq 0$, then

$$u_i(\mathtt{T}_k, \mathbf{a}_{-i}, \boldsymbol{\theta}) = 0. \tag{4}$$

C. Definition of Game $\mathbf{G}_1$

A game in normal form is defined by the set of players, action set of each player and utility of each player. In addition, when the game is Bayesian, we need to specify the type set and belief of each player and finally the system state that is given by the external random variables that affect the utilities.

Players	$\mathcal{N}$
Action	$\mathrm{a}_i \in \mathcal{A}_i$
Type	$\boldsymbol{\theta}_i \in \Theta_i$
Belief	$f_{\boldsymbol{\theta}_{-i}}$ over Θ_{-i}
System state	$\gamma_{km}\ \forall\ k \in \mathcal{K}$, $m \in \mathcal{M}$
Payoff	$u_i(\mathbf{a}, \boldsymbol{\theta})$, $\mathbf{a} \in \mathcal{A}$ and $\boldsymbol{\theta} \in \Theta$

In this paper the pairs of terms "player"-"SUE" and "payoff"-"utility" are synonymously used.

IV. Symmetric-Threshold Equilibrium of $\mathbf{G}_1$

The *ex interim* expected utility of player $i \in \mathcal{N}$ is defined as, $\mathbb{E}_{\gamma\boldsymbol{\theta}_{-i}|\boldsymbol{\theta}_i} u_i$ [35]. From the independence of random variables in symmetric-independent types, the conditional expectation $\mathbb{E}_{\gamma\boldsymbol{\theta}_{-i}|\boldsymbol{\theta}_i}$ simplifies to $\mathbb{E}_{\gamma\boldsymbol{\theta}_{-i}}$. For brevity, the rest of the paper refers to ex interim expected utility as the expected utility.

Next we introduce the standard definitions of pure strategies, best response (BR) strategy, and Bayesian-Nash equilibria [4], [35].

Definition 1. In a Bayesian game a pure strategy of a player i is a relation $s_i : \Theta_i \to \mathcal{A}_i$.

Following standard game-theoretic notation, the strategy vector of all players except SUE i is denoted by $\mathbf{s}_{-i} := (s_j, j \in \mathcal{N} \setminus \{i\})$ and the strategy profile of all players is denoted by $\mathbf{s} := (s_i, i \in \mathcal{N})$.

Definition 2. Given strategy vector $\mathbf{s}_{-i}$, a BR strategy of player i, denoted by $\tilde{s}_i$, is given by

$$\tilde{s}_i(\boldsymbol{\theta}_i) \in \arg\max_{s_i \in \mathcal{S}_i} \left\{ \mathbb{E}_{\gamma\boldsymbol{\theta}_{-i}} u_i\left(s_i(\boldsymbol{\theta}_i), \mathbf{s}_{-i}, \boldsymbol{\theta}\right) \right\}, \forall \boldsymbol{\theta}_i \in \Theta_i.$$

According to Definition 2, a strategy is a BR if the player cannot obtain a higher payoff by another strategy for any of its types.

Definition 3. The strategy profile $\tilde{\mathbf{s}} := (\tilde{s}_i, i \in \mathcal{N})$ is a Bayesian-Nash equilibrium if $\tilde{s}_i$ is a BR strategy for $\tilde{\mathbf{s}}_{-i}$ $\forall i \in \mathcal{N}$.

A. Threshold Strategies

This paper considers *threshold strategies*. Such strategies form a subset of the feasible strategies of the game. We define the threshold strategy of SUE i as follows:

$$s_{i\text{th}}(\boldsymbol{\theta}_i) := \begin{cases} \mathtt{T}_k & \text{if } \alpha_i = 1, h_i^k = \max_{k' \in \mathcal{K}} \left\{ h_i^{k'} \right\} \geq h_{i\text{th}}, \\ \mathtt{X} & \text{otherwise}, \end{cases} \tag{5}$$

where $h_{i\text{th}}$ is a non-negative real valued parameter. The threshold-strategy definition (5) states that a player transmits on channel k if the channel power gain h_i^k is the largest among all the channels and h_i^k is greater than a threshold $h_{i\text{th}}$. As a consequence of the special form of threshold strategies, we can denote the strategy profile of the players by simply specifying their threshold vector $\mathbf{s}_{\text{th}} := (h_{i\text{th}}, i \in \mathcal{N})$. Similarly, the threshold strategy vector of all players except i is denoted by $\mathbf{s}_{-i\text{th}} = (h_{j\text{th}}, j \in \mathcal{N} \setminus \{i\})$. If the threshold is *symmetric*, i.e., common to all players, then we denote the strategy profile by $\mathbf{s}_{\text{th}}^{\text{sym}} := (h_{\text{th}})$. Since we search for a unique BNSE in threshold strategies, according to Definition 3, our goal is to demonstrate that there is a unique threshold $h_{i\text{th}} = \tilde{h}_{\text{th}}$ $\forall\, i \in \mathcal{N}$ such that $\mathbf{s}_{\text{th}}^{\text{sym}} = (\tilde{h}_{\text{th}})$ is a mutual BR strategy profile. The question is weather the strategy space defined by (5) is large enough to contain a NE. We answer this question positively in the following.

When SUE i plays the threshold strategy (5) the probability that it transmits on channel k is,

$$\mathrm{q}_i^k(h_{i\text{th}}) = \Pr\Big(\alpha_j = 1, h_i^k = \max_{k' \in \mathcal{K}} \left\{ h_i^{k'} \right\}, h_i^k \geq h_{i\text{th}}\Big). \tag{6}$$

We observe that the probability $\mathrm{q}_i^k(h_{i\text{th}})$ is decreasing in $h_{i\text{th}}$. From independence of types, the probability that all the SUEs in $\mathcal{N}' \subseteq \mathcal{N}$ transmit on channel k is $\mathrm{p}_1^k(\mathcal{N}') := \prod_{j \in \mathcal{N}'} \mathrm{q}_j^k(h_{j\text{th}})$, and the probability that none of the SUEs in $\mathcal{N}'$ transmit on k is $\bar{\mathrm{p}}_1^k(\mathcal{N}') := \prod_{j \in \mathcal{N}'} \left(1 - \mathrm{q}_j^k(h_{j\text{th}})\right)$. Let $\mathcal{N}'_{-b_i k}$ denote the set of SUEs that belong to SAPs other than b_i and that *does not* transmit on channel k. The probability that i does not encounter a collision on k is $\bar{\mathrm{p}}_1^k(\mathcal{N}_{b_i} \setminus \{i\})$. The probability that the set $\mathcal{N}_{-b_i k}$ transmits is $\mathrm{p}_1^k(\mathcal{N}_{-b_i k})$ and the probability that the set $\mathcal{N}'_{-b_i k}$ does not transmit on channel k is $\bar{\mathrm{p}}_1^k(\mathcal{N}'_{-b_i k})$. These probabilities are used to define the expected utility.

The expected utility of player i when $\alpha_i = 1$ and $\mathrm{a}_i = \mathtt{T}_k$ is denoted by $\mathbb{E}_{\gamma\boldsymbol{\theta}_{-i}} u_i\left(h_i^k, \mathbf{s}_{-i\text{th}}, \boldsymbol{\theta}\right)$ and is given by (7). The power set of $\mathcal{N}_{-b_i}$ is denoted by $\mathbb{P}(\mathcal{N}_{-b_i})$. The integration region $\mathcal{D}$ is $\{(g_{jb_i}^k, j \in \mathcal{N}_{-b_i k}), \gamma_k : \frac{h_i^k}{\sum_{j \in \mathcal{N}_{-b_i k}} g_j^k + \gamma_k + \sigma^2} \geq \Gamma_{\text{th}}\}$ and $f_{\mathbf{g}}$ is the probability density of the random variable vector $\left(g_j^k, j \in \mathcal{N}_{-b_i k}\right)$.

