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Constrained MSSM and the electric dipole moment of the neutron

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Abstract

We study the constraints on the CP-violating soft-breaking phases in the minimal supersymmetric standard model using the limits on the chromoelectric dipole moment of the strange quark extracted from the neutron EDM experiment. Our investigation shows that the phase mediated by the gluino exchange diagram has to be very small, $\phi \leq 8 \cdot 10^{-4}$, for the common supersymmetric mass of the order of 100 GeV. Then, solving the renormalization group equations analytically by iterations, we calculate the electric dipole moment of the neutron in the MSSM with CP-conserving soft-breaking parameters for the case of three and four generations. For the three-generation case we resolve the apparent discrepancies between order-of-magnitude estimates and numerical calculations existing in the literature. In this case the EDM of the neutron does not exceed $10^{-32} e \cdot cm$. For the four-generation case we show that there is a significant enhancement which renders the EDM of the neutron at a measurable level of $10^{-26} e \cdot cm$.

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1 Introduction

The current experimental limit on the electric dipole moment (EDM) of the neutron [1], $d_N/e < 10^{-25}\,cm$, exceeds the realistic Standard Model prediction for this quantity by seven orders of magnitude. This gap between theory and experiment provides an excellent opportunity to limit a new CP-violating physics beyond the Standard Model (SM). The purpose of this work is to consider the electric dipole moment of the neutron in the Minimal Supersymmetric Standard Model (MSSM) with three and four generations of quarks.

The Minimal Supersymmetric Standard Model [2] looks nowadays as a very promising candidate for the physics beyond the SM. It can be probed experimentally through the high energy production of superpartners. Since no supersymmetric particles have been found so far, one can limit the parameter space of MSSM using the supersymmetric radiative corrections to the low-energy observables. The limits from the precise measurements of EDMs of the neutron, electron and heavy atoms are known to be a very good source of information in this respect [3]. Namely, the one-loop contribution to the electric dipole moments of quarks allows one to put stringent limits on the CP-violating phases in the soft-breaking sector, $\phi \leq$ few $\times\, 10^{-3}$[3]. Alternatively, the squark masses have to lie in the TeV range if we believe that the phases are not suppressed. This dilemma is often referred to as the Supersymmetric CP problem.

To avoid this problem from the very beginning, one can postulate that the soft-breaking sector as well as the μ-term in the superpotential are CP-conserving. This choice of parameters is often suggested by supergravity [4]. In addition to that, it is reasonable to assume the universality in the mixing of left- and right-handed scalar quarks to avoid unwanted FCNC at low energies. As a result, the only source of CP-violation resides in the Yukawa couplings and can be described at the electroweak scale by the Kobayashi-Maskawa (KM) phase. In principle, the prediction for the neutron EDM in MSSM can be different from the SM one due to the specific supesymmetric contributions and the KM-related phase which appears in the squark mass matrix [5].

The order-of-magnitude estimates of EDMs [4, 7], handling the main dependence on Yukawa couplings and mixing angles, suggest that the EDM of the neutron does not exceed $10^{-32} e \cdot cm$. This means that in this type of models the contribution from supersymmetric loops never exceeds the long-distance contribution to EDM from the usual SM [6]. To

improve the order-of-magnitude estimates, one has to solve the nonlinear renormalization group equations which is difficult to do analytically. Surprizingly enough, the attempts to solve these equations numerically by plugging in the values for quark masses and mixing angles measured at the electroweak scale give somewhat bigger results [8, 9, 10]. The discrepancy between [7] and [9], for example, constitutes six orders of magnitude so that it definitely requires an explanation. To resolve the apparent discrepancies and find the connection between the two approaches, we solve the renormalization group equations analytically, assuming that the renormalization group coefficient, $t = (4\pi)^{-2}\log(\Lambda^2/M_W^2)$, is small. In this case the equations are solvable by iterations and an analytical result for EDM can be obtained. It turns out that a finite result arises already after the first iteration, so that d_N is proportional to the first power of t. This result may be viewed as the intermediate step between parametrical estimates and numerical calculations. The fact that t is not small can change the final answer somehow, but not by many orders of magnitude if we believe that all Yukawa couplings remain in the perturbative regime in the whole interval from M_W to Λ. In any case, this question is more of methodological interest because of the expected smallness of the result.

As it was shown in [11, 12], the existence of a possible fourth generation in the framework of the usual SM gives a significant enhancement to EDM. The result, however, is still too small to be probed experimentally. With the natural assumptions about mixing angles it does not exceed $10^{-29} e \cdot cm$ [12]. In this paper we consider the MSSM with four generations and show that the value of the EDMs is significantly enhanced. This may give nontrivial constraints on the parameters of the model, complementary to those coming from the direct search of particles from the fourth generation and from the analysis of the renormalization group evolution of Yukawa couplings [13, 14].

We organize the paper as follows. In the next section we obtain new, apparently stronger, limits on CP-violating phases in the soft breaking sector mediated by the chromoelectric dipole moment (CEDM) of the strange quark and present the analytical calculation of EDM in the constrained MSSM with three generations. In Section 3 we estimate the EDMs in the MSSM with four generations. Our conclusions are sumarized in Section 4.

2 EDM in MSSM with three generations

We start by writing down the structure of the CP-odd effective Lagrangian dim$\leq$ 6 at the scale of 1 GeV which gives a major contribution to the EDM of the neutron:

$$\mathcal{L}_{eff}(x) = \theta\frac{\alpha_s}{8\pi}G^a_{\mu\nu}\tilde{G}^a_{\mu\nu} + i\frac{c_W}{6}g_s^3 f^{abc}\tilde{G}^a_{\alpha\beta}G^b_{\beta\mu}G^c_{\mu\alpha} + i\sum_i \frac{\tilde{d}_i}{2}\bar{q}_i t^a(G^a\sigma)\gamma_5 q_i + i\sum_i \frac{d_i}{2}\bar{q}_i(F\sigma)\gamma_5 q_i, \tag{1}$$

where the sum over i runs over light flavours, u, d and s. All other possible CP-odd operators dim=6 are irrelevant (see, for example, Ref. [11]). In SM we have to take into account also the so called long-distance contributions: the combination of two flavour-changing operators saturated by all possible hadronic states [6].

