UNIVERSITÉ DU QUÉBEC À MONTRÉAL

FULL-DUPLEX RELAYING WITH SELF-INTERFERENCE AVOIDANCE

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RELAYAGE FULL-DUPLEX AVEC PRÉVENTION DE L'AUTO-INTERFERENCE

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The master journey is a funny one. At the beginning, you have no idea where you are going, but you know that you have to get there. At the end, when all is said and done, you still have no idea where you are going. But you have become very good at getting there. It is not possible to completely express my gratitude to my advisors, Professor Wessam Ajib and Jean-François Frigon, for all the years of guidance and tolerance they have given me. I recognize that I am not the easiest student to work with. They fostered an environment where I could explore freely, find out what I liked, make attempts at what I fancied, and finally succeed in what I did. As a graduate student, I could not have asked for more. As a person, I am forever indebted.

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In memory of my father

To my mother, brother and *sister*

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ABBREVIATIONS

AWGN	Additive white Gaussian noise
AF	Amplify-and-forward
C	Capacity
CSI	Channel side information
D	Destination
DF	Decode-and-forward
FD	Full-duplex
FDAE	Full-duplex with Alamouti encoding
FDI	Full-duplex with interference
I	Mutual information
MFDAE	Modified full-duplex with Alamouti encoding
MIMO	Multiple-input multiple-output
R	Relay
S	Source
SIR	Signal to interference ratio
SNR	Signal to noise ratio

RÉSUMÉ

Transmettre et recevoir des signaux simultanément(communication *full−duplex*) sur la même bande de fréquence n'est possible que grâce à la prévention de l'auto-interférence avant d'atteindre le convertisseur analogique numérique de l'antenne de reception. Une contrainte sur la prévention de l'auto-interférence reçue doit être imposée sur le noeud *full − duplex* et cette contrainte doit être considérée dans l'analyse de la capacité d'un canal de relayage *full − duplex*. Ce travail étudie le problème d'évaluation de performance d'un système de relayage *full − duplex* sous la contrainte d'une auto-interférence nulle. Tout d'abord, nous calculons la capacité de canal de relayage *full − duplex*. En se basant sur ce résultat, nous obtenons une formule explicite de l'information mutuelle maximale d'un canal de relayage *full − duplex* "décoder-et-transférer" où l'état des canaux est disponible au niveau du relai.

En outre, nous proposons deux algorithmes de relayage, basés sur l'encodage distribué Alamouti, qui à la fois résolvent efficacement le problème de l'interférence due au lien direct et préviennent complètement l'auto-interférence. Nous montrons que les performances de ces deux algorithmes s'approchent de l'information mutuelle maximale. Ensuite, en supposant que le relai connaît parfaitement l'état des canaux ainsi que leurs statistiques, nous calculons la distribution de puissance optimale dans le temps qui permet d'atteindre l'information mutuelle maximale. Les résultats numériques prouvent que la connaissance de l'état des canaux et leurs statistiques fournit un gain d'information mutuelle important et réduit l'énergie consommée.

ABSTRACT

Full-duplex transmission is feasible only through self-interference avoidance, before reaching the analogue to digital convertor at the front-end of the received antenna. In order to grant feasibility, a constraint on the received self-interference power have to be imposed on the full-duplex node and this constraint should be considered in analyzing the full-duplex relay channel capacity. This thesis reconsiders the problem of evaluating the performance of full-duplex relay channel, with channel side information at the relay, under the constraint of null received self-interference power. First, the corresponding capacity question is formulated and answered. Based on this result, we derive an explicit formula for the maximum mutual information of full-duplex decode-and-forward relay channel when the relay has instantaneous channel state knowledge. Moreover, we propose two relay transmission schemes, based on distributed Alamouti encoding, which take into account the self-interference avoidance and efficiently mitigate the impact of the direct link interference to achieve near maximum mutual information performance. Next, assuming that the relay perfectly knows the channel state and the channel statistic, we derive the optimal power distribution in time that achieves the maximum mutual information. Numerical results show that this full state information provides significant mutual information gain and relay power saving compared to the case of limited instantaneous channel state knowledge.

INTRODUCTION

0.1 Background

Communication has always been a fundamental necessity through the course of human history. Because we are living in a society, contributing to society, sharing with the society, communication is a natural part of who we are. The efforts to make communication a better experience have always been with us. The limitations of communication have been continuously reduced by the technology. For instance, one innovation that makes the communication a better experience was wireless communication systems. It extended the maximum distance to communicate and ameliorated the life of mankind.

Different requirements of communication systems have come into play as we progressed. The extent of security, speed, accuracy and distance has put the challenge into different fields, compelling us to invent different forms of communication. Yet, there has always been one goal which is common for all forms of communication systems : achieving high data rate. We are living in a world in which wireless traffic grows increasingly and wireless applications are getting more and more greedy for data rate and accuracy. The challenge to design the next-generation of wireless systems, with high throughput and better coverage, becomes more and more difficult.

An important metric, known as channel capacity, is widely used to characterize wireless communication systems by giving the theatrical maximum reliable transmission rate. It is defined in information theory as the maximum rate of reliable communication that can be supported by a communication system. Since the channel capacity of a wireless communication system may point out if the system can handle more wireless traffic, analyzing the channel capacity is often the first step of new wireless system performance analysis. One of the wireless communication systems, with high channel capacity, is the relay channel.

A relay channel has a node, denoted by relay, that relays signals between a source node and a destination one. Relays are transceivers that have the ability to receive signals from a source node and forward them to a destination node. Relay channel has attracted attention due to its diverse applications and numerous advantages. The use of relays in wireless communication systems brings several benefits. For instance, relaying improves the system coverage by repeating the signal towards farther distances and lead to higher system throughput and higher efficient power consumption. A relay channel, with half-duplex (HD) relay that alternates between reception and transmission processes, is denoted by HD relay channel. In HD relay channel the relay receives and transmits signals in orthogonal channels on frequency or time domains. This results in an inefficient spectrum use and hence the end-to-end channel capacity degrades. When the relay is able to handle the two processes of transmission and reception simultaneously and over the same band (full-duplex node), the system is denoted by full-duplex (FD) relay channel. Using jointly the concepts of relaying and full-duplex communication within the same system is seen as a promising approach to enhance the system capacity and to improve the spectrum utilization efficiency. A relay channel is said degraded (see 1.3) when the channel capacity of the link between the source and the relay is more important than the channel capacity of the link between the source and the destination.

A basic perception of wireless communication is that a radio transmits and receives signals on channels that are orthogonal in frequency domain (frequency division duplexing) or in time domain (time division duplexing). Recent works have investigated the possibility to design a full-duplex radio that simultaneously communicates on both directions (i.e., receive and transmit) over the same frequency band. The fact that FD radio transmits and receives simultaneously over the same band allows the spectrum reuse which is an efficient way to combat the problem of spectrum scarcity. Full-duplex radio is a promising technology that provides physical layer gain and can mitigate many wireless networks problems. Indeed, full-duplex radio can help to address several challenges such as reducing the high end-to-end transmission delays of multi-hop wireless communication system. Note that full-duplex communication is used in this document to denote transmitting and receiving at the same time and on the same frequency band.