Claim 1. When $\alpha_i = 1$, for symmetric-independent types and strategy vector $\mathbf{s}_{-i\text{th}}$, it holds that

$$\arg\max_{k \in \mathcal{K}} \left\{ \mathbb{E}_{\gamma\boldsymbol{\theta}_{-i}} u_i\left(h_i^k, \mathbf{s}_{-i\text{th}}, \boldsymbol{\theta}\right) \right\} = \arg\max_{k \in \mathcal{K}} \left\{ h_i^k \right\}.$$

Proof: By symmetric-independent types, we have that interference channel power gains of a player j to SAP b_i, given by $g_{jb_i}^k$ are i.i.d. $\forall\ k \in \mathcal{K}$. From (6) we have that $\mathrm{q}_i^k(h_{i\text{th}}) = \mathrm{q}_i^{k'}(h_{i\text{th}})$ for $k, k' \in \mathcal{K}$. Then from (7) we have that $h_i^k \geq h_i^{k'}$ implies $\mathbb{E}_{\gamma\boldsymbol{\theta}_{-i}} u_i\left(h_i^k, \mathbf{s}_{-i\text{th}}, \boldsymbol{\theta}\right) \geq \mathbb{E}_{\gamma\boldsymbol{\theta}_{-i}} u_i\left(h_i^{k'}, \mathbf{s}_{-i\text{th}}, \boldsymbol{\theta}\right)$. Therefore, selecting the channel with best expected payoff is equivalent to selecting the channel with the highest channel power gain. ∎

Claim 1 essentially says that the sub-strategy space defined by (5) is large enough to contain a BR to itself. That is when players $\mathcal{N} \setminus \{i\}$ follow a strategy in the space defined by (5), player i can find a BR in (5) too. Consequently, when $\alpha_i = 1$, it brings down the choices of actions from set $\mathcal{A}_i$, to just 2 actions, namely $\arg\max_{k \in \mathcal{K}} \left\{ h_i^1, \ldots, h_i^K \right\}$ and $\mathtt{X}$. Without loss of generality we suppose that $k = \arg\max_{k \in \mathcal{K}} \left\{ h_i^1, \ldots, h_i^K \right\}$. Then to select the BR between $\mathtt{T}_k$ and $\mathtt{X}$, player i tests for the condition $\mathbb{E}_{\gamma\boldsymbol{\theta}_{-i}} u_i\left(h_i^k, \mathbf{s}_{-i\text{th}}, \boldsymbol{\theta}\right) \geq \mathbb{E}_{\gamma\boldsymbol{\theta}_{-i}} u_i\left(\mathtt{X}, \mathbf{s}_{-i\text{th}}, \boldsymbol{\theta}\right)$. If the condition is true then it chooses $\mathtt{T}_k$ otherwise $\mathtt{X}$. The threshold $h_{i\text{th}}$, that h_i^k must exceed in order to meet the above condition, is the solution to the following equation

$$\mathbb{E}_{\gamma\boldsymbol{\theta}_{-i}} u_i\left(h_i^k, \mathbf{s}_{-i\text{th}}, \boldsymbol{\theta}\right) = \mathbb{E}_{\gamma\boldsymbol{\theta}_{-i}} u_i\left(\mathtt{X}, \mathbf{s}_{-i\text{th}}, \boldsymbol{\theta}\right). \tag{9}$$

By (2) we observe that the right hand side of (9) is equal to ρ. The solution $h_i^k = h_{i\text{th}}$ defines the BR of player i in the set of threshold strategies defined by (5).

$$\mathbb{E}_{\gamma\boldsymbol{\theta}_{-i}} u_i\left(h_i^k, \mathbf{s}_{-i\mathrm{th}}, \boldsymbol{\theta}\right) = \bar{\mathrm{p}}_1^k\left(\mathcal{N}_{b_i} \setminus \{i\}\right) \sum_{\mathcal{N}_{-b_i k} \in \mathbb{P}\left(\mathcal{N}_{-b_i}\right)} \mathrm{p}_1^k\left(\mathcal{N}_{-b_i k}\right) \bar{\mathrm{p}}_1^k\left(\mathcal{N}'_{-b_i k}\right) \int_{\mathcal{D}} f_{\mathbf{g}} f_\gamma \log\Big(1 + \frac{h_i^k}{\sum\limits_{j \in \mathcal{N}_{-b_i k}} g_j^k + \gamma_k + \sigma^2}\Big) d\mathbf{g} d\gamma. \tag{7}$$

$$\mathbb{E}_{\gamma\boldsymbol{\theta}_{-i}} u_i\left(h_i^k, \mathbf{s}_{-i\mathrm{th}}^{\mathrm{sym}}, \boldsymbol{\theta}\right) = \bar{\mathrm{p}}_1^{\mathrm{sym}}\left(\mathcal{N}_{b_i} \setminus \{i\}\right) \mathbb{E}_X \int_{\mathcal{D}} f_{\mathbf{g}} f_\gamma \log\Big(1 + \frac{h_i^k}{\sum\limits_{j \in X} g_j^k + \gamma_k + \sigma^2}\Big) d\mathbf{g} d\gamma. \tag{8}$$

In the case of symmetric-independent types and strategy profile $\mathbf{s}_{\mathrm{th}}^{\mathrm{sym}} = (h_{\mathrm{th}})$, the event that player i transmits on k and the event that player j also transmits on k are independent and have equal probabilities given by (6). Therefore, let us define the unique probability that a player transmits on a channel by

$$\mathrm{q}_1\left(h_{\mathrm{th}}\right) = \mathrm{q}_i^k\left(h_{\mathrm{th}}\right), \forall i \in \mathcal{N}, \forall k \in \mathcal{K}. \tag{10}$$

From symmetric-independent types and strategy profile $\mathbf{s}_{\mathrm{th}}^{\mathrm{sym}} = (h_{\mathrm{th}})$, the probability that player i experiences no collisions on channel k is given by

$$\bar{\mathrm{p}}_1^{\mathrm{sym}}\left(\mathcal{N}_{b_i} \setminus \{i\}\right) = \left(1 - \mathrm{q}_1\left(h_{\mathrm{th}}\right)\right)^{N_{b_i} - 1}, \tag{11}$$

where N_{b_i} is the cardinality of $\mathcal{N}_{b_i}$.

Moreover, for symmetric-independent types and strategy profile $\mathbf{s}_{\mathrm{th}}^{\mathrm{sym}} = (h_{\mathrm{th}})$, the probability that the subset of SUEs $\mathcal{N}_{-b_i k} \subseteq \mathcal{N}_{-b_i}$ takes action $\mathtt{T}_k$ follows the binomial distribution of $N_{-b_i k}$ successes in a sequence of N_{-b_i} independent binary trials with success probability of one trial given by (10). Here $N_{-b_i k}$ and N_{-b_i} are the cardinalities of $\mathcal{N}_{-b_i k}$ and $\mathcal{N}_{-b_i}$, respectively. We denote this binomial distribution by $\mathscr{B}_1\left(N_{-b_i}, \mathrm{q}_1\left(h_{\mathrm{th}}\right)\right)$ and the probability of $N_{-b_i k}$ number of successes is denoted by $\mathrm{p}_{\mathscr{B}_1}\left(N_{-b_i k}\right)$. Due to symmetric-independent types the interference channel power gains g_j^k are i.i.d. and therefore the density $f_{\mathbf{g}}$ in (7) only depends on the cardinality of the set $\mathcal{N}_{-b_i k}$ (not on the exact SUEs in $\mathcal{N}_{-b_i k}$). Next we use these observations to simplify (7) [25]. Let us define the binomial random variable $X \sim \mathscr{B}_1\left(N_{-b_i}, \mathrm{q}_1\left(h_{\mathrm{th}}\right)\right)$, then the expected utility of player i for action $\mathtt{T}_k$ and $\mathbf{s}_{-i\mathrm{th}}^{\mathrm{sym}} = (h_{\mathrm{th}})$ is given by (8) where $\sum\limits_{j \in X} g_j^k$ is the sum of X number of i.i.d. random variables g_j^k.