Here we concentrate ourselves on the calculation of EDMs, d_i, and colour EDMs $\tilde{d}_i$. The relative meaning of these operators for the electric dipole moment of the neutron is quite a controversial subject. To get the final result, one has to know how to calculate the EDM of the neutron in terms of d_i and $\tilde{d}_i$. This is a very difficult task if one takes into account the strong dynamics at large distances. To simplify this task, the naive nonrelativistic quark model is used very often. It relates d_N with d_u and d_d in a simple manner:

$$d_N = \frac{4}{3}d_d - \frac{1}{3}d_u \tag{2}$$

At the same time, the chromoelectric dipole moments as well as the electric dipole moment of s-quark are believed to be suppressed. The importance of the information provided by the EDM data suggests, however, that the naive formula based on the non-relativistic quark model must be abandoned and replaced with more elaborated approaches handling long-distance dynamics.

Let us suppose for a moment that we have the approximate universality in the sector of EDMs and CEDMs, for example, $d_d/(em_d) \simeq d_s/(em_s) \simeq a$, $\tilde{d}_d/(m_d) \simeq \tilde{d}_s/(m_s) \simeq b$ and $a \sim b/g_s$. In a recent work [15], the contribution of different quark EDMs to the EDM of the neutron were treated using the proton spin experimental data implying a non-zero content of the strange quark spin in the nucleon. In other words, the authors of Ref. [15] assume the identity between axial, $(\Delta q)_A$, and tensor, $(\Delta q)_T$, charges. (The definitions for these quantities are: $\bar{N}\gamma_\mu\gamma_5 N(\Delta q)_A = \langle N|\bar{q}\gamma_\mu\gamma_5 q|N\rangle$; $\bar{N}\sigma_{\mu\nu}N(\Delta q)_T = \langle N|\bar{q}\sigma_{\mu\nu}q|N\rangle$[4].) As a

[4]The EDM Lorentz structure is reducible to O_T using the identity $2\sigma_{\mu\nu}\gamma_5 = i\epsilon_{\mu\nu\alpha\beta}\sigma_{\alpha\beta}$

result, a sort of cancellation between strange and down quark contributions to neutron EDM was observed. This leads to apparently milder limits on ϕ [15]. It is clear, however, that the axial and tensor charges correspond to different structure functions and do not have to coincide [16]. It is especially true for the tensor charge of the s-quark in the neutron. Representing basically the sea quark contribution, the matrix element of the strange quark should be especially sensitive to the property of the operator with respect to charge conjugation. Since $C(\bar{q}\gamma_\mu\gamma_5 q) = +\bar{q}\gamma_\mu\gamma_5 q$ and $C(\bar{q}\sigma_{\mu\nu}q) = -\bar{q}\sigma_{\mu\nu}q$, we deduce that $(\Delta s)_T < (\Delta s)_A$. The calculation in the instanton-inspired model shows, in fact, that $(\Delta s)_T \ll (\Delta s)_A$ [17]. Taking into account the results of the tensor charge calculation on the lattice [18], within QCD sum rules [19] and in instanton-inspired model [17], we conclude that in the quark EDM channel the naive formula (2) is roughly correct and, at the same time, the electric dipole moment of the s-quark is unimportant providing *no* conspiracy among different flavours.

In our opinion, the question of conspiracy among the EDMs of different flavours is not relevant in MSSM on account of large contributions coming from the chromoelectric dipole moments of quarks. We would like to apply here the results obtained in Refs. [20, 21, 22, 23] in the context of chiral perturbation theory and QCD sum rules. If the above mentioned universality is held, the contributions from the chromoelectric dipole moments to the neutron EDM tend to dominate over the quark EDM contributions. The resulting estimate is given by [20, 22, 23]:

$$d_N \simeq e(0.7\tilde{d}_d + 0.1\tilde{d}_s) \tag{3}$$

and normally $e\tilde{d}_i > d_i$ due to the large g_s factor in $\tilde{d}_i$. Moreover, the second term in the estimate (3) tends to dominate due to the large m_s. Simple arguments favoring the suppression of the strange quark contribution in the quark EDM channel do not work for CEDM contributions. (For a recent discussion on the strange quark matrix elements in the nucleon see, for example, Ref. [24].)

It was shown recently that this operator produces a large neutron EDM in the supersymmetric $SO(10)$ model [23]. In the Minimal Supersymmetric Standard Model with CP-violating phases in the soft-breaking sector, the gluino exchange diagram leads to the following CEDM of a quark:

$$\tilde{d}_i \simeq g_s \frac{5\alpha_s}{72\pi}\,\frac{m_i}{m^3}\,F(m_{\lambda_3}^2/m^2)\,(A\sin\phi_A + \mu\tan\beta\sin\phi_B), \tag{4}$$

where $\phi_A = \mathrm{Im}(Am_{\lambda_3}^*)$ and $\phi_B = \mathrm{Im}(\mu m_{\lambda_3}^*)$ are the specific CP-violating phases which

appear in this diagram. The invariant function in Eq. (4) contains the dependence of the supersymmetric masses in the loop. It is normalized in such a way that $F(1) = 1$. Eq. (4) gives us the limits on the CP-violating phases considerably stronger than those coming from the EDMs of quarks. Taking $m_s(100\,GeV) = 80\,MeV$, $\mu \sim A \sim m$ and using (3), (4) and experimental limits we obtain the following constraint on the CP-violating phase:

$$\phi_{A(B)} \left(\frac{100\text{GeV}}{m}\right)^2 \leq 8 \cdot 10^{-4}. \tag{5}$$

When obtaining the limit (5) we have assumed that both phases can be constrained independently, i.e. no fine tunning among A, $\mu \tan\beta$, ϕ_A and ϕ_B. It is interesting to note also that the function F reaches its maximum, $F \sim 2.7$ for $m_{\lambda_3} \simeq 0.2\,m$, so that the constraints on the phases in this case are almost three times stronger.