During full-duplex transmission, the FD node may receive its own transmitted signal (self-interference) interfered with the signal-of-interest sent by other distant nodes. One can think that a FD node with one transmit antenna and one received antenna can simply subtract the contribution of its own transmitted signal from the received signal. It can then process a free-interference signal-of-interest. However, the self-interference signal received from the nearby local transmit antenna has much higher power than the signal-of-interest coming from farther nodes. The strong self-interference spans most of the range of the analog to digital converter (ADC) in the received signal processing path which dramatically increases the quantization noise for the signal-of-interest. This results in a very low signal-of-interest power to noise ratio. Therefore, the key idea to make feasible the full-duplex nodes is to eliminate the self-interference before that the analog received signal is sampled by the ADC. Full-duplex transmission is feasible under the constraint of null (or very weak) self-interference power constraint. Recent works show that a multiple-input multiple-output (MIMO) node can use the spatial domain to eliminate self-interference. The idea behind the spatial suppression is to beamform the transmit signal such a way it is completely precanceled (or avoided as much as possible) on the direction of the local receiver which can completely or partialy eliminate the self-interference and thus makes possible full-duplex communication.

0.2 Research Problem

Full-duplex relaying is a promising technology that provides high system capacity and efficient spectrum utilization. Under the assumption of no self-interference, full-duplex relay channels can provide twice the rate of a half-duplex relay channel. FD relay channel is greatly affected by relay self-interference which makes the full-duplex relaying not feasible. The assumption of no self-interference is too strong and hence the previous results can be seen as capacity upper bounds. Full-duplex communication is feasible only under the constraint of null (or very weak) self-interference received power. The performance of full-duplex relay channel capacity should be reevaluated when a constraint on the received self-interference power is considered. Moreover, it is interesting to characterize the important decode-and-forward relay channel by giving the maximum rate of reliable communication (i.e., maximum mutual information) over the system with channel state information (CSI) under the constraint of null self-interference power.

In half-duplex relay channels the relay (R) and the source (S) transmit signals, toward the destination (D), in orthogonal channels on time. Thus, the destination receives free-interference signals. In full-duplex relay channel, the relay and the source transmit signals simultaneously over the same band and thus the signal transmitted by S interferes at D with the signal transmitted by R. This results in a low end-to-end achievable data rate compared to the maximum mutual information. It is thus challenging to design a transmission scheme that achieves the maximum mutual information.

In point-to-point system, when the transmitter has instantaneous channel state information and the channel statistics (i.e., probability density functions (PDFs)) knowledge, it should adapt the transmit power in time in order to achieve the capacity. But, what about the full-duplex decode-and-forward relay channel when the relay has instantaneous CSI and the channel statistics? The relay power optimization problem hence should be formulated and the optimal relay power distribution over time should be also derived in order to achieve the maximum mutual information.

0.3 Key Innovations in this Thesis

Recently, smart nodes, equipped with advanced radio technologies such as full-duplex communication nodes, have been introduced to help the other nodes. That is the case of the FD relay in our system model. This thesis focuses on full-duplex relay channel under the constraint of null self-interference received power. For convenience, we say that the relay has instantaneous CSI if it perfectly knows the instantaneous channel state of all the links. We also say that the relay has full CSI, if it perfectly knows the instantaneous channel state and the PDFs of the channel of all the links.

Our first contribution is to provide an explicit form of full-duplex degraded relay channel capacity under the constraint of null received self-interference power (Chraiti et al., 2014a). Based on the full-duplex degraded relay channel capacity results, we derive the maximum mutual information expression for full-duplex relay channel with decode-and-forward strategy and instantaneous CSI at the relay.

As our second major contribution, we propose two transmission schemes, based on the widely used space-time block coding MIMO technique named Alamouti encoding technique, that provide a near maximum mutual information performance. The proposed schemes mitigate the problem of direct link interference by appropriately combining at the destination the signal received via direct link and its delayed copy received via the full-duplex relay link. The first scheme is denoted by full-duplex with distributed Alamouti encoding (FDAE) (Chraiti et al., 2013a). The performance of FDAE depends on the processing delay and it is close to the maximum mutual information at low relay processing delay. Since the FDAE scheme does not always provide optimal performance, we propose a new transmission scheme denoted by modified FDAE (MFDAE) (Chraiti et al., 2014a). The second proposed scheme performance is independent of the processing delay. It provides near maximum mutual information. However, the FDAE scheme outperforms the MFDAE scheme at low relay processing delay.

Our third contribution is to address the problem of relay power allocation in time when full CSI knowledge is available at the relay. We formulate the problem of long-term relay power adaptation and we derive the optimal relay power distribution over time that achieves the maximum average mutual information. We propose an algorithm to derive the exact optimal power allocation for the important case of discrete channel distribution. The mutual information of full-duplex relay channel with full CSI is compared to the one with instantaneous CSI. Numerical results show that full CSI knowledge provides a significant mutual information gain and relay power saving compared to the instantaneous CSI case.

0.4 Outline of this Thesis

This thesis is organized as follows. In chapter 1, we review the literature related to full-duplex relay channel. In chapter 2, we analyze the performance of full-duplex relay channel under the constraint of null self-interference received power.

In chapter 3, we propose two transmission schemes that give near-optimal achievable data rate. In chapter 4, we derive the optimal power distribution in time that achieves the maximum mutual information when the relay has full CSI. Finally, we conclude and discuss the outcomes of the thesis.

CHAPTER I

LITERATURE REVIEW

This chapter introduces different wireless communication techniques and summarizes the work that has been done in earlier literature, mostly about MIMO, full-duplex nodes and relay channel. In the following sections, first, we define the basic concepts used in information theory to analyze the performance of a wireless channel. Moreover, we introduce MIMO systems and the performance analysis in information theory sense. Second, we introduce the full-duplex radio and the relay channel. Their benefits and drawbacks are pointed out. Later, the full-duplex relay channel is discussed. The previous efforts and various proposed solutions are presented.

1.1 MIMO and Information Theory

1.1.1 Capacity of wireless channel

The framework for studying the performance limits in wireless communication is the information theory. The basic measure of performance is the channel capacity which presents the maximum rate of reliable communication that can be supported by the channel. Reliable communication is the communication for which arbitrarily small error probability can be achieved. Two issues that significantly influence the capacity notions are the ratio of transmit symbol duration to the coherence time and the amount of channel state information available at the transmitters and receivers. Coherence time is the time duration over which the channel impulse response is considered to be not varying. Depending on the ratio of the transmit symbol duration to the coherence time duration the channel is called slow-fading channel or fast-fading channel. A channel is said to be slow-fading if the required transmission delay is shorter than the channel coherence time. Otherwise, the channel is said to be fast-fading channel. The channel state information is defined as the of knowledge about the channel realization throughout the system. Obviously, when a system has better channel state information knowledge, it provides better channel capacity. To briefly describe the channel capacity, we consider a simple system with one single-antenna source and one single-antenna destination and a slow-fading channel. Accurate channel state information is available at the destination but not at the source. In this subsection, we make use of on the familiar complex AWGN (additive white Gaussian noise) channel. The AWGN channel is represented by a series of a random Gaussian outputs $y[k]$ at a time k. $y[k]$ is the sum of a random Gaussian input $s[k]$ multiplied with the channel coefficient and a Gaussian independent and identically distributed (i.i.d) noise $n[k]$.

$$y[k] = hs[k] + n[k], \tag{1.1}$$

where h captures the effect of multipath fading. Without loss of generality we consider that $n[k]$ is an AWGN noise with zero mean and variance 1. When the destination can measure the fading process, i.e., the destination has instantaneous channel state information, with high accuracy, then the fading is considered as an additional channel output. The mutual information between S and D can be written as

$$\begin{aligned} I(s; y, h) &= I(s; h) + I(s; y|h) \\ &= I(s; y|h) \end{aligned} \tag{1.2}$$

where the second equality results from the fact that s and h are independent. We consider that $s[k]$ are i.i.d complex Gaussian variables with variance equal to P (i.e., the signal transmit power), then the received signal to noise ratio (SNR) is $P|h|^2$. We denote the channel gain by $\gamma = h^2$. The accurate CSI is available only at the destination. For a channel realization h, the mutual information in (1.2) becomes hence the channel capacity supported by this channel (Tse and Viswanath, 2005) :

$$\begin{aligned} C_{/\gamma} &= I(s; y|\gamma) \\ &= \log(1 + P\gamma), \end{aligned} \tag{1.3}$$

and the channel capacity of slow-fading channel is written as

$$\begin{aligned} C &= \mathbb{E}_\gamma[C_{/\gamma}] \\ &= \int_\Gamma \log(1 + P(\gamma)\gamma) f(\gamma) \, \mathrm{d}\gamma, \end{aligned} \tag{1.4}$$

where Γ is the set of γ and $f(\gamma)$ is the probability density function of γ.