In order to find the symmetric BR strategy for player i, we need to find the unique threshold $\tilde{h}_{\mathrm{th}}$ such that $h_i^k = \tilde{h}_{\mathrm{th}}$ and $\mathbf{s}_{-i\mathrm{th}}^{\mathrm{sym}} = \left(\tilde{h}_{\mathrm{th}}\right)$ solves equation (9). That is to say that

$$\mathbb{E}_{\gamma\boldsymbol{\theta}_{-i}} u_i\left(\tilde{h}_{\mathrm{th}}, \mathbf{s}_{-i\mathrm{th}}^{\mathrm{sym}}, \boldsymbol{\theta}\right) = \rho. \tag{12}$$

Since all players follow the common threshold $\mathbf{s}_{\mathrm{th}}^{\mathrm{sym}} = \left(\tilde{h}_{\mathrm{th}}\right)$ and as player i is arbitrary, this threshold defines the symmetric BR strategy for all players and by Definition 3 it is a unique BNSE.

Theorem 1. For symmetric-independent types and identically populated cells, game $\mathbf{G}_1$ has a unique threshold $h_{i\mathrm{th}} = \tilde{h}_{\mathrm{th}}$ $\forall i \in \mathcal{N}$, such that the BNSE is given by the profile $\mathbf{s}_{\mathrm{th}}^{\mathrm{sym}} = \left(\tilde{h}_{\mathrm{th}}\right)$.

Proof: See *Appendix A*. ■

At the BNSE, all players follow the threshold strategy defined by the symmetric-threshold profile $\mathbf{s}_{\mathrm{th}}^{\mathrm{sym}} = \left(\tilde{h}_{\mathrm{th}}\right)$. Furthermore, each player is able to calculate $\tilde{h}_{\mathrm{th}}$ individually.

When $\rho \leq 0$, we have $\tilde{h}_{\mathrm{th}} = 0$, hence, at all times each SUE with a symbol available, transmits over the channel on which it has the highest gain. On the other hand when, $\rho > 0$ we have $\tilde{h}_{\mathrm{th}} > 0$, therefore an SUE may not transmit, even if a symbol is available, if its maximum channel power gain is below the threshold. We draw the attention of the reader to the similarity of our mechanism to that of backoff probability in CSMA-collision detection (CD). In CSMA-CD the backoff decision is a result of a previous collision, whereas in our scheme the CSI determines the backoff probability.

V. Design of $\mathbf{G}_2$: A Game with Statistical CSIT

In the previous sections we developed the BNSE in threshold strategies for game $\mathbf{G}_1$. In $\mathbf{G}_1$ mixed-BNSEs do not exist for threshold strategies of the form (5) with probability 1. The reason being a fundamental result in game theory which states that in a mixed-strategy NE all the actions that are played with non zero probability must yield the same payoff [4]. In $\mathbf{G}_1$, two actions $\mathtt{T}_k$ and $\mathtt{T}_{k'}$, $k, k \in \mathcal{K}$ may yield the same expected utility if and only if (iff) $h_i^k = h_i^{k'}$, which has $\Pr\left(h_i^k = h_i^{k'}\right) = 0$. Similarly a player obtains equal expected utilities for $\mathtt{T}_k$ and $\mathtt{X}$ iff $h_i^k = h_{i\mathrm{th}}$, which has zero probability as well.

In this section we consider the situation where an SUE possesses statistical-CSIT. In (1) the channels $\bar{h}_i^k\ \forall\ k \in \mathcal{K}$, from SUE i to its home-SAP, are i.i.d. single tap Rayleigh. Then the statistical knowledge an SUE i must possess is the mean λ_i of the exponential power gains $h_i^k \sim \mathrm{Exp}\left(\frac{1}{\lambda_i}\right)$, $k \in \mathcal{K}$. Then the type vector of SUE i is $\boldsymbol{\omega}_i := \left(\lambda_i, \left(g_{im}^k\right)_{k \in \mathcal{K}, m \in \mathcal{M} \setminus \{b_i\}}, \alpha_i\right)$ and the type set is denoted by $\Omega_i \ni \boldsymbol{\omega}_i$. We also define the product sets $\Omega := \prod\limits_{i \in \mathcal{N}} \Omega_i$ such that $\boldsymbol{\omega} \in \Omega$ and $\Omega_{-i} := \prod\limits_{j \in \mathcal{N} \setminus \{i\}} \Omega_j$ such that $\boldsymbol{\omega}_{-i} \in \Omega_{-i}$. In the Bayesian setting λ_i is known only to player i. The other players hold a belief of λ_i that we denote by the probability density f_{λ_i}. Assuming independent types, the belief player i holds about the types of other players $\boldsymbol{\omega}_{-i}$ is given by the density function $f_{\boldsymbol{\omega}_{-i}} = \prod\limits_{\substack{k \in \mathcal{K} \\ j \in \mathcal{N} \setminus \{i\}}} \left(f_{\lambda_j} \cdot f_{g_{jm}^k}\right) \cdot \prod\limits_{j \in \mathcal{N} \setminus \{i\}} f_{\alpha_j}$. Due to the change in types and the beliefs of the players from those of $\mathbf{G}_1$ we introduce the new game $\mathbf{G}_2$ as follows.

Players	$\mathcal{N}$
Action	$\mathrm{a}_i \in \mathcal{A}_i$
Type	$\boldsymbol{\omega}_i \in \Omega_i$
Belief	$f_{\boldsymbol{\omega}_{-i}}$ over Ω_{-i}
System state	$\gamma_{km}\ \forall\ k \in \mathcal{K},\, m \in \mathcal{M}$
Payoff	$u_i(\mathbf{a}, \boldsymbol{\omega})$, $\mathbf{a} \in \mathcal{A}$ and $\boldsymbol{\omega} \in \Omega$

Game $\mathbf{G}_2$ also follows the model of symmetric-independent types that was discussed in Section *III-A*. Hence, $\lambda_i\ \forall\ i \in \mathcal{N}$ are i.i.d. Following analysis is valid for any distribution f_{λ_i} with mild conditions on the existence of finite expectation.

A. Mixed Threshold Strategies

A mixed strategy r_i of a player is a probability distribution over its pure-strategies [35]. Following the standard game-theoretic notations the mixed-strategy vector of all players except SUE i is denoted by, $\mathbf{r}_{-i} := (r_j, j \in \mathcal{N} \setminus \{i\})$, and the strategy profile of all players is denoted by $\mathbf{r} := (r_i, i \in \mathcal{N})$. This section considers pure-strategies where for $\lambda_i \geq \lambda_{i\text{th}}$ and $\alpha_i = 1$ player i transmits on channel $k \in \mathcal{K}$, where $\lambda_{i\text{th}}$ is a threshold. If $\lambda_i < \lambda_{i\text{th}}$ or $\alpha_i = 0$, does not transmit. There are K such pure strategies, one for each channel. There can be various probability distributions over the K pure strategies each of which corresponds to a mixed-strategy. Our interest is in a special subset of the feasible mixed-strategy space. We call this sub-strategy space *uniformly distributed threshold strategies* (UDTSs). It consists of strategies of the following form:

$$r_{i\text{th}}(\boldsymbol{\omega}_i) := \begin{cases} \Pr(\mathtt{T}_k) = \dfrac{1}{K}, \Pr(\mathtt{X}) = 0 & \text{if}\, \alpha_i = 1, \lambda_i \geq \lambda_{i\text{th}} \\ \Pr(\mathtt{T}_k) = 0, \Pr(\mathtt{X}) = 1 & \text{otherwise.} \end{cases} \tag{13}$$

The strategy (13) essentially means that an SUE i picks a channel uniformly at random, if $\lambda_i \geq \lambda_{i\text{th}}$ and $\alpha_i = 1$. Otherwise it does not transmit. Thus a UDTS of a player i is completely characterized by the threshold $\lambda_{i\text{th}}$. Thus in order to specify the strategy profile of the players, it is sufficient to provide the threshold vector. Let us define $\mathbf{r}_{\text{th}} := (\lambda_{i\text{th}}, i \in \mathcal{N})$ and $\mathbf{r}_{-i\text{th}} := (\lambda_{j\text{th}}, j \in \mathcal{N} \setminus \{i\})$.