These limits are very restrictive and from naturalness reasons we assume further on that the soft breaking sector does respect CP-symmetry. In what follows, we calculate SUSY contributions to the coefficients in (1) due to the presence of CP-violation in Yukawa couplings.

We take the superpotential of MSSM in the following standard form:

$$W = \bar{U}\mathbf{Y}_u Q H_u + \bar{D}\mathbf{Y}_d Q H_d + \bar{E}\mathbf{Y}_e L H_d + \mu H_u H_d \tag{6}$$

The soft breaking sector at the unification scale comprises flavour-blind scalar mass terms:

$$m_{H_d}^2 |H_d|^2 + m_{H_u}^2 |H_u|^2 + m_U^2 \tilde{U}^\dagger \tilde{U} + m_D^2 \tilde{D}^\dagger \tilde{D} + m_Q^2 \tilde{Q}^\dagger \tilde{Q} \ + H.c.; \tag{7}$$

so called A-terms proportional to Yukawa couplings

$$A\left(\tilde{U}_R^\dagger \mathbf{Y}_u \tilde{Q}_L H_u + \tilde{D}_R^\dagger \mathbf{Y}_u \tilde{Q}_L H_d + \tilde{D}_R^\dagger \mathbf{Y}_u \tilde{Q}_L H_d\right) \ + H.c.; \tag{8}$$

B term,

$$B\mu H_u \cdot H_d + H.c.; \tag{9}$$

and the gaugino masses

$$\sum_{i=1,2,3} m_{\lambda_i} \bar{\lambda}_{iR} \lambda_{iL} \ + H.c. \tag{10}$$

In formal language, the absence of additional CP-violating phases means the following:

$$A = A^*, \ B = B^*, \ \mu = \mu^*, \ m_{\lambda_i} = m_{\lambda_i}^*. \tag{11}$$

The squark mass matrices at the unification scale

$$M^2_{\tilde{u}} = \begin{pmatrix} m^2_Q + \mathbf{M}^\dagger_u \mathbf{M}_u & (A + \mu \cot\beta)\mathbf{M}^\dagger_u \\ \mathbf{M}_u(A + \mu\cot\beta) & m^2_U + \mathbf{M}_u\mathbf{M}^\dagger_u \end{pmatrix} \tag{12}$$

$$M^2_{\tilde{d}} = \begin{pmatrix} m^2_Q + \mathbf{M}^\dagger_d \mathbf{M}_d & (A + \mu \tan\beta)\mathbf{M}^\dagger_d \\ \mathbf{M}_d(A + \mu\tan\beta) & m^2_D + \mathbf{M}_d\mathbf{M}^\dagger_d \end{pmatrix} \tag{13}$$

do not lead to any flavour-changing effects, not mentioning CP-violation, since they are diagonalizable in the generation space by the same bi-unitary transformation as quark mass matrices. As a result, quark-squark-neutralino or quark-squark-chargino interactions do not develop flavour-changing vertices. The terms bilinear in $\mathbf{M}_u$ and $\mathbf{M}_d$ in the squark mass matrices (12) and (13) originate from the F-term in the superpotential. Since they are not related to the renormalization group running from large scale down to M_W, we shall refer to them as the tree-level terms. The renormalization group evaluation of the soft-breaking parameters and Yukawa couplings down to the electroweak scale indroduces flavour-changing entries and brings, for example, the dependence of $\mathbf{Y}_d$ in $M^2_{\tilde{u}}$ [5]. At the scale of the electroweak symmetry breaking the squark mass matrices can be parametrized as follows:

$$M^2_{\tilde{u}} = \begin{pmatrix} M^2_{uLL} & M^2_{uLR} \\ M^2_{uRL} & M^2_{uRR} \end{pmatrix} \; M^2_{\tilde{d}} = \begin{pmatrix} M^2_{dLL} & M^2_{dLR} \\ M^2_{dRL} & M^2_{dRR} \end{pmatrix} \tag{14}$$

where different blocks have the following meaning:

$$\begin{aligned} M^2_{uLL} &= m^2_{Q1} + \mathbf{M}^\dagger_u\mathbf{M}_u + c_1\mathbf{Y}^\dagger_u\mathbf{Y}_u + f_1\mathbf{Y}^\dagger_d\mathbf{Y}_d + c_2(\mathbf{Y}^\dagger_u\mathbf{Y}_u)^2 + f_2(\mathbf{Y}^\dagger_d\mathbf{Y}_d)^2 + \\ &\quad + c'_2\{\mathbf{Y}^\dagger_u\mathbf{Y}_u, \mathbf{Y}^\dagger_d\mathbf{Y}_d\} + ... \\ M^2_{uLR} &= (M^2_{uRL})^\dagger = (A_1 + \mu\cot\beta)\mathbf{M}^\dagger_u + A(h_1\mathbf{Y}^\dagger_d\mathbf{Y}_d + k_1\mathbf{Y}^\dagger_u\mathbf{Y}_u + ...)\mathbf{M}^\dagger_u \\ M^2_{uRR} &= m^2_{u1} + \mathbf{M}_u\mathbf{M}^\dagger_u + l_1\mathbf{Y}_u\mathbf{Y}^\dagger_u + l_2(\mathbf{Y}_u\mathbf{Y}^\dagger_u)^2 + l'_2\mathbf{Y}_u\mathbf{Y}^\dagger_d\mathbf{Y}_d\mathbf{Y}^\dagger_u + ... \end{aligned} \tag{15}$$