If the destination can share the CSI with the source, then the source can adapt the transmit signal to the channel states. Thus, the capacity in (1.4) does not give any more the maximum reliable transmission rate. The source should optimally distribute the transmit power over time. The source would adapt the transmit power in time to conserve the power by allocating low (or null) power at low SNR, and transmitting at high power at high SNR. The system is subject to an average power constraint $\overline{\mathcal{P}}$

$$\int_\Gamma \mathcal{P}(\gamma) f(\gamma) \, \mathrm{d}\gamma \leq \overline{\mathcal{P}}. \tag{1.5}$$

The channel capacity becomes

$$\begin{aligned} C &= \max_{\int_\Gamma \mathcal{P}(\gamma) f(\gamma) \, \mathrm{d}\gamma \leq \overline{\mathcal{P}}} \mathbb{E}_\gamma[\log(1 + P(\gamma)\gamma)] \\ &= \max_{\int_\Gamma \mathcal{P}(\gamma) f(\gamma) \, \mathrm{d}\gamma \leq \overline{\mathcal{P}}} \int_\Gamma \log(1 + P(\gamma)\gamma) f(\gamma) \, \mathrm{d}\gamma, \end{aligned} \tag{1.6}$$

where $\mathcal{P}(.)$ is the power allocation function.

The optimal power distribution, that achieves the channel capacity, maximizes the average mutual information subject to the average power constraint $\overline{\mathcal{P}}$

$$\begin{aligned} \underset{\mathcal{P}(\gamma)}{\text{maximize}} \quad & \int_{\Gamma} \log\left(1+\mathcal{P}(\gamma)\gamma\right) f\left(\gamma\right)\, \mathrm{d}\gamma, \\ \text{subject to} \quad & \int_{\Gamma} \mathcal{P}(\gamma) f\left(\gamma\right)\, \mathrm{d}\gamma \leq \overline{\mathcal{P}}, \\ & \mathcal{P}(\gamma) \geq 0, \;\; \forall \gamma \in \Gamma, \end{aligned} \tag{1.7}$$

In the important case of discrete channel state (i.e., the channel state follows a stationary and ergodic stochastic process) the optimal power distribution that maximizes (1.7) is obtained by using the Lagrangian method and the Waterfilling algorithm as detailed in the next section.

1.1.2 Mutiple-Input Multiple-Output wireless system

There are two major problems in wireless communication namely the spectral bandwidth scarcity and the multipath effect. These problems make extremely difficult to design reliable wireless system with high rate transmission. To overcome these challenges, the MIMO technology was conceived. Indeed, MIMO offers the advantage to exploit the spatial dimension, in addition to the time and frequency dimensions, by using multiple antennas at the source and/or at the destination. MIMO systems introduce a new form of diversity known as space diversity. This technique exploits the multipath phenomena which was previously regarded as handicap. Indeed, the destination receives multiple copies of the same signal sent from multiple antennas and propagated over independent fading channels. The destination can hence take benefit from the received mutiple message copies to combat fading. MIMO technologies can be also used to enhance the transmission rate (keeping the same bandwidth and the same power) which is known as spatial multiplexing.

MIMO systems can provide a multiplexing gain and/or diversity gain :

- **Diversity gain** : in wireless communication, the received signal strength fluctuates randomly due to the multipath effect. If the destination receives multiple independent copies of the same signal, the probability to receive all signal copies with a deep fading decreases exponentially which improves the communication reliability. There are two widely used types of diversity techniques : time diversity (where multiple copies are sent at different times) and frequency diversity (where multiple copies are issued on several frequency bands). These two diversity techniques provides an inefficient spectrum use (Gesbert et al., 2003). The MIMO technology introduces the space diversity. The source uses the multiple transmit antennas to transmit various coded copies of the same message. The probability to detect at least one signal, with high SNR, hence increases. Hence, the spatial diversity combats appropriately the multipath fading without reducing the transmission rate. To define diversity quantitatively, we use the relationship between the received SNR, denoted by γ, and the probability of error, denoted by f_e. A tractable definition of the diversity order, or diversity gain, is

$$G_D = \lim_{\gamma \to \infty} \frac{\log(f_e(\gamma))}{\log \gamma} \tag{1.8}$$

  In other words, diversity order is the slope of the error probability curve in terms of the received SNR in a log-log scale. A wireless system consists of M-antenna source and N-antenna destination offers a spatial diversity on the order of $M \times N$.

- **Multiplexing gain** : a MIMO system with M-antenna source and N-antenna destination allows to transmit $\min(M, N)$ free-interference independent messages simultaneously on the same frequency. Since the destination receives a vector of N independent linear combinations of $\min(M, N)$ messages ($N >$

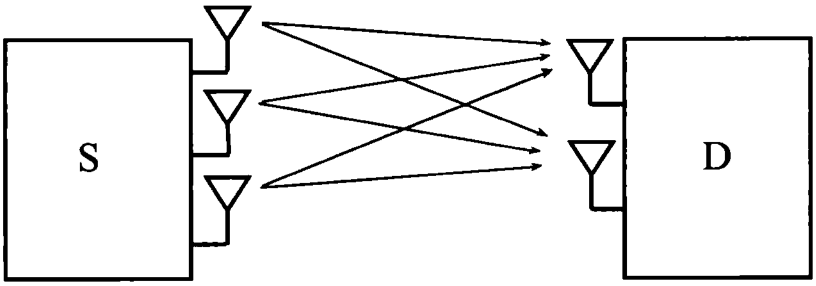


Figure 1.1: Point-to-point MIMO system

$\min(M, N)$), it can thus separate messages i.e., get free-interference messages. The destination can simply multiply the received vector of signals by the pseudo-inverse of the channel matrix in order to separate the messages i.e., get free-interference messages. The system becomes equivalent to $\min(M, N)$ parallel channels system. A metric, known as multiplexing gain, is defined in information theory to provides the number of free-interference independent messages that can be simultaneously transmitted over a MIMO system. The multiplexing gain or pre-log factor represents the rate of growth of the data rate with $\log(\gamma)$ when the signal to noise ratio tends to infinity. The $\log(\gamma)$ follows from the well known formula of the capacity for the single-users additive Gaussian noise channel, namely $\log(1 + \gamma)$, where γ tends to infinity.

$$M_G = \lim_{\gamma \to \infty} \frac{\text{Data rate}}{\log(\gamma)} \tag{1.9}$$

MIMO can be used to enhance the performance of the single-user and multi-user systems.