By (13) and i.i.d. channels, the probability that player i transmits on channel k is

$$\mathrm{q}_i^k(\lambda_{i\text{th}}) = \frac{1}{K}\Pr(\lambda_i \geq \lambda_{i\text{th}}, \alpha_i = 1)\,. \tag{14}$$

We observe that $\mathrm{q}_i^k(\lambda_{i\text{th}})$ is decreasing in $\lambda_{i\text{th}}$. Due to independence of types, the probability that a subset of SUEs $\mathcal{N}' \subset \mathcal{N}$ takes action $\mathtt{T}_k$ is $\mathrm{p}_2^k(\mathcal{N}') := \prod_{j \in \mathcal{N}'} \mathrm{q}_j^k(\lambda_{j\text{th}})$ and the probability that none of the SUEs in $\mathcal{N}'$ takes action $\mathtt{T}_k$ is $\bar{\mathrm{p}}_2^k(\mathcal{N}') := \prod_{j \in \mathcal{N}'}\left(1 - \mathrm{q}_j^k(\lambda_{j\text{th}})\right)$. These probabilities are next used to define the expected utility. The expected utility of i when $\alpha_i = 1$ and $\mathrm{a}_i = \mathtt{T}_k$ is denoted by $\mathbb{E}_{\gamma h_i^k \boldsymbol{\omega}_{-i}} u_i(\mathtt{T}_k, \mathbf{r}_{-i\text{th}}, \boldsymbol{\omega})$ and is given by (15).

B. Best Response Strategies

The UDTSs in (13) form a strict subset in the space of all possible strategies. Hence, it is important to demonstrate that this subset is sufficient to contain a BR. In this section we demonstrate that when the set of SUEs $\mathcal{N} \setminus \{i\}$ is playing UDTSs, the SUE i can find a BR also within the UDTSs.

Claim 2. For symmetric-independent types, when $\alpha_i = 1$, the expected utility given in (15) is increasing in λ_i almost surely.

Proof: Let us consider two exponential random variables $h_i^{k1} \sim \text{Exp}(\lambda_{i1})$ and $h_i^{k2} \sim \text{Exp}(\lambda_{i2})$, such that $\lambda_{i1} < \lambda_{i2}$. By stochastic coupling [30] $h_i^{k1} < h_i^{k2}$ almost surely. Now let us consider the expected utility conditioned on h_i^k, i.e., $\mathbb{E}_{\gamma\boldsymbol{\omega}_{-i}|h_{ik}} u_i(h_i^k, \mathbf{r}_{-i\text{th}}, \boldsymbol{\omega})$ and observe from (15) that it is increasing in h_i^k. Thus $\mathbb{E}_{\gamma\boldsymbol{\omega}_{-i}|h_i^{k1}} u_i\left(h_i^{k1}, \mathbf{r}_{-i\text{th}}, \boldsymbol{\omega}\right) < \mathbb{E}_{\gamma\boldsymbol{\omega}_{-i}|h_i^{k2}} u_i\left(h_i^{k2}, \mathbf{r}_{-i\text{th}}, \boldsymbol{\omega}\right)$ holds almost surely. Moreover, from properties of expectation, the inequality is preserved when expectation is taken on both sides with respect to h_i^{k1} and h_i^{k2}. By the law of total expectation, $\mathbb{E}_{h_i^{k1}}\mathbb{E}_{\gamma\boldsymbol{\omega}_{-i}|h_i^{k1}} u_i\left(h_i^{k1}, , \mathbf{r}_{-i\text{th}}, \boldsymbol{\omega}\right) = \mathbb{E}_{\gamma h_i^k \boldsymbol{\omega}_{-i}} u_i(\mathtt{T}_k, \mathbf{r}_{-i\text{th}}, \boldsymbol{\omega})$. Therefore, (15) is increasing in λ_i almost surely. ∎

The expected utility of SUE i when $\alpha_i = 1$ and $\mathrm{a}_i = \mathtt{X}$ is $\mathbb{E}_{\gamma\boldsymbol{\omega}_{-i}} u_i(\mathtt{X}, \mathbf{r}_{-i\text{th}}, \boldsymbol{\omega}) = \rho$. Next we demonstrate the form of the BR strategy of SUE i when all other SUEs are playing UDTS $\mathbf{r}_{-i\text{th}}$.

Lemma 1. For $\mathbf{r}_{-i\text{th}}$ and $\alpha_i = 1$, a BR mixed strategy of player i, denoted by $\tilde{r}_i$, is given by

$$\tilde{r}_i = \begin{cases} \begin{cases} \Pr(\mathtt{T}_k \mid \alpha_i = 1) = p_k \\ \Pr(\mathtt{X} \mid \alpha_i = 1) = 0 \end{cases} & \text{if } \lambda_i \geq \lambda_{i\text{th}}, \\ \begin{cases} \Pr(\mathtt{T}_k \mid \alpha_i = 1) = 0 \\ \Pr(\mathtt{X} \mid \alpha_i = 1) = 1 \end{cases} & \text{if } \lambda_i < \lambda_{i\text{th}}, \end{cases}$$

where $0 \leq p_k \leq 1$ are probabilities s.t., $\sum_{k \in \mathcal{K}} p_k = 1$.

Proof: Let U be an exponential random variable with mean 1. Since (15) is increasing in λ_i as proved in Claim 2, there exists a $\lambda_{i\text{th}}$ such that for channel k

$$\mathbb{E}_{\gamma h_i^k \boldsymbol{\omega}_{-i}} u_i(\lambda_{i\text{th}} U, \mathbf{r}_{-i\text{th}}, \boldsymbol{\omega}) = \rho. \tag{17}$$

Accordingly, SUE i transmits iff $\lambda_i \geq \lambda_{i\text{th}}$. Since the exponential distribution is completely characterized by the mean, for symmetric-independent types the expected payoff for two actions $\mathrm{a}_i = \mathtt{T}_k$ and $\mathrm{a}_i = \mathtt{T}_{k'}$ $k, k' \in \mathcal{K}$, are equal. Consequently, if $\lambda_i \geq \lambda_{i\text{th}}$ at the BR, SUE i may play *any* probability distribution $(p_k, k \in \mathcal{K})$ and obtains the same expected payoff. ∎

Lemma 1 essentially says that when other SUEs play (13) the BR of SUE i is *any distribution* (not necessarily uniform) over the set of channels, provided that the mean of the channel power gain is above a threshold. Therefore, player i may as well play the UDTS $r_{i\text{th}} = \lambda_{i\text{th}}$. Thus we have demonstrated that the subset of UDTSs is sufficiently large to hold a BR to itself.

A symmetric threshold is one that is common to all players and we denote a symmetric UDTS by $\mathbf{r}_{\text{th}}^{\text{sym}} = (\lambda_{\text{th}})$.