and similarly for the blocks of $M^2_{\tilde{d}}$. The explicit form of all coefficients can be found in the Appendix. In the expression (15) all quark masses and Yukawa couplings are taken at the electroweak scale. The dots stand for other terms in the series of the increasing power of Yukawa couplings and renormalization group factor t. The subscripts of the coefficients $c_1,\ c_2,\ f_1, ...$ denote the smallest power of t in which these entries to the mass matrix can

arise, i.e. $f_1 \sim t^1$, $f_2 \sim t^2$, etc. Please, note also that c_2, f_2, etc. *are not* the contributions from the two-loop beta function. Finally, A_1, m^2_{Q1} and m^2_{U1} are soft-breaking parameters renormalized by gauge interactions and by Yukawa interactions effectively conserving flavour, i.e. proportional to $\mathrm{Tr}[\mathbf{Y}_u \mathbf{Y}_u^\dagger]$.

As a result, one-loop diagrams with chargino and gluino inside the loop, Fig. 1 and Fig. 2, can develop imaginary parts and thus lead to a nonvanishing EDM. This effect, however, is severely suppressed by the combination of Yukawa couplings and mixing angles. The extraction of the imaginary part gives almost the structure of the Jarlskog invariant [25] and leads to the following simple estimate [4]:

$$d_d \sim e(\text{loop factors}) \times \frac{1}{m^2} \mathrm{Im}[\mathbf{M}_d \mathbf{Y}_u^4 \mathbf{Y}_d^2 \mathbf{Y}_u^2]_{11} \sim e(\text{loop f.}) \times J_{CP} \frac{m_d}{m^2} Y_t^4 Y_c^2 Y_b^2. \qquad (16)$$

where $J_{CP} = \mathrm{Im}(V^*_{td} V_{tb} V^*_{cb} V_{cd})$; m^2 is the characteristic momentum in the loop of the order of supersymmetric masses; "loop factors" denotes numerical coefficients reflecting the loop origin of the effect and the subscript 11 denotes the projection on the initial flavour d. Even with optimistic expectations for the numerical coefficients in Eq. (16), the result can hardly exceed $10^{-34} e \cdot cm$ for $\tan\beta \sim 1$. At the same time pure gluonic operators, as well as the EDM of the u-quark, are further suppressed by Yukawa couplings coming from down quark sector.

To resolve apparent discrepancies with the numerical calculations, we solve the renormalization group equation analytically by iterations retaining the smallest power of t. This procedure is very simple; it brings analytical answers for all operators in Eq. (1) and at the same time it gives the possibility to check all numerical calculation in an easy way.

In both papers quoting the estimate (16), Ref.[4] and Ref.[7], the effect is claimed to arize in the third or fourth order in renormalization group coefficient t. Surprizingly enough, we arrived to somewhat different conclusion. The chargino exchange diagram gives a nonvanishing d_d in the lowest possible (first) order in t. Clearly, this contribution is related with the $\mathbf{M}_u$-dependence of the u-squark mass matrix (14) coming from the tree-level and is not associated with t. The gluino exchange diagram yields EDMs in the down-quark sector in the third order in t.

Chargino contribution

The chargino exchange diagram which contributes to EDMs and CEDMs in the down-quark

sector contains the u-squark line. The result of the renormalization group running, up to the first power in t is summarized by the set of formulae displayed in the Appendix. Here we give the truncated form and list only relevant entries in (15) with Y_d^2-dependence:

$$M_{\tilde{u}}^2 = \begin{pmatrix} m_{Q1}^2 + \mathbf{M}_u^\dagger \mathbf{M}_u + f_1 \mathbf{Y}_d^\dagger \mathbf{Y}_d & (A_1 + \mu \cot\beta)\mathbf{M}_u^\dagger + A h_1 \mathbf{Y}_d^\dagger \mathbf{Y}_d \mathbf{M}_u^\dagger \\ (A_1 + \mu \cot\beta)\mathbf{M}_u + A h_1 \mathbf{M}_u \mathbf{Y}_d^\dagger \mathbf{Y}_d & m_{U1}^2 + \mathbf{M}_u \mathbf{M}_u^\dagger \end{pmatrix} \quad (17)$$

where

$$f_1 = -\frac{\log(\Lambda^2/m^2)}{16\pi^2}\left[m_Q^2 + m_U^2 + m_{H_u}^2 + A^2\right]; \quad h_1 = -\frac{\log(\Lambda^2/m^2)}{16\pi^2} \quad (18)$$

Assuming the equality between m_U and m_Q at the high-energy scale, $m_U = m_Q = m$, we can conclude that the difference of m_{U1} and m_{Q1} is also proportional to t.

The next step could be the expansion of the squark propagator in $A\mathbf{M}_u$ and $\mathbf{M}_u^2$ and the extraction of the CP-violating part from the string of Yukawa couplings and mass matrices [4]. This procedure is completely justified for the charm quark mass since $m_c^2,\ Am_c \ll m^2$ and seems to be ill-defined for the top flavour since we expect $m_t^2,\ Am_t \sim m^2$. Fortunately, there is no need to expand the squark propagator in terms of m_t^2. Let us take the first term of expansion in A. Then the resulting flavour structure of the squark line has the following simple form:

$$\begin{aligned} \mathrm{Im}\Bigg[& \frac{1}{p^2 - m_{Q1}^2 - \mathbf{M}_u^\dagger \mathbf{M}_u} f_1 \mathbf{Y}_d^\dagger \mathbf{Y}_d \frac{A_1 + \mu\cot\beta}{p^2 - m_{Q1}^2 - \mathbf{M}_u^\dagger \mathbf{M}_u} \mathbf{M}_u^\dagger \frac{1}{p^2 - m_{U1}^2 - \mathbf{M}_u \mathbf{M}_u^\dagger} \mathbf{M}_u + \\ & \frac{1}{p^2 - m_{Q1}^2 - \mathbf{M}_u^\dagger \mathbf{M}_u} A h_1 \mathbf{Y}_d^\dagger \mathbf{Y}_d \mathbf{M}_u^\dagger \frac{1}{p^2 - m_{U1}^2 - \mathbf{M}_u \mathbf{M}_u^\dagger} \mathbf{M}_u \Bigg]_{11} \simeq \\ & \simeq -J_{CP} m_c^2 m_t^4 Y_b^2 \frac{f_1(A + \mu\cot\beta) - A h_1 (m_{Q1}^2 - m_{U1}^2)}{(p^2 - m^2)^3 (p^2 - m^2 - m_t^2)^2} \end{aligned} \quad (19)$$

where we took the difference of the two soft-breaking masses to be much smaller than the masses themselves and neglected m_u^2 and m_c^2 in the denominator. It is easy to see that the term proportional to h_1 cannot develop any CP-violating part unless $m_{U1} \neq m_{Q1}$. However, since we are interested only in the first order in t, this contribution can be neglected being of the order t^2. In order to get the exact dependence of m_t, one has to sum up the series in $(A + \mu\cot\beta)m_t$, i.e. to take into account large $\tilde{t}_L - \tilde{t}_R$ mixing.

This complication as well as the effect of wino-higgsino mixing are irrelevant for the question under study and we would like to neglect them. Assuming also for simplicity that char-

gino mass is equal to the scalar quark mass m and taking $f_1 \simeq -3m^2(16\pi^2)^{-1}\log(\Lambda^2/m^2)$, after the trivial integration, we arrive at the following simple result:

$$\begin{aligned} d_d &\simeq \frac{1}{5}\frac{1}{16\pi^2}(\frac{5}{7}e_d - e_{\tilde{\chi}})\frac{\log(\Lambda^2/m^2)}{16\pi^2}\, J_{CP}\frac{m_d m_c^2 m_t^4 Y_b^2}{m^7}\,\frac{A+\mu\cot\beta}{v_u v_d} \quad \text{for } m \gg m_t \\ d_d &\simeq \frac{1}{2}\frac{1}{16\pi^2}(\frac{3}{5}e_d - e_{\tilde{\chi}})\frac{\log(\Lambda^2/m^2)}{16\pi^2}\, J_{CP}\frac{m_d m_c^2 Y_b^2}{m^3}\,\frac{A+\mu\cot\beta}{v_u v_d} \quad \text{for } m \ll m_t \end{aligned} \tag{20}$$

Here we keep explicit the loop origin of Y_b^2 and the tree-level origin of $m_c^2 m_t^4$. In a more general case with different supersymmetric masses, chargino mixing and realistic m_t-dependence taken into account, the numerical coefficients and the dependence of m_t^2 and m^2 should be substituted by some more complicated invariant function. Its exact form is beyond the scope of our interest. For the chromoelectric dipole moment we have a similar result:

$$\begin{aligned} \tilde{d}_d &\simeq \frac{g_3}{7}\frac{1}{16\pi^2}\frac{\log(\Lambda^2/m^2)}{16\pi^2}\, J_{CP}\frac{m_d m_c^2 m_t^4 Y_b^2}{m^7}\,\frac{A+\mu\cot\beta}{v_u v_d} \quad \text{for } m \gg m_t \\ \tilde{d}_d &\simeq \frac{3g_3}{10}\frac{1}{16\pi^2}\frac{\log(\Lambda^2/m^2)}{16\pi^2}\, J_{CP}\frac{m_d m_c^2 Y_b^2}{m^3}\,\frac{A+\mu\cot\beta}{v_u v_d} \quad \text{for } m \ll m_t \end{aligned} \tag{21}$$

Numerically, the EDM of d-quark (20) constitutes

$$|d_d| \sim 6\cdot 10^{-34}\left(\frac{\tan\beta}{10}\right)^3 e\cdot cm \tag{22}$$

were we took $J_{CP} \simeq 2\times 10^{-5}$, $\tan\beta > 1$, $(16\pi^2)^{-1}\log(\Lambda^2/m^2) \sim 1/3$ and $m \sim A \sim 100$ GeV. It should be noted here that all Yukawa couplings and quark masses are taken at the scale of 100 GeV so that the result is further suppressed by the QCD evaluation of $m_c^2 m_b^2$ from 1 GeV to this scale. Even taking into account the enhancement associated with the CEDM of the strange quark we cannot obtain the EDM of the neutron bigger than $10^{-32} e\cdot cm$.

The numerical enhancement of d_d, up to the level of $10^{-28} e\cdot cm$, observed in Ref. [9] is nothing but an artifact resulting from a quite surprizing choice of the parameters $m_{scalar} =$ 100 GeV and $A = 5$ TeV, so that $A/m_{scalar} = 50$. (This was noted also in Ref. [10].) We believe, however, that this choice of parameters is highly unnatural if not strictly forbidden. Here we would like to show how it could change the result numerically. In this case, the expansion in Am_c/m_{scalar}^2 is not associated with any numerical smallness since this factor is now roughly of the order 1. In consequence, the result would not be suppressed in m_c^2 at all, leading to an enhancement of the order 10^4.

Gluino contribution

It is clear that the result can arise here only in the third order in t. The simple solution for

the down squark mass matrix, analogous to (15) is not sufficient, though. Indeed, it can be shown that the flavour-diagonal projection of the combination

$$\frac{1}{p^2 - m_{Q1}^2 - (\frac{v_d^2}{2} + c_1)\mathbf{Y}_d^\dagger \mathbf{Y}_d - f_1 \mathbf{Y}_u^\dagger \mathbf{Y}_u}(A' + \mu \cot\beta + h_1 \mathbf{Y}_u^\dagger \mathbf{Y}_u)\mathbf{M}_d^\dagger \frac{1}{p^2 - m_{U1}^2 - (\frac{v_d^2}{2} + l_1)\mathbf{Y}_d \mathbf{Y}_d^\dagger} \tag{23}$$

does not develop any CP-violating part at all. This shows that the simple estimate presented in Refs. [4] and [7] based on the quadratic anzatz of the mass matrices are incorrect since the exact result in this approximation is just zero in any order in t. This situation resembles the cancellation of the two-loop contribution to the EDM of a quark in the SM. The combination of different diagrams, nonzero by themselves, vanishes after the complete summation over all flavours [26].