Single-user MIMO system

A single-user MIMO system consists of one multi-antenna source and one multi-antenna destination as illustrated in Fig. 1.1. In 1999, Telatar proved the benefits of single-user MIMO systems by providing the channel capacity of a multiple antenna system (Telatar, 1999) based on information theory. The given results show the capability of the MIMO technology to enhance the transmission rate. Since that, MIMO technology attracted the attention of wireless communication research community. In the following, we analyze the channel capacity of MIMO point-to-point system. The system consists of M-antenna source and N-antenna destination. At a time k, the source transmits a signal vector $\mathbf{s[k]} = \left(s[1,k], \;\; s[2,k], \;\; \ldots s[M,k]\right)^T$. Considering slow-fading, the signal model can be expressed in vector form as

$$\mathbf{y[k]} = \mathbf{Hs[k]} + \mathbf{n[k]}, \tag{1.10}$$

where $\mathbf{H}$ is $N \times M$ matrix of the fading channel coefficients and $[\mathbf{H}]_{nm} = h_{nm}$ is the channel coefficient of the link between the transmit antenna m and the received antenna n. $\mathbf{n[k]}$ is an $N \times 1$ Gaussian noise vector with identity covariance matrix (AWGN vector).

In (Telatar, 1999), Telatar derived the MIMO channel capacity for both cases when the channel state is perfectly known only at the destination and when the channel state is perfectly known at the source and at the destination. Considering that the channel state is only available at the destination, MIMO channel capacity can be achieved by equally distributing the transmit power over the transmit antennas. The MIMO channel capacity is then written as

$$\begin{aligned} C &= \log\left[\det\left(I_N + \frac{P}{M}\mathbf{HH}^*\right)\right] \\ &= \sum_{i=1}^{\min(M,N)} \log\left(1 + \frac{P}{M}\sigma_i^2\right), \end{aligned} \tag{1.11}$$

where P is the source transmit power and $\{\sigma_1, \sigma_2, \ldots, \sigma_{\min(M,N)}\}$ are the eigenvalues of $\mathbf{H}$. If the channel state is available at both the source and the destination, the source should optimally distribute the transmit power P over the transmit antennas. The optimal power distribution that maximizes the reliable communition rate can be obtained using the Waterfilling algorithm. The capacity formula becomes

$$C = \max_{\sum_{i=1}^{\min(M,N)} q_i = M} \sum_{i=1}^{\min(M,N)} \log\left(1 + \frac{P}{M}\sigma_i^2 q_i\right) \quad (1.12)$$

where $\mathbf{q} = \{q_1, q_2, \ldots, q_{\min(M,N)}\}$ is the power fraction vector. The optimal values of $\mathbf{q}$ can be derived using the Lagrangian method,

$$q_i = \left(\mu - \frac{M}{P\sigma_i^2}\right)^+, \forall i \in \{1, 2, \ldots, \min(M,N)\}, \quad (1.13)$$

where $(a)^+ = \max(0, a)$ and μ is the solution of $\sum_{i=1}^{\min(M,N)} \left(\mu - \frac{M}{P\sigma_i^2}\right)^+ = M$.

Anyhow, it is challenging to provide practical transmission techniques with near channel capacity performance and low complexity.

In general, MIMO schemes can be classified into two categories : a multiplexing gain schemes and diversity gain schemes. The first spatial multiplexing technique have been proposed in (Foschini, 1996) under the name of Bell Laboratories Layered Space-Time" (BLAST). Since that, Several variants of BLAST have been developed such us the vertical BLAST (VBLAST), the horizontal BLAST (HBLAST) and the diagonal BLAST (DBLAST).

The second category of MIMO schemes aims to enhance the diversity and thus to reduce the transmission error probability. A diversity technique that attracted a lot of attention is the space-time block coding (STBC). The first STBC technique is proposed in (Alamouti, 1998) and is named Almouti encoding. The main objective is to provide a diversity order equal to two for a system with two-antenna source and single-antenna destination with CSI knowledge only at the destination. This transmission technique is widely used in several standards due to its simplicity and efficiency. Several research groups have also been interested in Alamouti encoding technique and they extended it to the case of (Tarokh et al., 1999) where the

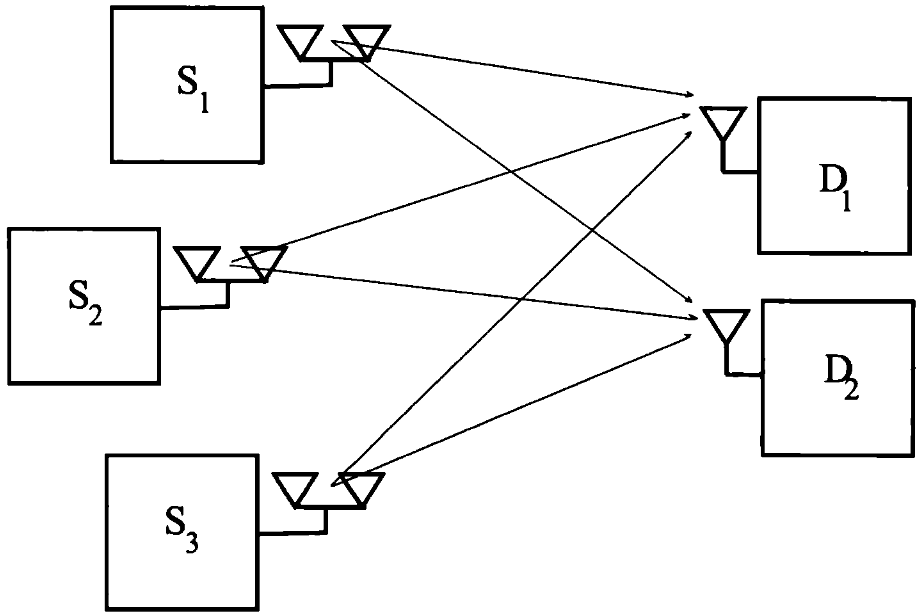


Figure 1.2: Multi-user MIMO system

Alamouti coding is applied for transmitters with more than two antennas.

Multi-user MIMO system

A multi-user system consists of multiple sources and multiple destinations as depicted in Fig. 1.2. MIMO technology allows a multi-antenna node to simultaneously communicate with multiple users. We focus, in the following, on the two important multi-user systems namely the multi-user access channel and the broadcast channel. A multi-user access channel consists of multiple transmitters that communicate with a single destination. When communications are done on the reverse direction, i.e., from one source to multiple destination, the system is said to be broadcast channel.

In the literature, the use of multi-antenna destination, in a MAC channel, is often called space-division multiple access (SDMA). The destination receives multiple linear combinations of transmitted signals and hence it can get free-interference signals. Many works deal with multiplexing gain of MAC channel. (Tse et al., 2004) shows that a MAC system, consisting of one N-antenna destination and two sources with M_1 and M_2 antennas, has a multiplexing gain equal to min $\left(M_1 + M_2, \quad N\right)$.

In (Jindal et al., 2004), the authors define a duality between the MAC and the broadcast channels (BCs). They show that the capacity region of the broadcast channel can be written in terms of the capacity region of the MAC channel, and vice versa. As a result, the multiplexing gain of the broadcast channel is equal to the one of the MAC channel. The two-user broadcast channel system, with M-antenna source and two destinations equipped with $\left(N_1, \quad N_2\right)$ antennas, has a multiplexing gain equal to min $\left(M, \quad N_1 + N_2\right)$ (Yu and Cioffi, 2004),(Viswanath and Tse, 2003). When sufficient knowledge of CSI are available at the source, the latest can perform the transmitted signal in the desired directions and null it out in the directions of other antennas or nodes.

1.2 Full-Duplex Radio

1.2.1 Main challenge in full-duplex transmission

During full-duplex transmission, the FD node receives its own transmitted signal (self-interference) interfered with the signal-of-interest transmitted by other distant nodes. As discussed in the introduction, the key idea to make full-duplex nodes feasible is to eliminate the self-interference before that the analog received signal is sampled by the ADC at the destination.