For symmetric-independent types and UDTS profile $\mathbf{r}_{\text{th}}^{\text{sym}} = (\lambda_{\text{th}})$, the event that player i transmits on channel k and the event that player j transmits on channel k are independent and have equal probabilities and hence is denoted by

$$\mathrm{q}_2(\lambda_{\text{th}}) := \mathrm{q}_{ik}(\lambda_{\text{th}}), \forall i \in \mathcal{N}, \forall k \in \mathcal{K}.$$

$$\mathbb{E}_{\gamma h_i^k \boldsymbol{\omega}_{-i}} u_i\left(\mathtt{T}_k, \mathbf{r}_{-i\text{th}}, \boldsymbol{\omega}\right) = \bar{\mathrm{p}}_2^k\left(\mathcal{N}_{b_i} \setminus \{i\}\right) \sum_{\mathcal{N}_{-b_i k} \in \mathbb{P}\left(\mathcal{N}_{-b_i}\right)} \mathrm{p}_2^k\left(\mathcal{N}_{-b_i k}\right) \bar{\mathrm{p}}_2^k\left(\mathcal{N}'_{-b_i k}\right) \int_{\mathcal{D}} f_{\mathbf{g}} f_\gamma \mathbb{E}_{h_i^k} \log\Big(1 + \frac{h_i^k}{\sum\limits_{j \in \mathcal{N}_{-b_i k}} g_j^k + \gamma_k + \sigma^2}\Big) d\mathbf{g} d\gamma. \tag{15}$$

$$\mathbb{E}_{\gamma h_i^k \boldsymbol{\omega}_{-i}} u_i\left(\mathtt{T}_k, \mathbf{r}_{-i\text{th}}^{\text{sym}}, \boldsymbol{\omega}\right) = \bar{\mathrm{p}}_2^{\text{sym}}\left(\mathcal{N}_{b_i} \setminus \{i\}\right) \mathbb{E}_X \int_{\mathcal{D}} f_{\mathbf{g}} f_\gamma \mathbb{E}_{h_i^k} \log\Big(1 + \frac{h_i^k}{\sum\limits_{j \in X} g_j^k + \gamma_k + \sigma^2}\Big) d\mathbf{g} d\gamma. \tag{16}$$

Moreover, analogous to (11) the probability that no collision is encountered by player i on channel k is given by $\bar{\mathrm{p}}_2^{\text{sym}}\left(\mathcal{N}_{b_i k}\right) := \left(1 - \mathrm{q}_2\left(\lambda_{\text{th}}\right)\right)^{N_{b_i} - 1}$. Also consequently, the probability that the subset of SUEs $\mathcal{N}_{-b_i k} \subseteq \mathcal{N}_{-b_i}$ takes action $\mathtt{T}_k$ follows the binomial distribution $\mathscr{B}_2\left(N_{-b_i}, \mathrm{q}_2\left(\lambda_{\text{th}}\right)\right)$.

Similar to the discussion in Section IV-A, when $X \sim \mathscr{B}_2\left(N_{-b_i}, \mathrm{q}_2\left(\lambda_{\text{th}}\right)\right)$, the expected utility of SUE i, for action $\mathrm{a}_i = \mathtt{T}_k$ and $\mathbf{r}_{-i\text{th}}^{\text{sym}} = \left(\lambda_{\text{th}}\right)$ is given by (16) where $\sum\limits_{j \in X} g_j^k$ is the sum of X number of i.i.d. random variables g_j^k.

If a BNSE in UDTSs exists, then there must be a unique $\lambda_{i\text{th}} = \tilde{\lambda}_{\text{th}}\ \forall\ i \in \mathcal{N}$ that defines a mutual BR $\mathbf{r}_{\text{th}}^{\text{sym}} = \left(\tilde{\lambda}_{\text{th}}\right)$. That is to say that $\mathbf{r}_{-i\text{th}}^{\text{sym}} = \left(\tilde{\lambda}_{\text{th}}\right)$ and $\lambda_i = \tilde{\lambda}_{\text{th}}$ solves the following equation:

$$\mathbb{E}_{\gamma h_i^k, \boldsymbol{\omega}_{-i}} u_i\left(\mathtt{T}_k, \mathbf{r}_{-i\text{th}}^{\text{sym}}, \boldsymbol{\omega}\right) = \rho. \tag{18}$$

Theorem 2. For symmetric-independent types and identically populated cells, $\mathbf{G}_2$ has a unique threshold $\lambda_{i\text{th}} = \tilde{\lambda}_{\text{th}}\ \forall\ i \in \mathcal{N}$, such that the BNSE in UDTSs is given by the profile $\mathbf{r}_{\text{th}}^{\text{sym}} = \left(\tilde{\lambda}_{\text{th}}\right)$.

Proof: The method is similar to the proof of Theorem 1, therefore it is omitted. ■

Theorem 2 states that when only mean of the CSI is known, the channel access game can still achieve a BNSE in mixed threshold strategies. The equilibrium strategy of an SUE i is to pick a channel uniformly at random if and only if it possesses a symbol and its mean channel power gain, λ_i, is above the threshold $\tilde{\lambda}_{\text{th}}$. This equilibrium strategy is extremely efficient to implement. Once an SUE obtains ρ, the beliefs, and system state distributions, it is able to compute the common threshold $\tilde{\lambda}_{\text{th}}$, without interaction, by solving (18). Such a scheme can be used for distributed resource allocation in dense SC zones. Furthermore, in both $\mathbf{G}_1$ and $\mathbf{G}_2$ the network administrator may control the number of SUEs that simultaneously transmit, and thus control collisions, by manipulating the parameter ρ. The higher the ρ is, the higher the thresholds and therefore the lesser the probability that an SUE transmits.

VI. Discussions and Numerical Results

A. Non-symmetric Games

$\mathbf{G}_1$ and $\mathbf{G}_2$ are symmetric games due to the assumptions of symmetric types, symmetric MUE load over channels, and identically populated SCs. The symmetric assumptions hold for a localized overlapping set of SCs, as justified in the above development. We identified this cases as a dense SC zone as depicted in Fig. 1. However, once the SCs are no longer clustered in localized zones and instead are dispersed in a larger area, such as the home SCs in a residential area, the assumption of symmetric independent types does not hold. That situation gives rise to a non-symmetric game.

The following discussion on non-symmetric case is carried out with respect to $\mathbf{G}_1$. Two levels of asymmetry can be observed. Firstly asymmetry among UEs and secondly asymmetry among channels of a given UE. Let us first consider asymmetry only among UEs. Thus we assume that the channel power gains are i.i.d. from a given UE to a given BS, i.e., $\forall\ k \in \mathcal{K}$, $g_{jm}^k = g_{jm}$, but a UE may have nonidentical (yet independent) channel distributions to different BSs. Also we keep the assumption that the MUE load is balanced across the channels. SUEs no longer have identical types nor is the number of SUEs in SCs identically distributed. Then at the equilibrium of $\mathbf{G}_1$, from (9) there must be thresholds $\forall\ i \in \mathcal{N}$, $h_{i\text{th}} = \tilde{h}_{i\text{th}}$, which are the solution to the system of nonlinear equations:

$$\mathbb{E}_{\gamma \boldsymbol{\theta}_{-i}} u_i\left(h_{i\text{th}}, \mathbf{s}_{-i\text{th}}, \boldsymbol{\theta}\right) = \rho,\ \forall i \in \mathcal{N}, \tag{19}$$

where $\mathbf{s}_{-i\text{th}} = \left(h_{j\text{th}}, j \in \mathcal{N} \setminus \{i\}\right)$. In the symmetric case this system condensed to a single equation $\mathbb{E}_{\gamma \boldsymbol{\theta}_{-i}} u_i\left(h_{\text{th}}, \mathbf{s}_{-i\text{th}}^{\text{sym}}, \boldsymbol{\theta}\right) = \rho$. The asymmetric game has a equilibrium in threshold strategies iff the system (19) has a solution.

