UQAM-PHE-94/08

FLAVOUR CHANGING TOP QUARK DECAY WITHIN THE MINIMAL SUPERSYMMETRIC STANDARD MODEL

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ABSTRACT

We present the results of the gluino and scalar quarks contribution to the flavour changing top quark decay into a charm quark and a photon, gluon or a Z^0 boson within the minimal supersymmetric standard model. We include the mixing of the scalar partners of the left and right handed top quark. This mixing has several effects, the most important of which are to greatly enhance the c Z decay mode for large values of the soft SUSY breaking scalar mass m_S and to give rise to a GIM–like suppresion in the c γ mode for certain combinations of parameters.

September 1994

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Recent experimental evidence of the top quark [1] makes its rare decay modes a promising test ground for the standard model (SM) and physics beyond the SM. The flavour–changing decay mode of the top quark was calculated within the SM in [2-5] and shown to be far away from experimental reach; this makes it an excellant probe for models beyond the SM. Two-Higgs-doublet models (THDM)were considered in [6,7], where it was shown that the decay rate is enhanced by several (3–4) orders of magnitude. Recently [8], the $t \to cV$ decay was considered within the minimal supersymmetric SM (MSSM) and the authors obtained the same enhancement as in the THDM's. However they did not include the mixing of the scalar partners of the left and right handed top quark, they omitted one diagram in the $c\ g$ decay mode and their current was not gauge invariant.

In this paper we present the QCD loop corrections to the $t \to cV$ decay in the MSSM with gluinos and scalar quarks running on the loop, as shown in fig.1. Throughout the calculation we neglect all quark masses besides the top quark mass and include the mixing of the scalar partners of the left and right handed top quark, which is proportional to the top quark mass.

In supersymmetric QCD it was shown that there occur flavour changing strong interactions between the gluino, the left handed quarks and their supersymmetric scalar partners, whereas the couplings of the gluino to the right handed quarks and their partners remains flavour diagonal [10-16]. Since the mixing of $\tilde{t}_L$ and $\tilde{t}_R$ is proportional to the top quark mass we have to include the full scalar top quark matrix which is given by [9]:

$$M_{\tilde{t}}^2 = \begin{pmatrix} m_{\tilde{t}_L}^2 + m_{\rm top}^2 + 0.35 D_Z^2 & -m_{\rm top}(A_{\rm top} + \mu \cot\beta) \\ -m_{\rm top}(A_{\rm top} + \mu \cot\beta) & m_{\tilde{t}_R}^2 + m_{\rm top}^2 + 0.16 D_Z^2 \end{pmatrix} \tag{1}$$

where $D_Z^2 = m_Z^2 \cos 2\beta$, $m_{\tilde{t}_{L,R}}^2$ are soft SUSY breaking masses, $A_{\rm top}$ is a trilinear scalar interaction parameter and μ is the supersymmetric mass mixing term of the Higgs bosons. The mass eigenstates $\tilde{t}_1$ and $\tilde{t}_2$ are related to the current eigenstates $\tilde{t}_L$ and $\tilde{t}_R$ by:

$$\tilde{t}_1 = cos\Theta_t \tilde{t}_L + \sin\Theta_t \tilde{t}_R \qquad \tilde{t}_2 = -\sin\Theta_t \tilde{t}_L + \cos\Theta_t \tilde{t}_R \tag{2}$$

In the following we take $m_{\tilde{t}_L} = m_{\tilde{t}_R} = m_S = A_{\rm top}$ (global SUSY). The gluino mass $m_{\tilde{g}}$ is a free parameter, which in general is supposed to be larger than 100 GeV, although there is still the possibility of a small gluino mass window in the order of 1 GeV [17,18].

To calculate the 1 loop diagrams shown in fig.1 we need the couplings of the gluon to the gluinos, of the scalar partners of the left handed quarks to the gluon, photon and Z^0 boson and of the gluino to the left handed quark and its scalar partner. The first one* is given by Eq.C92 in [19]:

$$\mathcal{L}_{g\tilde{g}\tilde{g}} = \frac{i}{2} g_s f_{abc} \overline{\tilde{g}_a} \gamma_\mu \tilde{g}_b G_c^\mu \qquad (3),$$

* In order to shorten the notation we will use $cos\Theta = c_\Theta$, $sin\Theta = s_\Theta$, and $s_W = sin\Theta_W$ where Θ_W is the weak mixing angle.