1.2.2 Self-interference mitigation

Several groups in academia and industry are interested to design and to implement full-duplex radio. Prior works have made significant progress on the self-interference cancellation problem. In (Choi et al., 2010)-(Jain et al., 2011), the authors design a new full-duplex radio. They propose analog and digital cancellation techniques that reduce the self-interference power. However, they provide at best 85dB of cancellation, which still leave about 25dB of residual self-interference. Recent works show that multi-antenna nodes can exploit space domain (spatial suppression) to eliminate the self-interference before reaching the received antenna and thus to perform full-duplex transmission.

The idea behind the spatial suppression is to beamform the transmit signal such that it is orthogonal to the local received antenna direction which completely eliminate the self-interference. Different methods were used for spatial suppression, some of them are described below. In (Suraweera et al., 2013), the authors propose antenna selection technique to partially eliminate the self-interference. The multi-antenna FD node selects the best receiver and transmit antennas combination in order to produce the lowest interference. In (Senaratne and Tellambura, 2011)-(Riihonen et al., 2011), the authors showed that FD nodes can completely avoid the self-interference using null-space beamforming techniques. In (Senaratne and Tellambura, 2011), the authors showed how a finite computational error can affect the FD transmission feasibility. A broad range of self-interference mitigation techniques, combining spatial processing, null-space beamforming, and time domain processing through a minimum mean square error filtering, have been investigated in (Riihonen et al., 2011). The authors showed that self-interference can also be eliminated when side information knowledge is imperfect.

1.3 Relay Channel

A relay is a wireless transceiver that has the ability to receive a wireless signal from one node then to forward it toward another node. In general, a relay channel consists of a source (S), a relay (R) and a destination (D) as shown in Fig. 1.3. The transmission process is performed in two phases. In the first phase, the source transmits a message to the relay. Then, the relay processes the received signal and forwards it to the destination in the second phase. The message s transmitted by the source is received by both the relay (y_R) and the destination (y_D). The destination receives a combination of the message transmitted by the source (s) and the one transmitted by the relay (r). The relay channel can be then represented by a channel output y_D, the channel input s, the relay's output y_R and the relay's input r. The relay channel was first introduced by Van Der Meulen in 1971 (Meulen, 1971). The author considers a relay channel with negligible direct link (i.e., the link between S and D), called two-hop relay channel. Since then, relay channels become an interesting subject in information theoretic perspective for a long time. Relay channel capacity was extensively studied. In (Cover and Gamal, 1979), the authors developed general strategies for relay networks and provided the relay channel capacity with channel side information knowledge. They defined the degraded relay channel as follows : "a relay channel is degraded if $f(y_D \mid s, r, y_R) = f(y_D \mid r, y_R)$,i.e., $s \to (r, y_R) \to y_D$ form a Markov chain". They examined some non-faded relay channels and they gave the capacity of the degraded relay channel. Moreover, they presented an upper and lower bounds for the general relay channel. In (Wang et al., 2005), the authors studied the capacity of MIMO relay channel. They proposed an algorithm to derive numerically an upper bound and a lower bound for MIMO relay channel capacity.

Relay channel has attracted attention due to its diverse and numerous practical

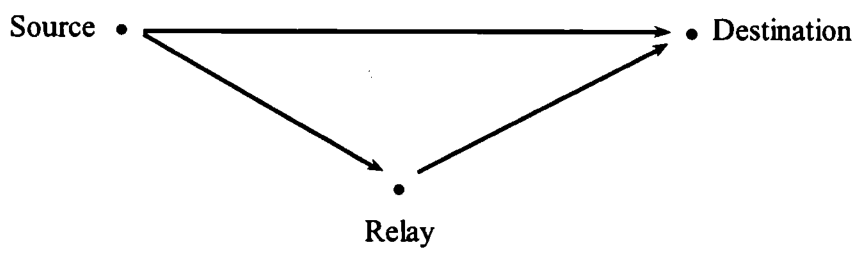


Figure 1.3: Relay channel

advantages. Relaying can be employed to divert traffic from congested area of cellular systems to cells with lower traffic load (Wu et al., 2001). Thanks to relay channels, ad-hoc networks provide higher network capacity proportional to the logarithm of the relay numbers (Gupta and Kumar, 2000)-(Gastpar and Vetterli, 2002).

Taking advantage of the broadcast nature of wireless channel and wireless multipath propagation, the relay channel can also enhance the transmission diversity (Sendonaris et al., 2003). Indeed, the destination can take benefit from the relayed message copy propagated over multiple paths channel independent of the direct link channel (i.e., between the source and the destination). The destination then may combine the redundant message copies which provides a diversity gain. The relay channel can be used to extend cell coverage (Pabst et al., 2004a)-(Host-Madsen and Zhang, 2005)-(Pabst et al., 2004b) and filling uncovered territories by forwarding the data signal to the areas on which the signal coming directly from the source cannot reach. All in all, relay channels are efficient in power consumption, and they lead to higher throughput. Fig. 1.3 depicted the basic system model of a relay channel.

In literature, several relaying algorithms have been proposed, called relaying strategies. Each one has advantages and disadvantages over the others. The most common relaying strategies are amplify-and-forward and decode-and-forward.

- **Amplify-and-forward (AF) :** The received signal by the relay do not go through the decoding process. The relay simply amplifies the received signal subject to relay power constraint then it retransmits the amplified signal. The noise signal associated to the desired signal is also amplified, which cause noise power amplification. AF requires low computational power and the shortest processing delay compared to the other relaying strategies. However, AF strategy is not efficient for multi-hop relay channel, due to noise power amplification at each relay node.
- **Decode-and-forward (DF) :** The relay decodes the received signal before retransmission and hence it requires higher processing delay than AF. The relay gives high SNR performance. It outperforms the other relaying strategies (Laneman et al., 2004). Therefore, DF strategy is widely preferred.

1.4 Full-Duplex Relaying

As defined above, a relay is a transceiver that has the ability to receive a signal from a node and forward it to another node. The deployment of a relay that handles the two processes simultaneously or alternate between them is the main difference between the FD and HD relay channels. In HD and FD relay channels, the same frequency band is used for both transmissions from S to R and from R to D. In HD relay channel, different time slots are occupied by each transmission (Gatzianas et al., 2007a) and hence the destination receives two various copies, orthogonal in time, of the same signal. This provides a diversity gain on the order of two. However, half of the time spent on the communication process is wasted in HD relay channel. This results in an inefficient spectrum use. A full-duplex relay node has the ability to receive and to transmit simultaneously over the same spectrum band. Therefore, full-duplex relaying is needed to enhance the system capacity and to improve the spectrum utilization efficiency .

Full-duplex relay channel capacity was thoroughly analyzed in (Cover and Gamal, 1979) under the assumption of no self-interference. The authors provide the degraded relay channel capacity and also a general upper bound on the relay channel capacity known as cut-set-bound. In (Simoens et al., 2009), the authors consider a MIMO (multiple input multiple output) relay channel and give upper and lower bounds of the capacity. Under the assumption of no self-interference, FD relay channel provides twice as much capacity as HD relay channel(Riihonen et al., 2009). In all those works, a key assumption is that the relay operates in FD mode with no self-interference. This assumption is too strong since FD relaying is greatly affected by relay self-interference. The self-interference should be eliminated before reaching the received antenna. Therefore the previous results can be seen as capacity upper bounds and the FD relay channel capacity should be reevaluated when a constraint on the received self-interference power is considered.

HD relaying and FD relaying performances have been compared in (Riihonen et al., 2009)-(Kang and Cho, 2009), without considering the effect of the direct link, showing that FD relaying achieves higher end-to-end data rates. In FD relay channel, the direct link between the source and the destination may not be negligible and then the signal transmitted by the source also acts, at the destination, as interference to the desired signal transmitted by the relay. In (Kwon et al., 2010), the authors showed the harmful effect of the direct link interference problem on the system outage probability. They also showed that, due to this interference, FD relaying only outperforms HD relaying in high signal to interference ratio (SIR) of the relay link to the direct link. For this reason, it is challenging to provide a transmission scheme that achieves the maximum mutual information in all SIR scenarios.