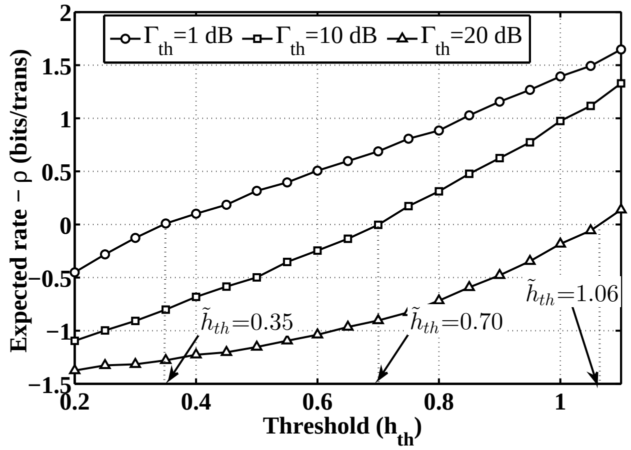
Now let us consider asymmetry among SUEs together with asymmetry among channels. Then the MUE load need not be balanced among the channels and the channel power gain from a SUE to a BS may depend on the channel index. Then for an equilibrium to exist, we seek $K \times N$ threshold values $\forall\ i \in \mathcal{N}$, $h_{i\text{th}}^k = \tilde{h}_{i\text{th}}^k$, which are the solution to the system of nonlinear equations:

$$\mathbb{E}_{\gamma \boldsymbol{\theta}_{-i}} u_i^k\left(h_{i\text{th}}^k, \mathbf{s}_{-i\text{th}}, \boldsymbol{\theta}\right) = \rho,\ \forall i \in \mathcal{N}, \forall k \in \mathcal{K}, \tag{20}$$

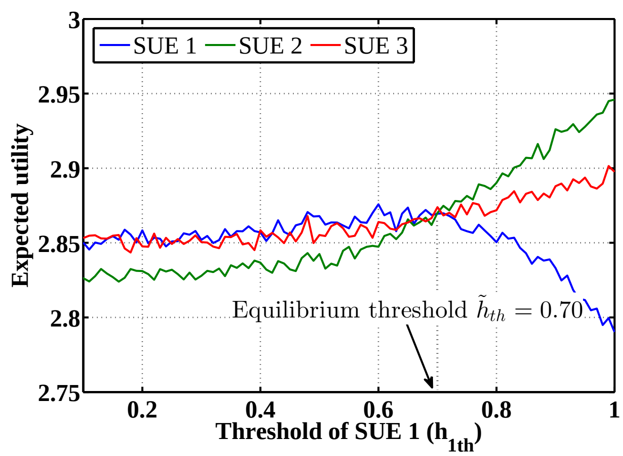
where $\mathbf{s}_{-i\text{th}} = \left(h_{j\text{th}}^k, j \in \mathcal{N} \setminus \{i\}, k \in \mathcal{K}\right)$ and $\mathbb{E}_{\gamma \boldsymbol{\theta}_{-i}} u_i^k\left(h_{i\text{th}}^k, \mathbf{s}_{-i\text{th}}, \boldsymbol{\theta}\right)$ is the expected rate over channel k. SUE i transmits on the channel with the highest $\mathbb{E}_{\gamma \boldsymbol{\theta}_{-i}} u_i^k$, when it has a channel power gain above $h_{i\text{th}}^k$, else does not transmit. A general existence result of threshold based NE for the asymmetric case requires to establish the existence of a solution to systems (19) or (20) and is not considered in this paper.

B. Identically Distributed Populations

We have mentioned that the theory developed in this paper applies to a situation where the population size of the SUEs in each SC are i.i.d. We carry out the following discuss

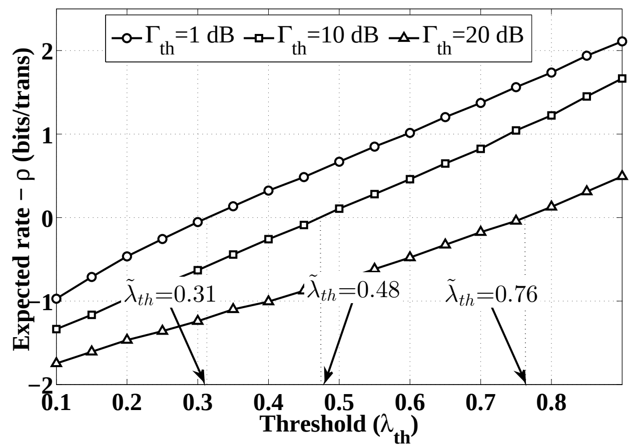


(a) Equilibrium threshold computation of pure-strategy BNSE.

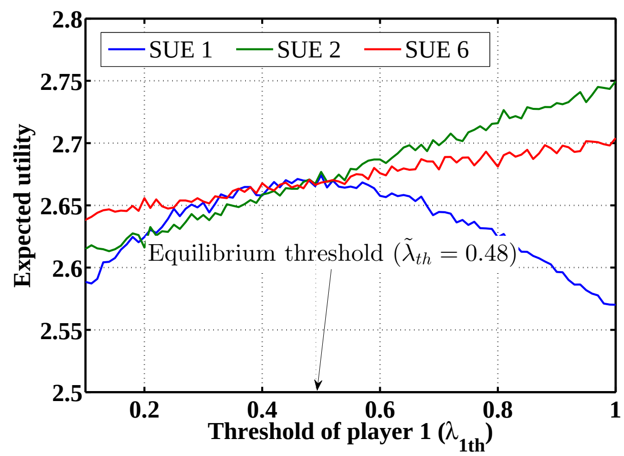


(b) The effect of SUE 1's deviation from equilibrium for $\Gamma_{\text{th}} = 10$ dB

Figure 2: The existence and uniqueness of pure-strategy BNSE of $\mathbf{G}_1$.



(a) Equilibrium threshold computation of mixed-strategy BNSE



(b) The effect of SUE 1's deviation from equilibrium for $\Gamma_{\text{th}} = 10$ dB

Figure 3: The existence and uniqueness of mixed-strategy BNSE of $\mathbf{G}_2$.

with respect to $\mathbf{G}_1$, but it applies to $\mathbf{G}_2$ as well. Suppose the cardinality of the set $\mathcal{N}_{b_i} \setminus \{i\}$, $b_i \in \mathcal{M}$, given player $i \in \mathcal{N}_{b_i}$ exists has probability $\Pr(\mathcal{N}_{b_i} \setminus \{i\} \mid \{i\})$. Let $\Pr(\mathcal{N}_{-b_i})$ denote the joint probability of the cardinality of the sets $(\mathcal{N}_{m'})_{m' \in \mathcal{M} \setminus \{b_i\}}$. Then (7) has to be averaged over conditional distribution of the population given i exists $\mathbb{E}_{\mathcal{N}|\{i\}}\mathbb{E}_{\gamma\boldsymbol{\theta}_{-i}} u_i\left(h_i^k, \mathbf{s}_{-ith}, \boldsymbol{\theta}\right) = \sum_{\mathcal{N}_{b_i} \setminus \{i\}, \mathcal{N}_{-b_i}} \Pr(\mathcal{N}_{b_i} \setminus \{i\} \mid \{i\}) \Pr(\mathcal{N}_{-b_i}) \mathbb{E}_{\gamma\boldsymbol{\theta}_{-i}} u_i(h_i^k, \mathbf{s}_{-ith}, \boldsymbol{\theta})$,where the summation is over all possible values of cardinalities of sets $\mathcal{N}_{b_i} \setminus \{i\}$ and $\mathcal{N}_{-b_i}$. Then we note that Claim 1 holds when $\mathbb{E}_{\gamma\boldsymbol{\theta}_{-i}} u_i\left(h_i^k, \mathbf{s}_{-ith}, \boldsymbol{\theta}\right)$ is replaced with $\mathbb{E}_{\mathcal{N}|\{i\}}\mathbb{E}_{\gamma\boldsymbol{\theta}_{-i}} u_i\left(h_i^k, \mathbf{s}_{-ith}, \boldsymbol{\theta}\right)$. Then from the i.i.d. assumption of population size among SCs, it is verified that all other results follow and we find the BNSE in threshold strategies by solving $\mathbb{E}_{\mathcal{N}|\{i\}}\mathbb{E}_{\gamma\boldsymbol{\theta}_{-i}} u_i\left(h_{\text{th}}, \mathbf{s}_{-ith}^{\text{sym}}, \boldsymbol{\theta}\right) = \rho$.