To be rigorous, however, we have to go further and calculate quartic combinations of Yukawa couplings in the squark masses and A-parameter proportional to the second power of t, i.e the coefficients c_2, d_2, etc. It can be shown also that the t^3-order entries in the quark mass matrices contribute to EDMs in the next (fourth) order in t and thus are irrelevant. The calculation is very simple and all relevant coefficients are listed in the Apendix. Omitting all intermediate steps, we would like to quote here the final result:

$$d_d \;=\; e_d\, m_d\, m_b^2 Y_c^2 Y_t^4\, J_{CP}\, \frac{2\alpha_s}{3\pi}\, \frac{t^3\, A^2}{m^7} \left[\frac{4}{30} A - (A + \mu \tan\beta) \frac{8A(A + \mu \tan\beta) - (3m^2 + A^2)}{42m^2} \right] \tag{24}$$

It is obtained with the same simplification as Eq. (20) and for $m_t \ll m$. Numerically, for relistic values of t and soft-breaking parameters, this result does not differ very much from Eq. (22) and finally we conclude that the EDM of the neutron in constrained MSSM with three generations does not exceed the nonsupersymmetric SM predictions.

3 MSSM with four generations and neutron EDM

The extreme smallness of the result (22) kills any hopes of getting any observable effect in the model. Moreover, it never exceeds the non-supersymmetric SM result. The situation is somehow different in the constrained MSSM with four generations of fermions. This model acquired significant attention in recent years, since its parameter space is already severely restricted by the existing data [13, 14]. The detailed analysis of the renormalization group equations in Ref. [13] showed that the condition of perturbative evolution for all Yukawas

requires the masses of the fourth generation quarks to be under 200 GeV. The similarity between m_t and $m_{t'}$ suggests that the third and fourth generations are strongly mixed, possibly with the angle $V_{tb'} \sim \mathcal{O}(\lambda)$ or even $\sim \mathcal{O}(1)$. This means that possible CP-odd invariant combination of angles involving second, third and fourth generations, $\mathrm{Im}(V_{ts}^* V_{tb'} V_{cb'}^* V_{cs})$ is bigger than J_{CP} in SM by a factor of the order $\lambda^{-1} - \lambda^{-2}$ (See also Ref. [12]).

Clearly, this is not the main source of enhancement for CP-odd observables in the model. A tremendous enhancement is associated now with the change of masses and Yukawa couplings. Instead of being proportional to $m_c^2 m_t^4 Y_b^2$, all the results for EDMs contain now the factor $m_t^2 m_{t'}^2 (m_t^2 - m_{t'}^2) Y_{b'}^2$. Obviously, this factor is comparable with m_{scalar}^6 and independent from all "light" flavours: u, d, s, c and b.

The detailed calculation of EDMs in the constrained MSSM with four generations is more difficult than in the conventional three-generation model. The main complication now is that not only m_t, but also $m_{t'}$ and $m_{b'}$ are comparable with m, so that we have to hold the exact dependence of these masses. The language of mass insertions is completely inadequate now. Instead, we have to perform the analysis in the mass eigenstate approach. In the limit of small t it is still possible to obtain the result in a closed analytical form. Bearing in mind, however, the big degree of uncertainty related with unknown masses and mixing angles, we believe that simple estimates are sufficient in this case. In particular, we can say that the renormalization group evolution of the Yukawa couplings and soft breaking parameters from the unification scale introduces at the electroweak scale CP-violating phases of the order $\mathrm{Im}(V_{ts}^* V_{tb'} V_{cb'}^* V_{cs}) Y_t^2 Y_{t'}^2 (Y_t^2 - Y_{t'}^2) Y_{b'}^2$. Thus, we can use the estimates of the EDM in the presence of CP-violating phases with the simple substitution: $\sin(\phi_{A(B)}) \to \mathrm{Im}(V_{ts}^* V_{tb'} V_{cb'}^* V_{cs})$. Then for the chromoelectric dipole moment of the s-quark we have:

$$\tilde{d}_s \sim g_s \frac{5\alpha_s}{72\pi} \frac{m_s}{m^2} \mathrm{Im}(V_{ts}^* V_{tb'} V_{cb'}^* V_{cs}) \tag{25}$$

The resulting estimate for the electric dipole moment of the neutron now is:

$$d_N \sim \mathrm{Im}(V_{ts}^* V_{tb'} V_{cb'}^* V_{cs}) \left(\frac{200\mathrm{GeV}}{m}\right)^2 2.5 \cdot 10^{-23} e \cdot cm \tag{26}$$

where we took the supersymmetric mass to be of the order of 200 GeV. The overall enhancement factor in the MSSM with four generations in comparison with the three-generation case is enormous, being roughly of the order 10^8. As a result, the EDM of the neutron is predicted in this model to be at the measurable level since we expect $\mathrm{Im}(V_{ts}^* V_{tb'} V_{cb'}^* V_{cs}) \sim$

$\lambda^5 - \lambda^4 \sim 10^{-4} - 10^{-3}$. The current experimental limit on the EDM of the neutron translates Eq. (26) into the following constraint on the CP-odd combination of matrix elements:

$$\mathrm{Im}(V_{ts}^* V_{tb'} V_{cb'}^* V_{cs}) \left(\frac{200\mathrm{GeV}}{m}\right)^2 \leq 4 \cdot 10^{-3} \tag{27}$$

It is instructive to compare MSSM with four generations and nonsupersymmetric SM with the same number of generations. It turns out that in SM with four generations the result arises at three-loop level and these loops are not supported by the large logarithmic factors. It means that the phase space factor in the denominator is large, confining neutron EDM to lie within $10^{-29} e \cdot cm$ [11, 12]. From the point of view of the parameters defined at the electroweak scale, the EDM in the MSSM with four generations arises just at one-loop level and therefore is considerably larger than in the non-supersymmetric case.