On the other hand, it is well known that when the transmitter has channel side information (CSI), such as the channel state and its probability density function (PDF) knowledge, it can adapt the transmit power in time in order to achieve capacity. In (Goldsmith and Varaiya, 1997), the authors showed how to optimally distribute the transmission power in time to achieve the fading-channel capacity with channel state knowledge and long-term average power constraint. Optimal adaptive relay power allocation in time for amplify-and-forward HD relay channel have also been studied in (Gatzianas et al., 2007b)-(Rodriguez et al., 2013), respectively. For a decode-and-forward FD relay channel, the relay should adapt its transmit power in time in order to achieve the maximum mutual information. Hence, the problem of power allocation in time should be addressed in order to find the optimal power distribution when the relay has full CSI.

1.5 Notations

Throughout this thesis, we use R, S and D to denote relay, source and destination respectively. $\|.\|$ denotes the 2-norm, $[.]^T$ and $[.]^*$ denote the transpose and the conjugate transpose operators respectively. $E[.]$ denotes the expectation operator. $|.|$ denotes the modulus operator of the complex number (.). h_{XY} denotes the channel coefficient between two antennas X and Y, $\gamma_{XY} = |h_{XY}|^2$ and $\overline{\gamma}_{XY} = E[\gamma_{XY}]$. The argument of a complex number is denoted by arg(.). $[x]^+$ denotes $\max(0, x)$. The PDF of a random variable x is denoted by $f(x)$. $C^I_{X,Y}$ (respectively, $I^I_{X,Y}$) denotes the channel capacity (respectively, maximum mutual information) when instantaneous CSI is available at the node X and Y. C^{R-D}_R denotes the channel capacity when instantaneous CSI of R–D link is available at R.

CHAPTER II

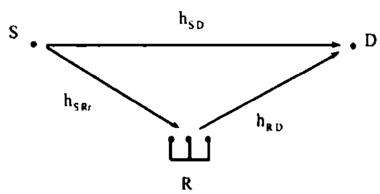
ON THE PERFORMANCE OF FULL-DUPLEX RELAY CHANNEL UNDER THE CONSTRAINT OF NULL SELF-INTERFERENCE POWER

Full-duplex transmission is feasible only through self-interference avoidance, before reaching the ADC at the front-end of the received antenna. In order to grant feasibility, a constraint on the received self-interference power have to be imposed on the full-duplex node and this constraint should be considered in analyzing the full-duplex relay channel capacity. The channel capacity is often studied under transmit power constraints. The channel capacity under receive power constraints was introduced in (Gastpar, 2007). Our contribution in this chapter is to provide an explicit form of FD degraded relay channel capacity under the constraint of null received self-interference power. Based on this result, we derive the maximum mutual information expression for the FD relay channel with the important decode-and-forward strategy and instantaneous CSI at the relay. We provide that the maximum mutual information of FD relay channel is twice the one of HD relay channel with single-antenna relay when the channel coefficients follows Rayleigh distribution.

The contents of this chapter have been submitted for publication in IEEE Transaction on Wireless Communication (TWC) (Chraiti et al., 2014a).

2.1 System Model

We consider a classical relay degraded channel models which consists of one single-antenna transmitter, one single-antenna destination and one relay. The relay takes advantage of MIMO technology to control the received self-interference power which typically requires two transmit antennas (R_{t1} and R_{t2}) more than the received antenna (R_r). The time is divided in fixed-size time slots (denoted by TS). The TS size is small enough so that the channel stays constant within a TS but varies independently from TS to another. Fig. 2.1 shows the relaying system considered in this paper, where $s[k]$ and $r[k]$ capture the source and the relay complex Gaussian inputs respectively, during k^{th} TS. The relay and the source transmitted signals are subject to an instantaneous power constraints P_R and P_S respectively.



	TS 1	TS 2	...	TS k	...
Source	s[1]	s[2]	...	s[k]	...
Relay	r[1]	r[2]	...	r[k]	...

Figure 2.1: Full duplex transmission process

2.1.1 Signal model

At a given TS (when not required, the TS index k is dropped), S transmits a codeword s while the relay transmits a codeword r which is depends on the previously received signals from the source. The relay may use a MIMO precoding technique to control the self-interference. The received signals at the relay and at the destination can then be written as

$$y_R = \sqrt{P_S} h_{SR_r} s + \sqrt{P_R} \mathbf{h}_R . \mathbf{w}^* r + n_R \quad (2.1)$$

$$y_D = \sqrt{P_R} \mathbf{h}_{RD} . \mathbf{w}^* r + \sqrt{P_S} h_{SD} s + n_D, \quad (2.2)$$

where $\mathbf{h}_R = \left(h_{R_{t1}R_r}, \ h_{R_{t2}R_r}, \right)$, $\mathbf{h}_{RD} = \left(h_{R_{t1}D}, \ h_{R_{t2}D} \right)$ and $\mathbf{w}$ is a precoding vector used to control the received self-interference power. n_R and n_D are the AWGN at R and D respectively. A part of the noise n_R depends on the received self-interference power. Without loss of generality, n_R is also assumed AWGN with unit variance when R_r does not receive any self-interference signal.

2.1.2 Self-interference pre-nulling

The relay can completely eliminate the self-interference when perfect CSI is available at the relay (Riihonen et al., 2010), i.e., the relay received antenna receives null self-interference power. Indeed, the relay can beamform the transmitted signal in the desired direction and null it out on relay received antenna direction. A simple beamforming technique can be used such as zero forcing beamforming. In this case, the signal received by the relay is written as

$$y_R = \sqrt{P_S} h_{SR_r} s + n_R. \quad (2.3)$$

2.2 Performance Analysis Under Null Received Self-Interference Power Constraint

In the following, we derive an explicit form of the channel capacity for FD degraded relay channel capacity, under the constraint of null received self-interference power. Based on the capacity formula, we then derive an explicit form of the maximum mutual information for a FD decode-and-forward relay channel with instantaneous CSI available only at the relay. We obviously assume that the channel state of S–D and R–D links is known at the destination. The system is subject to transmit power constraints (P_R and P_S) and a receive power constraint (null self-interference power).

2.2.1 Capacity analysis

We consider an FD degraded relay channel with instantaneous CSI at the source and the relay. Degraded relay channel was studied in (Cover and Gamal, 1979) considering just channel input constraints,i.e., the source and the relay transmissions are subject to transmit power constraints. The authors examined non-faded relay channels and they gave the capacity of the degraded relay channel when S, R and D are single-antenna nodes. From [Theorem 1, (Cover and Gamal, 1979)], the general form of the FD degraded relay channel capacity, with instantaneous CSI at the source and the relay, is written as

$$C^{I}_{S,R}(1) = \max_{0 \leq f(s,r) \leq 1} \min \left(I(s; y_R | r), I(s, r; y_D) \right). \tag{2.4}$$

where $f(s,r)$ is the joint probability of s and r. We denote $f(s,r)$ by β. Under transmit signal power constraint, the capacity obtained in (2.4) becomes

$$\begin{aligned} C^{I}_{S,R}(1) = \max_{0 \leq \beta \leq 1} \min \Bigg(& \log\left(1 + P_S(1-\beta)\gamma_{SR_r}\right), \\ & \log\left(1 + P_S\gamma_{SD} + P_R|\gamma_{RD}|^2 + 2\sqrt{\beta P_S \gamma_{SD} P_R |\gamma_{RD}|^2}\right) \Bigg). \end{aligned} \tag{2.5}$$

In order to present the next theorem, we present the QR decomposition of the channel matrix as $\mathbf{H} = \mathbf{G}\mathbf{Q}$, where $\mathbf{H} = \begin{pmatrix} h_{R_{t1}R_r} & h_{R_{t2}R_r} \\ h_{R_{t1}D} & h_{R_{t2}D} \end{pmatrix}$, $\mathbf{G} = \begin{pmatrix} G_1 & 0 \\ G_2 & G_{QR} \end{pmatrix}$ is a $\mathbb{C}_{2\times2}$ lower triangle matrix and $\mathbf{Q}$ is a unitary matrix. It is to be highlighted that the capacity formula given in the next theorem considers a constraint on the received self-interference power in addition to the constraint on the transmit power, whereas the formula obtained in (2.4) considers only a constraint on the transmit signal power.