which is multiplied by 2 to obtain the Feynman rules. The interactions of the gluon, photon and the Z^0 boson with squark are given by Eq.6–8 in [9]:

$$\mathcal{L}_{\tilde{q}\tilde{q}V} = - ieA^\mu \sum_{i=L,R} e_{q_i} \tilde{q}_i^* \overleftrightarrow{\partial}_\mu \tilde{q}_i - \frac{ie}{s_W \ c_W} Z^\mu \sum_{i=L,R} \Big(T_{3q_i} - e_{q_i} s_W \Big) \tilde{q}_i^* \overleftrightarrow{\partial}_\mu \tilde{q}_i \qquad (4)$$
$$- ig_s T^a G^{a\mu} \sum_{i=L,R} \tilde{q}_i^* \overleftrightarrow{\partial}_\mu \tilde{q}_i$$

After the introduction of nontrivial squark mixing this becomes

$$\mathcal{L}_{\tilde{q}\tilde{q}V} = - ieA^\mu \sum_{i=1,2} e_{q_i} \tilde{q}_i^* \overleftrightarrow{\partial}_\mu \tilde{q}_i - ig_s T^a G^{a\mu} \sum_{i=1,2} \tilde{q}_i^* \overleftrightarrow{\partial}_\mu \tilde{q}_i \qquad (5)$$
$$- \frac{ie}{s_W c_W} Z^\mu [(T_{3L} c_\Theta^2 - e_q s_W^2) \tilde{q}_1^* \overleftrightarrow{\partial}_\mu \tilde{q}_1 + (T_{3L} s_\Theta^2 - e_q s_W^2) \tilde{q}_2^* \overleftrightarrow{\partial}_\mu \tilde{q}_2$$
$$- T_{3L} c_\Theta s_\Theta (\tilde{q}_1^* \overleftrightarrow{\partial}_\mu \tilde{q}_2 + \tilde{q}_2^* \overleftrightarrow{\partial}_\mu \tilde{q}_1)]$$

Finally the coupling that leads to flavour changing is given by Eq.1 in [16]:

$$\mathcal{L}_{\mathcal{FC}} = -\sqrt{2} g_s T^a K \overline{\tilde{g}} P_L q (c_\Theta \tilde{q}_1 - s_\Theta \tilde{q}_2) + h.c. \qquad (6)$$

Here K is the supersymmetric version of the Kobayashi–Maskawa matrix whose form will appear later. Flavour changing couplings occur only in the left handed scalar quark sector; the right handed sector does not contribute to our process.

After summation over all diagrams, we obtain the following effective tcV vertex:

$$M_{\mu V}^\alpha = - i \frac{\alpha_s}{2\pi} K_{\alpha t} K_{\alpha c} \overline{u}_{p_2} [\gamma_\mu P_L V_V^\alpha + \frac{P_\mu}{m_{\text{top}}} P_R T_V^\alpha] u_{p_1} \qquad (7)$$
$$V_\gamma^\alpha = e e_q C_2(F) [c_{\Theta_\alpha}^2 (C_\epsilon^{11\alpha} - C_{SE}^{1\alpha}) + s_{\Theta_\alpha}^2 (C_\epsilon^{22\alpha} - C_{SE}^{2\alpha})]$$
$$T_\gamma^\alpha = e e_q C_2(F) [c_{\Theta_\alpha}^2 C_{\text{top}}^{11\alpha} + s_{\Theta_\alpha}^2 C_{\text{top}}^{22\alpha}]$$
$$V_g^\alpha = g_s T^a \{ [-\frac{1}{2} C_2(G) + C_2(F)] [c_{\Theta_\alpha}^2 C_\epsilon^{11\alpha} + s_{\Theta_\alpha}^2 C_\epsilon^{22\alpha}] - C_2(F) [c_{\Theta_\alpha}^2 C_{SE}^{1\alpha} + s_{\Theta_\alpha}^2 C_{SE}^{2\alpha}]$$
$$+ \frac{1}{2} C_2(G) \{ c_{\Theta_\alpha}^2 [C_\epsilon^{\tilde{g}1\alpha} + C_{\tilde{g}}^{1\alpha} + C_{q^2}^{1\alpha} + C_t^{1\alpha}] + s_{\Theta_\alpha}^2 [C_\epsilon^{\tilde{g}2\alpha} + C_{\tilde{g}}^{2\alpha} + C_{q^2}^{2\alpha} + C_t^{2\alpha}] \} \}$$
$$T_g^\alpha = g_s T^a \{ [-\frac{1}{2} C_2(G) + C_2(F)] [c_{\Theta_\alpha}^2 C_{\text{top}}^{11\alpha} + s_{\Theta_\alpha}^2 C_{\text{top}}^{22\alpha}] - \frac{1}{2} C_2(G) [c_{\Theta_\alpha}^2 C_t^{1\alpha} + s_{\Theta_\alpha}^2 C_t^{2\alpha}] \}$$
$$V_Z^\alpha = \frac{e}{s_W c_W} C_2(F) \{ (T_{3L} c_{\Theta_\alpha}^2 - e_q s_W^2) c_{\Theta_\alpha}^2 C_\epsilon^{11\alpha} + (T_{3L} s_{\Theta_\alpha}^2 - e_q s_W^2) s_{\Theta_\alpha}^2 C_\epsilon^{22\alpha}$$
$$+ T_{3L} c_{\Theta_\alpha}^2 s_{\Theta_\alpha}^2 (C_\epsilon^{12\alpha} + C_\epsilon^{21\alpha}) - (T_{3L} - e_q s_W^2) [c_{\Theta_\alpha}^2 C_{SE}^{1\alpha} + s_{\Theta_\alpha}^2 C_{SE}^{2\alpha}] \}$$
$$T_Z^\alpha = \frac{e}{s_W c_W} C_2(F) \{ (T_{3L} c_{\Theta_\alpha}^2 - e_q s_W^2) c_{\Theta_\alpha 2} C_{\text{top}}^{11\alpha} + (T_{3L} s_{\Theta_\alpha}^2 - e_q s_W^2) s_{\Theta_\alpha}^2 C_{\text{top}}^{22\alpha}$$
$$+ T_{3L} c_{\Theta_\alpha}^2 s_{\Theta_\alpha}^2 (C_{\text{top}}^{12\alpha} + C_{\text{top}}^{21\alpha}) \}$$

$$C_\epsilon^{kl\alpha} = \int_0^1 d\alpha_1 \int_0^{1-\alpha_1} d\alpha_2 [\frac{1}{\epsilon} - \gamma + \ln(4\pi\mu^2) - \ln(f_{kl}^\alpha)]$$

$$C_{\rm top}^{kl\alpha} = \int_0^1 d\alpha_1 \int_0^{1-\alpha_1} d\alpha_2 \frac{m_{\rm top}^2 \alpha_1 (1-\alpha_1-\alpha_2)}{f_{kl}^\alpha}$$

$$C_{SE}^{k\alpha} = \int_0^1 d\alpha_1 \alpha_1 [\frac{1}{\epsilon} - \gamma + \ln(4\pi\mu^2) - \ln(g_k^\alpha)]$$

$$C_\epsilon^{\tilde{g}k\alpha} = \int_0^1 d\alpha_1 \int_0^{1-\alpha_1} d\alpha_2 [\frac{1}{\epsilon} - \gamma + 1 + \ln(4\pi\mu^2) - \ln(h_k^\alpha)]$$

$$C_{\tilde{g}}^{k\alpha} = \int_0^1 d\alpha_1 \int_0^{1-\alpha_1} d\alpha_2 \frac{m_{\tilde{g}}^2}{h_k^\alpha}$$

$$C_{q^2}^{k\alpha} = \int_0^1 d\alpha_1 \int_0^{1-\alpha_1} d\alpha_2 \frac{q^2 \alpha_1 \alpha_2}{h_k^\alpha}$$

$$C_t^{k\alpha} = \int_0^1 d\alpha_1 \int_0^{1-\alpha_1} d\alpha_2 \frac{m_{\rm top}^2 \alpha_1 (1-\alpha_1-\alpha_2)}{h_k^\alpha}$$

$$f_{kl}^\alpha = m_{\tilde{g}}^2 - (m_{\tilde{g}}^2 - m_{\tilde{q}_k^\alpha}^2)\alpha_1 - (m_{\tilde{g}}^2 - m_{\tilde{q}_l^\alpha}^2)\alpha_2 - m_{\rm top}^2 \alpha_1(1-\alpha_1-\alpha_2) - q^2\alpha_1\alpha_2$$