C. Numerical Results

Let us consider a scenario where, in a hotel lobby, 4 service providers have deployed 1 SAP each. This scenario is depicted in Fig. 1. There are $K = 8$ channels in the uplink and it is noted when the number changes. The rest of the variables are as follows: $\Pr(\alpha_i = 1) = 0.9$, SINR threshold $\Gamma_{\text{th}} = 20$ dB, distribution of MUE interference γ is $f_\gamma = \delta(\gamma - 0.001)$, where $\delta(\cdot)$ is the impulse function, channel power gains h_i^k and g_{im}^k follow exponential distribution with means $\mathbb{E}\left(h_i^k\right) = 0.5$, $\mathbb{E}\left(g_{im}^k\right) = 0.05$ respectively $\forall\ i \in \mathcal{N}$, $\forall\ k \in \mathcal{K}$, and $m \in \mathcal{M}$, and parameters $\rho = 2$ bits/s/Hz and noise power $\sigma^2 = 10^{-6}$. Recall that the symmetric model assumed i.i.d. number of SUEs among service providers. For simplicity the simulation considers 5 SUEs for each service provider with probability one. The SUEs 1 to 5 belong to SC_1 and SUEs 6 to 10 belong to SC_2 and so on and so forth.

Fig. 2a depicts (expected rate -ρ) vs. symmetric threshold h_{th} for SUE 1. The expected rate of an SUE in $\mathbf{G}_1$ is given by (8). The value of h_{th} for which the expected rate is equal to ρ, is the solution of (12) and defines the unique equilibrium $\mathbf{s}_{\text{th}}^{\text{sym}} = \left(\tilde{h}_{\text{th}}\right)$ of $\mathbf{G}_1$ according to Theorem 1. Three SINR thresholds $\Gamma_{\text{th}} \in \{5, 10, 20\}$ dB are considered. As Γ_{th} increases, the expected rate (8) decreases since the probability of violation of SINR threshold increases and hence the channel power gain required to achieve a rate of ρ increases leading to a higher equilibrium threshold.

Fig. 2b demonstrates that the root obtained in the Fig 2a is indeed the equilibrium point. To this end, we let SUE 1 deviate from the symmetric equilibrium threshold while all other SUEs follow the symmetric equilibrium threshold strategy $\mathbf{s}_{-ith}^{\mathrm{sym}} = \left(\tilde{h}_{\mathrm{th}}\right)$. As can be seen, SUE 1 is unable to achieve strictly better performance by unilateral deviation i.e., $h_{1\mathrm{th}} \gtrless \tilde{h}_{\mathrm{th}} = 0.7$.

Next let us consider $\mathbf{G}_2$. The simulation setup assumes that λ_i is uniformly distributed in the interval $(0, 2)$. The rest of the simulation parameters are kept the same as in the previous section. The expected rate of an SUE in $\mathbf{G}_2$ is given by (16). Fig. 3a depicts the (expected rate$-\rho$) vs. symmetric threshold λ_{th} for SUE 1. It demonstrates the existence and uniqueness of solution to (18) which defines the symmetric threshold $\tilde{\lambda}_{\mathrm{th}}$ of the mixed BNSE. Observe that as Γ_{th} increases, the mean channel power gain required to achieve an expected rate of ρ increases according to (18) resulting in a higher $\tilde{\lambda}_{\mathrm{th}}$ at equilibrium.

Fig. 3b demonstrates the payoffs of SUEs as SUE 1 deviates from the equilibrium threshold value while all other SUEs follow the equilibrium strategy $\mathbf{r}_{-ith}^{\mathrm{sym}} = \left(\tilde{\lambda}_{\mathrm{th}}\right)$. For clarity utilities of only three SUEs are depicted. As expected, SUE 1 is unable to achieve strictly better performance by unilateral deviation, therefore $\mathbf{r}_{\mathrm{th}}^{\mathrm{sym}} = \left(\tilde{\lambda}_{\mathrm{th}}\right)$ defines the NE.

The above results consider a fixed number of SUEs per cell. Next we consider the effect of the distribution of the number of users. Let us construct the distribution of cardinality as follows. Let the maximum number of players in SC m be $N_m > 0$ and let each player i have an independent probability of existence p_{exist}. Then the cardinality of $\mathcal{N}_{b_i} \setminus \{i\}$ has the binomial distribution with $N_{b_i} - 1$ trials and p_{exist} success probability. Similarly the cardinality of $\mathcal{N}_{-b_i}$ has binomial distribution with parameters N_{-b_i} and p_{exist}. In (8) there are two values that depend on the distribution of cardinalities $\mathcal{N}_{b_i} \setminus \{i\}$ and $\mathcal{N}_{-b_i}$. The probability of no collision $\bar{\mathrm{p}}_1^{\mathrm{sym}}\left(\mathcal{N}_{b_i k}\right)$ and the feasible SINR region $\mathcal{D}$, both of which decrease as the number of SUEs in a cell increases. Thus as p_{exist} grows the binomial distributions $\left(N_{b_i} - 1, p_{\mathrm{exist}}\right)$ and $\left(N_{-b_i}, p_{\mathrm{exist}}\right)$, increase the probabilities given to higher cardinalities and thus the expected utility $\mathbb{E}_{\mathcal{N}|\{i\}}\mathbb{E}_{\gamma\boldsymbol{\theta}_{-i}} u_i\left(h_{\mathrm{th}}, \mathbf{s}_{-ith}^{\mathrm{sym}}, \boldsymbol{\theta}\right)$, decreases and therefore the equilibrium threshold h_{th} increases. Fig. 4 shows this phenomenon where as p_{exist} tends to 1 the threshold approached 0.70 which is the value obtained when all players exists with probability 1 for $\Gamma_{\mathrm{th}} = 10$ dB in Fig. 2a.

The configurable network parameter available to the administrator is ρ. It is proven under Theorem 1 that as ρ is increased the collisions in the network must reduce. Fig. 5 demonstrates this fact in the simulation environment. To emphasize on number of collisions, a reduced channel number of $K = 4$ is used in this simulation.

D. Fairness and Benchmark

As a result of the symmetric-independent types, all SUEs achieve equal expected utility in both games $\mathbf{G}_1$ and $\mathbf{G}_2$ at the equilibrium. Hence, fairness among the SUEs is ensured by both Games. This can be observed in Fig. 2b and Fig. 3b. When the thresholds of all SUEs coincide at the equilibrium threshold, they all achieve equal expected utility.

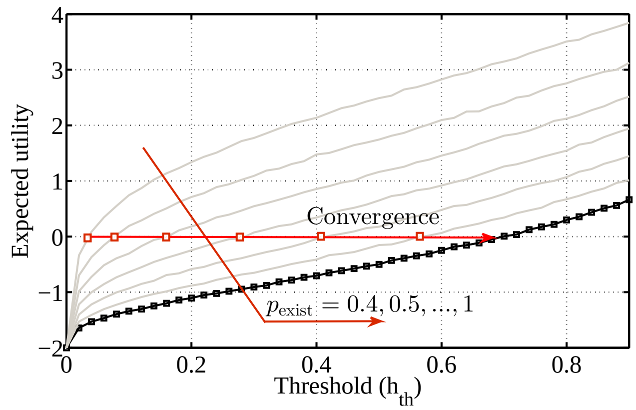


Figure 4: Behavior of threshold as p_{exist} increases for $\Gamma_{\mathrm{th}} = 10$ dB.