Theorem 1. *Considering that the source and the relay have instantaneous CSI, for a given channel realization, the channel capacity of the FD degraded relay channel under null received self-interference power constraint can be written as*

$$C^{I}_{S,R}(1) = \max_{0\leq\beta\leq 1} \min\Bigg(\log\left(1 + P_S(1-\beta)\gamma_{SR_r}\right), \\ \log\left(1 + P_S\gamma_{SD} + P_R|G_{QR}|^2 + 2\sqrt{\beta P_S \gamma_{SD} P_R |G_{QR}|^2}\right)\Bigg). \quad (2.6)$$

Démonstration. To prove (2.6), we first prove that the capacity is higher or equal than the explicit form given in (2.6) and second we prove that it is lower or equal.

First, let us consider the relay precoded technique, based on QR decomposition, defined by the following precoded signal

$$\mathbf{u} = \mathbf{Q}^{*}\mathbf{P}\begin{pmatrix} r_1 \\ r_2 \end{pmatrix}, \quad (2.7)$$

where $\mathbf{P}$ is the relay power allocation matrix and r_1 and r_2 denote the two codewords transmitted to R_r and D, respectively. Especially, when $\mathbf{P} = \begin{pmatrix} 0 & 0 \\ 0 & \sqrt{P_R} \end{pmatrix}$ and $r_2 = r$, the received signals by R_r and D from the relay transmit antennas

(R_{t1} and R_{t2}), in noiseless environment, are as follows

$$\begin{aligned}\begin{pmatrix} y_{R_r} \\ y_D \end{pmatrix} &= \mathbf{H} \times \mathbf{u} \\ &= \begin{pmatrix} G_1 & 0 \\ G_2 & G_{QR} \end{pmatrix} \times \mathbf{Q} \times \mathbf{Q}^* \times \begin{pmatrix} 0 & 0 \\ 0 & \sqrt{P_R} \end{pmatrix} \begin{pmatrix} 0 \\ r \end{pmatrix} \\ &= \begin{pmatrix} 0 \\ \sqrt{P_R} G_{QR} r \end{pmatrix}. \end{aligned} \tag{2.8}$$

The QR decomposition with these previous parameters allow the relay to completely eliminate the self-interference and thus to satisfy the constraint of null received self-interference power. We get the following inequality

$$\begin{aligned}\max_{0\le\beta\le 1} \min \Bigg(\log\Big(1 + P_S(1-\beta)\gamma_{SR_r}\Big), \log\Big(1 + P_S\gamma_{SD} + \\ P_R|G_{QR}|^2 + 2\sqrt{\beta P_S \gamma_{SD} P_R |G_{QR}|^2}\Big)\Bigg) \le C^I_{S,R}(1)\end{aligned}, \tag{2.9}$$

which follows from the definition of the capacity obtained in (2.4).

Next, to prove the inequality in the reverse direction, a key step is to treat the transmission system formed by the relay and the destination as a broadcast channel system with two single-antenna receivers {R_r, D} and a two-antenna transmitter (R_{t1}, R_{t2}) as shown in Fig. 2.2. The link between the two-antenna transmitter and R_r (resp. D) will be denoted the link R-R_r (resp. R-D).

The capacity region of the MIMO broadcast channel is provided in (Weingarten et al., 2006). The authors showed that dirty paper coding (DPC) region coincides with the capacity region. In the case of a two-antenna transmitter and two single-antenna users, a transmission scheme that achieves capacity has been proposed in (Caire and Shamai, 2001). It is denoted by modified ranked known interference. This transmission scheme is based on the QR technique, DPC technique and power

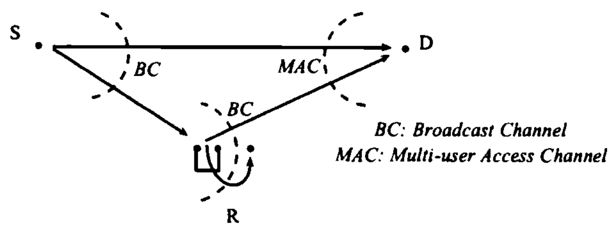


Figure 2.2: Full duplex relay system

allocation. The form of the precoded signal is $\mathbf{u} = \mathbf{Q}^* \begin{pmatrix} \sqrt{P_{R1}} & \sqrt{P_{R3}} \\ 0 & \sqrt{P_{R2}} \end{pmatrix} \begin{pmatrix} r_1 \\ r_2 \end{pmatrix}$. The resulting two received signals are

$$y_{R_r} = \sqrt{P_{R1}} G_1 r_1 + \sqrt{P_{R3}} G_1 r_2 + n_{R_r} \tag{2.10}$$

$$y_D = (\sqrt{P_{R3}} G_2 + \sqrt{P_{R2}} G_{QR}) r_2 + \sqrt{P_{R1}} G_2 r_1 + n_D \tag{2.11}$$

The signals r_1 and r_2 are destined to R_r and D respectively. Using the coding for known interference technique, the destination is able to decode r_2 as if the interference signal $\sqrt{P_{R1}} G_2 r_1$ is not present (Caire and Shamai, 2003). Let a_1 and a_2 denote two positive real variables. At a given transmit power matrix and fixed values of a_1 and a_2, the mutual information of links R–R_r and R–D are (Caire and Shamai, 2003)

$$I_D = \log\left(1 + P_{R3} a_2 |G_2|^2 + P_{R2} a_2 |G_{QR}|^2\right) \tag{2.12}$$

$$I_{R_r} = \log\left(1 + \frac{P_{R1} a_1 |G_1|^2}{1 + P_{R3} a_2 |G_1|^2}\right). \tag{2.13}$$

The capacity of the broadcast channel is obtained by solving the following optimization problem (Caire and Shamai, 2003)

$$\begin{aligned}
&\underset{\mathbf{P},a_1,a_2}{\text{maximize}} && I_D + I_{R_r} && (2.14a)\\
&\text{s.t} && P_{R1}a_1 + (P_{R2} + P_{R3})a_2 \leq P_R && (2.14b)\\
& && a_1 \geq 0 \,\text{and}\, a_2 \geq 0 \ . && (2.14c)
\end{aligned}$$

From (2.13) and (2.10), R_r receives a desired signal and an interference signal. Considering the constraint of null received self-interference imposed on the relay (i.e., null out both signals on R_r direction), we add the following two constraints to the optimization problem in (2.14),

$$\begin{cases} P_{R3}a_2|G_1|^2 = 0 \\ P_{R1}a_1|G_1|^2 = 0 \end{cases} . \tag{2.15}$$