$$g_k^\alpha = m_{\tilde{g}}^2 - (m_{\tilde{g}}^2 - m_{\tilde{q}_k^\alpha}^2)\alpha_1 - m_{\rm top}^2 \alpha_1(1-\alpha_1)$$

$$h_k^\alpha = m_{\tilde{q}_k^\alpha}^2 - (m_{\tilde{q}_k^\alpha}^2 - m_{\tilde{g}}^2)(\alpha_1+\alpha_2) - m_{\rm top}^2 \alpha_1(1-\alpha_1-\alpha_2) - q^2\alpha_1\alpha_2$$

where $\epsilon = 2 - d/2$, $C_2(F) = 4/3$ and $C_2(G) = 3$ for SU(3). If $\alpha \neq$ top we have $c_{\Theta_\alpha} = 1$. Using the spin-1 condition ($q_\mu = (p_1 - p_2)_\mu = 0$) we can write $P_\mu = (p_1 + p_2)_\mu = 2p_{1\mu}$. $K_{\alpha q}$ is the SUSY–Kobayashi–Maskawa matrix whose form is as follows:

$$K_{ij} = \begin{pmatrix} 1 & \varepsilon & \varepsilon^2 \\ -\varepsilon & 1 & \varepsilon \\ -\varepsilon^2 & -\varepsilon & 1 \end{pmatrix} \quad (8)$$

Here ε is a small number (not to be confused with the ϵ above) to be taken as $\varepsilon^2 = 1/4$ [16,8]. It is straightforward at this point to verify that all divergent terms cancel exactly, without the GIM mechanism.

A crucial test is also provided by the nature of the current. Using the following identity:

$$\overline{u}_{p_2} \frac{P^\mu}{m_{\rm top}} P_R u_{p_1} \equiv \overline{u}_{p_2} [\gamma_\mu P_L + i\sigma_{\mu\nu} \frac{q^\mu}{m_{\rm top}} P_R] u_{p_1} \quad (9)$$

we can show that the quantity in front of the γ^μ term vanishes in the limit $q^2 \to 0$, as required by gauge invariance.

When summing over all scalar quarks within the loops the scalar up quark cancels out because of the unitarity of K_{ij} and with $K_{23} = -K_{32}$ the mass splitting of the scalar top quark and the scalar charm quark comes into account, which was taken to be $m_{\tilde{c}} = 0.9\ m_{\tilde{t}}$ in [8] and therefore too small for a top quark mass of 174 GeV. If all scalar quark masses would be the same the decay rate of $t \to cV$ would be identical to 0. As a final result we obtain:

$$\Gamma_S(t \to cV) = \frac{\alpha_s^2}{128\pi^3}\ m_{\rm top}\left(1 - \frac{m_V^2}{m_{\rm top}^2}\right)^2 \varepsilon^2 \left[V_V^2\left(2 + \frac{m_{\rm top}^2}{m_V^2}\right) - 2V_V T_V\left(1 - \frac{m_{\rm top}^2}{m_V^2}\right) - T_V^2\left(2 - \frac{m_V^2}{m_{\rm top}^2} - \frac{m_{\rm top}^2}{m_V^2}\right)\right] \quad (11)$$

where $V_V = V_V^{\tilde{t}} - V_V^{\tilde{c}}$ and $T_V = T_V^{\tilde{t}} - T_V^{\tilde{c}}$. For $V = \gamma, g$ we have $V_V = -T_V$ and all terms containing m_V^2 are absent.
We define [6]: $B(t \to cV) = \Gamma_S(t \to cV)/\Gamma_W(t \to bW^+)$ where

$$\Gamma_W(t \to bW^+) = \frac{\alpha}{16\sin^2\Theta_W} m_{\rm top}\left(1 - \frac{m_{W+}^2}{m_{\rm top}^2}\right)^2\left(2 + \frac{m_{\rm top}^2}{m_{W+}^2}\right) \quad (12)$$

Our input parameters are $m_{\rm top} = 174$ GeV and the strong coupling constant $\alpha_s = 1.4675/\ln(\frac{m_{\rm top}^2}{\Lambda_{\rm QCD}^2}) = 0.107$ with $\Lambda_{\rm QCD} = 0.18$ GeV [6].