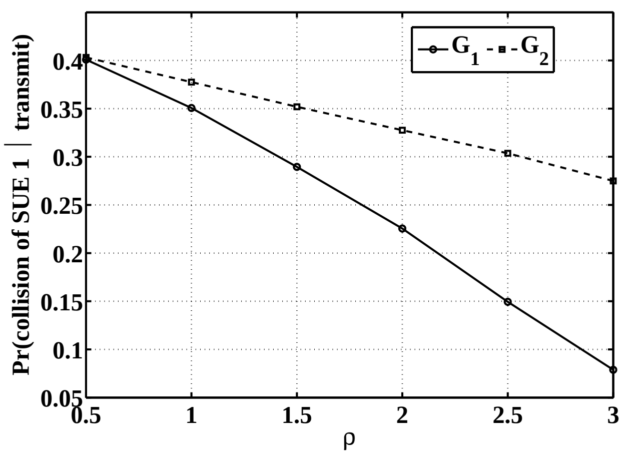


Figure 5: Empirical probability of a collision experienced by SUE 1, given it transmits

In order to compare the system throughput, this paper implements the benchmark decentralized scheduling scheme where each SAP schedules its SUE who has a symbol and has the highest channel power gain. Thus this scheme requires a message to be sent to the selected SUE of each SC in each time slot. Then there are no intracell collisions but only intercell interference. When the scheduled SUEs of the SCs satisfy the SINR threshold Γ_{th}, they achieve the rates given by (3). While this benchmark scheme is not globally optimal, it is locally optimal at each SAP to maximize its uplink throughput, when there is no intercell CSI exchange. We present the results for CSIT case of the benchmark and the related CSIT game $\mathbf{G}_1$ in Fig. 6. The rate distribution of $\mathbf{G}_1$ performs close to the benchmark. Therefore for a dense SC zone as in Fig. 1, employing the proposed game models, rather than the benchmark is reasonable, as the proposed games have the added advantage of being fully distributed, once the parameter ρ has been broadcast to the SUEs.

VII. Conclusion

This paper analyzed the distributed uplink channel access problem of a cluster of dense underlay SCs. The analysis

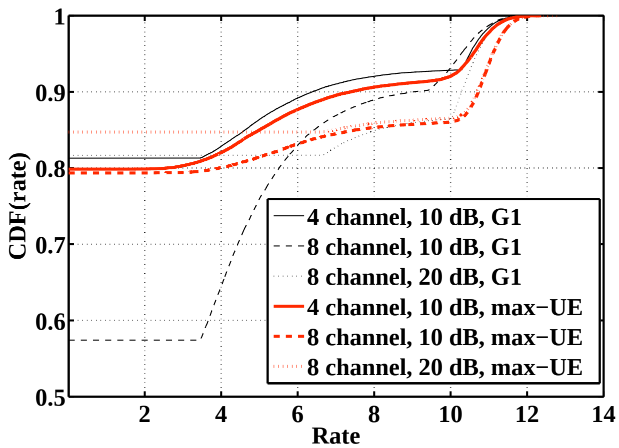


Figure 6: Empirical CDF for rate (bits/trans) comparison

was carried out using the theory of Bayesian games. The system model was chosen to be sufficiently general and it includes multiple cells and channels, intercell interference, intracell collisions and random symbol availability, which are important parameters in modeling picocells, femtocells, and wireless local area networks. Two CSI availability models are used resulting in two games. The first game, $\mathbf{G}_1$ assumes CSIT and we solve it for pure-strategy symmetric equilibrium. At the equilibrium each SUE transmits on the highest gain channel if that gain is above a threshold. The second game, $\mathbf{G}_2$, only assumes statistical CSIT. $\mathbf{G}_2$ is proved to possess an interesting symmetric mixed-strategy equilibrium where an SUE uniformly distributes channel access if mean channel gains is above a threshold. The two pure- and mixed-strategy equilibria, are particularly interesting for distributed systems as at the equilibrium, the best response strategy is defined by a single threshold parameter and both equilibria can be achieved without interaction among the SUEs. The key extension that remains is to explore nonsymmetric equilibria.

Appendix A
Proof of Theorem 1

Proof: Consider the strategy profile $\mathbf{s}_{\text{th}}^{\text{sym}} = (h_{\text{th}})$. Define the random variable $X(h_{\text{th}}) \sim \mathscr{B}_1(N_{-b_i}, \mathrm{q}_1(h_{\text{th}}))$ and let $z(X(h_{\text{th}}), h_{\text{th}}) = \int_{\mathcal{D}} f_{\mathbf{g}} f_{\gamma} \log(1 + \dfrac{h_{\text{th}}}{\sum_{j \in X(h_{\text{th}})} g_j^k + \gamma_k + \sigma^2} d\mathbf{g}\gamma)$. Then the expected payoff in (8) is restated as $\mathbb{E}_{\boldsymbol{\theta}_{-i}} u_i\left(\tilde{h}_{\text{th}}, \mathbf{s}_{-i\text{th}}^{\text{sym}}, \boldsymbol{\theta}\right) = \bar{\mathrm{p}}_1^{\text{sym}}(\mathcal{N}_{b_i} \setminus \{i\}) \mathbb{E}_{X(h_{\text{th}})} z(X(h_{\text{th}}), h_{\text{th}})$. Note that $z(X(h_{\text{th}}), h_{\text{th}})$ is increasing in h_{th} (as the $\log(\cdot)$ and integration region $\mathcal{D}$ both grows with h_{th}). Also observe that $z(X(h_{\text{th}}), h_{\text{th}})$ is decreasing in $X(h_{\text{th}})$ (the number of interfering SUEs grows as $X(h_{\text{th}})$ increases). We can also observe by (6) and (10) that $\mathrm{q}_1(h_{\text{th}})$ is decreasing in h_{th}. Therefore, if $h_{\text{th}}^1 < h_{\text{th}}^2$, then $\mathrm{q}_1(h_{\text{th}}^2) < \mathrm{q}_1(h_{\text{th}}^1)$. Through stochastic coupling theory [30] $X(h_{\text{th}}^2) < X(h_{\text{th}}^1)$ a.s. Therefore, $z(X(h_{\text{th}}^1), h_{\text{th}}^1) < z(X(h_{\text{th}}^2), h_{\text{th}}^2)$ a.s. Taking expectations gives $\mathbb{E}_{X(h_{\text{th}}^1)} z(X(h_{\text{th}}^1), h_{\text{th}}^1) < \mathbb{E}_{X(h_{\text{th}}^2)} z(X(h_{\text{th}}^2), h_{\text{th}}^2)$ a.s. From (11) and (6) we observe that the probability of no collision $\bar{\mathrm{p}}_1^{\text{sym}}(\mathcal{N}_{b_i k})$ is also increasing in h_{th}. Consequently $\mathbb{E}_{\gamma\boldsymbol{\theta}_{-i}} u_i\left(\tilde{h}_{\text{th}}, \mathbf{s}_{-i\text{th}}^{\text{sym}}, \boldsymbol{\theta}\right)$ is increasing in h_{th}. Hence there exists unique $\tilde{h}_{\text{th}}$ such that $\mathbb{E}_{\gamma\boldsymbol{\theta}_{-i}} u_i\left(\tilde{h}_{\text{th}}, \mathbf{s}_{-i\text{th}}^{\text{sym}}, \boldsymbol{\theta}\right) = \rho$. ■

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