4 Conclusions

We have shown that the limit on the colour EDM of the strange quark obtained in [20] implies that the constraints on the phases ϕ_A and ϕ_B in the soft-breaking sector of MSSM are stronger than those from the quark EDM channel [3]. This strengthens the so called supersymmetric CP-problem and may have an impact on some applications of the specific SUSY CP-violation which use the phases on the edge of the previous limits [27]. The solution to the supersymmetric CP-problem may be related, see for example Ref. [28], with the dynamical suppression of gaugino masses and/or A parameter. Alternatively, all phases can be chosen zero at the unification scale [4].

In the MSSM with the CP-violation coming from the Yukawa matrices simple order-of-magnitude estimates and the numerical calculations are known to disagree by many orders of magnitude. We demonstrate that analytical results are possible for EDMs in the limit of small t. The chargino-exchange diagram yields non-vanishing EDMs for the down-quark sector in the lowest possible (first) order in t. The result shows explicitly the suppression factor associated with Yukawa couplings and basically confirms the order-of-magnitude estimates [4, 7]. It provides the possibility to check all numerical calculations by lowering the high-energy scale. We believe, however, that even for $t \simeq 1/2$, corresponding to $\Lambda \sim 10^{19}$ GeV the results cannot differ from ours by many orders of magnitude. The contrary would be possible only if a kind of singularity exists in the solutions of the renormalization group equations at

some point between M_W and Λ. We reject this possibility and therefore conclude that the EDM of the neutron in the constrained MSSM does not exceed its SM prediction [6].

The four-generation modification of the MSSM can be viewed as an interesting generalization of the conventional MSSM and at the same time it can be checked and/or ruled out experimentally in the nearest future [13, 14]. We believe that the similarity between m_t and $m_{t'}$ suggests also non-zero, presumably large, mixing between the third and the fourth generations. At the same time, the relevant combination of masses is much bigger in this case and the enhancement factor is given by $(m_{b'}^2 m_t^2)/(m_b^2 m_c^2) \sim 10^8$. This leads to a large EDM of the neutron at the measurable level of $10^{-26} e \cdot cm$.

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Appendix

A) Assuming that the renormalization group coefficient t is small, the renormalization group equations can be solved analyticaly by iterations. In this case, the coefficients which appear in equation (15) are found to have the following explicit form:

$$\begin{aligned} c_1 &= -t\,[m^2_{Q_0} + m^2_{u_0} + m^2_{H_{u_0}} + A^2] + \mathcal{O}(t^2) + \ldots \\ f_1 &= -t\,[m^2_{Q_0} + m^2_{d_0} + m^2_{H_{d_0}} + A^2] + \mathcal{O}(t^2) + \ldots \\ h_1 &= -t + \mathcal{O}(t^2) + \ldots \\ k_1 &= -3t + \mathcal{O}(t^2) + \ldots \\ l_1 &= -2t[m^2_{Q_0} + m^2_{u_0} + m^2_{H_{u_0}} + A^2] + \mathcal{O}(t^2) + \ldots \end{aligned} \tag{28}$$

These quantities arise after the first iteration and are the relevant entries to the u-squark mass matrix for the calculation of chargino-exchange contribution to quark EDM.

Since gluino-induced quark EDMs appear only in the third power in the renormalization group coefficient t, we have to perform also the second iteration. The results for c_2, f_2, c_2', l_2 and l_2' are:

$$\begin{aligned} c_2 &= 3\,t^2\,A^2 + \mathcal{O}(t^3) + \ldots \\ f_2 &= 3\,t^2\,A^2 + \mathcal{O}(t^3) + \ldots \\ c_2' &= t^2\,A^2 + \mathcal{O}(t^3) + \ldots \\ l_2 &= 6\,t^2\,A^2 + \mathcal{O}(t^3) + \ldots \\ l_2' &= t^2\,[m^2_{Q_0} + m^2_{d_0} + m^2_{H_{d_0}} + 3A^2] + \mathcal{O}(t^3) + \ldots \end{aligned} \tag{29}$$

These expressions are obtained starting with diagonal soft-breaking mass parameters at the unification scale; the index 0 refers to the parameters at that scale. In order to obtain the coefficients for the entries in the d-squark mass matrix, one has only to interchange the indices u and d.

B) We would also like to present the complete solutions up to the second order in t for the renormalization group equations associated with the soft-breaking parameters. We used the notations and the one-loop β-functions given in [29].

$$\begin{aligned} \mathbf{m}^2_{Q_2} = {} & m^2_{Q_2} + a^q \mathbf{Y}^\dagger_u \mathbf{Y}_u + b^q \mathbf{Y}^\dagger_d \mathbf{Y}_d + \\ & + t^2\,A^2 [3(\mathbf{Y}^\dagger_{u_2}\mathbf{Y}_{u_2})^2 + 3(\mathbf{Y}^\dagger_{d_2}\mathbf{Y}_{d_2})^2 + \{\mathbf{Y}^\dagger_{u_2}\mathbf{Y}_{u_2},\, \mathbf{Y}^\dagger_{d_2}\mathbf{Y}_{d_2}\}] \end{aligned}$$