In fig. 2 we present the branching ratio $B(t \to cZ)$ as a function of the scalar mass m_S for a gluino mass of 100 GeV. We see that without mixing, the branching ratio decreases rapidly with increasing scalar mass. The mixing has a drastic effect. It enhances the branching ratio by up to 5 orders of magnitude for large m_s. Higher values of $\tan\beta$ diminish the branching ratio. The gluino mass hardly affects the decay rate. Even for a small gluino mass of the order of 1 GeV the branching ratio remains of the same order.

In fig. 3 we consider the same cases as in fig. 2 but for $B(t \to cg)$. The effect of the mixing is not as drastic as in the previous case. It decreases the branching ratio generally by 1–2 orders of magnitude. This reduction is larger for larger scalar masses. Increasing $\tan\beta$ diminishes the branching ratio in general, an exception is the case $\mu = 100$ GeV and $m_{\tilde{g}} = 500$ GeV. Increasing the gluino mass diminishes the branching ratio by several orders of magnitude for lower values of the scalar mass whereas lower values of the gluino mass enhances the ratio. The shape of the figures remains the same.

In figs. 4 and 5 we consider the branching ratio $B(t \to c\gamma)$ We notice first that the effect of the mixing is rather small for small values of m_s. We also note that the sensitivity of the branching ratio to $\tan\beta$ is greatly increased. Thirdly, one sees that the mixing generally reduces the branching ratio. This is true generally but might not hold for some regions of parameter space, as can be seen on fig. 4, when some combinations of parameters can greatly increase the branching ratio. Most interesting, the mixing gives rise to a GIM-like suppression where the contribution of the top quark exactly cancels the contribution from the c-quark. This dramatic cancellation is also seen on fig. 4. Such a cancellation is not *isolated* as seen on fig. 5*. We have tried many different combinations of μ and $m_{\tilde{g}}$ and we found a *rift* similar to the one visible on fig. 5 with all the combinations. Such a cancellation does not occur for the gluon and Z decay modes. In the first case, the $g - \tilde{g} - \tilde{g}$ vertex spoils it while in the second case it seems to be the $q^2 \neq 0$ that does it. In this paper we presented the supersymmetric QCD 1 loop correction to the flavour changing decay rate $t \to cV$. We have shown that the $t \to cZ$ decay rate is enhanced by several orders of magnitude compared to the standard model. If we include the mixing of the scalar partners of the top quark we do get a further enhancement and the decay rate remains relatively large for a very wide range of gluino and scalar masses. For the $t \to cg$ decay rate we have shown that the mixing reduces generally the branching ratio. Larger values for $\tan\beta$ also diminish the branching ratio. In the $t \to c\gamma$ decay mode, the most dramatic effect of this mixing is to give rise to a GIM-like cancellation for some combinations of parameters. It also reduces the branching ratio and greatly increases the sensitivity to $\tan\beta$.

One of us (H.K.) would like to thank the physics department of Carleton university and Université de Montréal for the use of their computer facilities as well as M. Boyce for computerial advices. The figures were done with the very user friendly program PlotData from TRIUMF. This work was partially funded by N.S.E.R.C. of Canada and Les Fonds F.C.A.R. du Québec.

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* This figure is intended to give a very good idea of the global behaviour but not to be read numerically.

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FIGURE CAPTIONS

Fig.1 The diagrams with scalar quarks and gluinos within the loop, which contribute to the top quark decay into a charm quark and a Z boson, photon or gluon.

Fig.2 The ratio Γ_S/Γ_W of the the top quark decay into a charm quark and Z^0 boson as a function of the scalar mass m_S. The gluino mass was taken to be 100 GeV. The solid line is the unphysical case with no mixing ($\mu = 0 = A_{\rm top}$) and $\tan\beta = 1$, the dotted line the same case with $\tan\beta = 10$. The other cases are with mixing ($A_{\rm top} = m_S$). The dashed lines are with $\mu = 100$ GeV and the dashed–dotted ones with $\mu = 500$ GeV. The shorter ones are with $\tan\beta = 1$ and the longer ones with $\tan\beta = 10$.

Fig.3 The same as Fig.2 but for the decay of the top quark into a charm quark and a gluon.

Fig.4 The same as in Fig.2 but for the decay of the top quark into a charm quark and a photon. The solid line with a sharp dip corresponds to $\mu = 100\ GeV$ and $\tan\beta = 2$.

Fig.5 $Log_{10}(t \to c\gamma)$ as a function of m_s and $tan\beta$ for $m_{\tilde{g}} = 100\ GeV = \mu$. The vertical scale is about the same as on fig. 4.