$$
\begin{aligned}
\mathbf{m}^2_{u_2} &= m^2_{u_2} + a^u \mathbf{Y}_u \mathbf{Y}^\dagger_u + \\
&\quad + \frac{1}{2} t^2 [12 A_0^2 \mathbf{Y}_{u_2} \mathbf{Y}^\dagger_{u_2} \mathbf{Y}_{u_2} \mathbf{Y}^\dagger_{u_2} + (4A_0^2 + b^u) \mathbf{Y}_{u_2} \mathbf{Y}^\dagger_{d_2} \mathbf{Y}_{d_2} \mathbf{Y}^\dagger_{u_2}] \\
\mathbf{m}^2_{d_2} &= m^2_{d_2} + a^d \mathbf{Y}_d \mathbf{Y}^\dagger_d + \\
&\quad + \frac{1}{2} t^2 [12 A_0^2 \mathbf{Y}_{d_2} \mathbf{Y}^\dagger_{d_2} \mathbf{Y}_{d_2} \mathbf{Y}^\dagger_{d_2} + (4A_0^2 + b^d) \mathbf{Y}_{d_2} \mathbf{Y}^\dagger_{u_2} \mathbf{Y}_{u_2} \mathbf{Y}^\dagger_{d_2}] \\
\mathbf{h}_{u_2} &= \mathbf{Y}_{u_2} \Big[A_u + a^{h_u} \mathbf{Y}^\dagger_u \mathbf{Y}_u + b^{h_u} \mathbf{Y}^\dagger_d \mathbf{Y}_d \Big] \\
\mathbf{h}_{d_2} &= \mathbf{Y}_{d_2} \Big[A_d + a^{h_d} \mathbf{Y}^\dagger_d \mathbf{Y}_d + b^{h_d} \mathbf{Y}^\dagger_u \mathbf{Y}_u \Big]
\end{aligned}
\tag{30}
$$

In the above formulae we used the following notations:

$$
\begin{aligned}
a^q &= -t\,[m^2_{Q_0} + m^2_{u_0} + m^2_{H_{u_0}} + A^2] + \frac{1}{4}\Big[\int_0^{-t} (m^2_{H_{u1}} - m^2_{H_{u0}}) dt + \\
&\quad + t^2 [\rho^2_{Q_0} + \rho^2_{u_0} + A\eta_{u_0}] - t^2 [2m^2_{Q_0} + 2m^2_{u_0} + 2m^2_{H_{u_0}} + 3A_0^2] \Big] \\
b^q &= -t\,[m^2_{Q_0} + m^2_{d_0} + m^2_{H_{d_0}} + A^2] + \frac{1}{4}\Big[\int_0^{-t} (m^2_{H_{d1}} - m^2_{H_{d0}}) dt + \\
&\quad + t^2 [\rho^2_{Q_0} + \rho^2_{d_0} + A\eta_{d_0}] - t^2 [2m^2_{Q_0} + 2m^2_{d_0} + 2m^2_{H_{d_0}} + 3A^2] \Big] \\
a^{u,d} &= -2t\,[m^2_{Q_0} + m^2_{u,d_0} + m^2_{H_{u,d_0}} + A^2] + \int_0^{t} (m^2_{H_{u,d_1}} - m^2_{H_{u,d_0}}) dt + \\
&\quad + \frac{1}{2} t^2 \Big[\rho^2_{Q_0} + A\eta_{u,d_0} - 2\beta_{u,d_0} [m^2_{Q_0} + m^2_{u,d_0} + m^2_{H_{u,d_0}} + A^2] \Big] \\
b^{u,d} &= 2\,[m^2_{Q_0} + m^2_{d,u_0} + m^2_{H_{d,u_0}} + A^2] \\
a^{h_{u,d}} &= -3t\,A + \frac{1}{4} t^2\,[-8A\beta_{u,d_0} + 3\eta_{u,d_0}] \\
b^{h_{u,d}} &= -tA + \frac{1}{4} t^2\,[A(\beta_{u,d_0} - 4\beta_{d,u_0}) + \frac{1}{2}(2\eta_{d,u_0} - \eta_{u,d_0})]
\end{aligned}
\tag{31}
$$

Finally, $A_{u,d}$, m_{Q_2}, m_{u_2} and m_{d_2} are soft-breaking parameters renormalized by gauge interactions and flavour-conserving Yukawa interactions. The indices 0, 1 and 2 refer to the iteration to be considered for the corresponding quantity. The functions η, ρ and β are scalar quantities involving gauge couplings, gaugino masses and $\mathrm{Tr}[\mathbf{Y}^\dagger_i \mathbf{Y}_i]$. Their exact form can be extracted from Ref.[29]. Using these solutions, one can identify all the coefficients in equation (15) up to the second power in t.

Figure captions

Fig. 1. Quark self-energy involving chargino exchange, generating EDM (CEDM) of quark in the external electromagnetic (color) field.

Fig. 2. Quark self-energy involving gluino exchange, generating EDM (CEDM) of quark in the external electromagnetic (color) field.

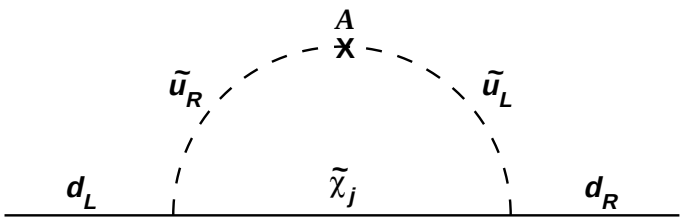

Fig. 1.

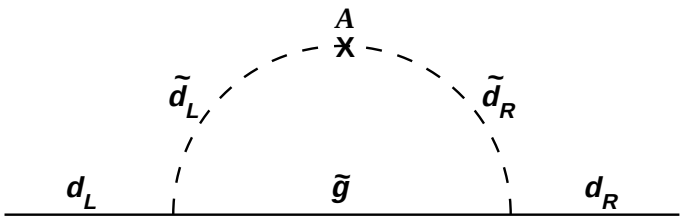

Fig. 2.