Obviously, the channel gains in (2.15) are strictly greater than zero. Therefore, $P_{R3}a_2 = 0$ and $P_{R1}a_1 = 0$. Hence, the optimization problem obtained in (2.14) becomes

$$\begin{aligned}
&\underset{P_{R2},a_2}{\text{maximize}} && \log\left(1 + P_{R2}a_2|G_{QR}|^2\right) && (2.16a)\\
&\text{s.t} && P_{R2}a_2 \leq P_R && (2.16b)\\
& && a_2 \geq 0 \ . && (2.16c)
\end{aligned}$$

The optimal solution, $P_{R2}a_2 = P_R$, is trivial to obtain. Then, the capacity of the broadcast channel between the transmit antennas of R (R_{t_1} and R_{t_2}) and D, under null received self-interference power constraint, is the channel capacity of the link R–D which is written as

$$C_B = \log\left(1 + P_R|G_{QR}|^2\right) . \tag{2.17}$$

It is important to note that the capacity in (2.17) can be achieved by using the QR decomposition and power allocation. Considering the null received self-interference power and transmit power constraints, the channel capacity of the R–D link is obtained by C_B in (2.17) which is equivalent to the capacity of a Gaussian single-user system with no received power constraint when the channel coefficient is equal to G_{QR}. The system becomes equivalent to a relay channel with single-antenna relay and with no self-interference where the channel coefficient between the relay and the destination is equal to G_{QR}. Form (Cover and Gamal, 1979), we get an upper bound of the relay channel capacity under the constraint of null received self-interference power

$$
\begin{aligned}
C^I_{S,R}(1) \leqslant \max_{0\leq\beta\leq 1} \min\Bigg(& \log\left(1 + P_S(1-\beta)\gamma_{SR_r}\right), \\
& \log\left(1 + P_S\gamma_{SD} + P_R|\gamma_{RD}|^2 + 2\sqrt{\beta P_S\gamma_{SD}P_R|\gamma_{RD}|^2}\right)\Bigg).
\end{aligned} \tag{2.18}
$$

From inequalities (2.9) and (2.18), one can find (2.6). This completes the proof. □

Note that the previous results are obtained assuming that the signal is always transmitted via the direct and relay links. However, when the channel gain of the direct link is better than the one of the relay link, it is optimal for the relay to not relay signals. In this case, the above capacity reduces down to the capacity of the direct channel between the source and the destination. The capacity of this systems can then be expressed as

$$
C^I_{S,R} = \max\left(\log\left(1 + P_S\gamma_{SR_r}\right), C^I_{S,R}(1)\right). \tag{2.19}
$$

We will now derive the FD degraded relay channel capacity, under the constraint of null received self-interference, for the case where state of channels is not available at the source. Moreover, we consider that the relay has only knowledge of the channel state of the R–D link but not the channel state of the S–D link. This result is crucial to derive, in the next subsection, the explicit form of the maximum mutual information of FD relaying with the decode-and-forward strategy.

Theorem 2. *Considering that the relay has CSI of R-D link but not the CSI of the S-D link, the channel capacity of the FD degraded relay channel under the constraint of null received self-interference can be written as*

$$C_R^{R-D} = \min\left(\log\left(1 + P_S\gamma_{SR_r}\right), \log\left(1 + P_S\gamma_{SD} + P_R|G_{QR}|^2\right)\right). \tag{2.20}$$

Démonstration. We need to prove that when the CSI of the direct link is not available to the relay, the capacity is obtained by (2.6) with $\beta = 0$, i.e., r and s are independent.

We denote by $y_D^{'}$ the received signal by the destination multiplied with the factor $\frac{h_{SD}^*}{|h_{SD}|}$,

$$y_D^{'} = \sqrt{P_S}|h_{SD}|s + \sqrt{P_R}|G_{QR}|e^{j\theta}r + n_D^{'}, \tag{2.21}$$

where $\theta = \arg(h_{SD}^* G_{QR})$ and $n_D^{'} = \frac{h_{SD}^*}{|h_{SD}|}n_D$. The channel state of the link S–D is unknown at the source and at the relay, then θ is a random variable uniformly distributed in $[0, 2\pi[$. The channel capacity is then written as

$$C_R^{R-D} = \max_{0\leq\beta\leq 1} \min\left(I(s; y_R|r), E_\theta[I(s, r; y_D^{'} \mid \theta)]\right). \tag{2.22}$$

The second term of the min function in (2.22) is developed in (2.23).

$$
\begin{aligned}
&E_\theta[I(s,r;y_D^{'} \mid \theta)] \\
&= \log\left(1 + E_{s,r,\theta}[y_D^{'}(y_D^{'})^*]\right) \\
&= \log\left(1 + \begin{pmatrix} |h_{SD}| & |G_{QR}| \end{pmatrix} \begin{pmatrix} P_S & \sqrt{P_S P_R}\beta E_\theta\left[e^{-j\theta}\right] \\ \sqrt{P_S P_R}\beta E_\theta\left[e^{j\theta}\right] & P_R \end{pmatrix} \begin{pmatrix} |h_{SD}| \\ |G_{QR}| \end{pmatrix}\right) \\
&= \log\left(1 + P_S|h_{SD}|^2 + P_R|G_{QR}|^2\right).
\end{aligned} \tag{2.23}
$$

It is hence independent of β. Moreover, the first term $I(s;y_D|r) = \log(1 + P_S(1-\beta)|h_{SD}|^2)$ is maximum for $\beta = 0$ (i.e., s and r are independent). Thus, the capacity is achieved when the source transmits a codeword s independent of the codeword r transmitted by the relay. This completes the proof. □

2.2.2 Maximum Mutual Information Analysis

In this subsection, we consider that instantaneous CSI knowledge is only available at the relay. At a given time slot (k^{th} TS), the source transmits a new codeword, independent of the previous transmitted codewords. Simultaneously, the decode-and-forward FD relay decodes the k^{th} source codeword and transmits an estimated codeword from the signal received during the previous TS ($(k-1)^{th}$ TS). The maximum mutual information is given in the following theorem.

Theorem 3. *Considering that the relay has instantaneous CSI, for a given channel realization, the maximum mutual information of a decode-and-forward FD relay channel under null received self-interference power constraint can be written as*

$$I_R^I(1) = \min\left(\log\left(1 + P_S\gamma_{SR_r}\right), \log\left(1 + P_S\gamma_{SD} + P_R|G_{QR}|^2\right)\right). \tag{2.24}$$

Démonstration. The capacity obtained in (2.20) assumes that the relay has only the CSI of R-D and R-R_r links. Moreover, it is achieved when S and R transmit two independent codewords. Therefore, when the relay has the CSI of R-D, R-R_r and S-D links and when S and R transmit two independent codewords, then the system provides a maximum mutual information higher than the one when the CSI of the direct link is not available

$$\begin{aligned} I_R^I(1) &\geq C_R^{R-D} \\ &= \min\left(\log\left(1 + P_S\gamma_{SR_r}\right), \log\left(1 + P_S\gamma_{SD} + P_R|G_{QR}|^2\right)\right). \end{aligned} \tag{2.25}$$

On the other hand, when both S and R have instantaneous CSI and S and R transmit two independent codewords, the maximum mutual information is obtained by(2.6) with $\beta = 0$. When only the relay has instantaneous CSI, the system provides a lower maximum mutual information than when instantaneous CSI is available at S and R. We therefore have that

$$\begin{aligned} I_R^I(1) &\leq C_{S,R}^I(1)_{|\beta=0} \\ &= \min\left(\log\left(1 + P_S\gamma_{SR_r}\right), \log\left(1 + P_S\gamma_{SD} + P_R|G_{QR}|^2\right)\right). \end{aligned} \tag{2.26}$$

The inequalities (2.25) and (2.26) lead to (2.24). This completes the proof. □

It is optimal for the relay to not relay signals when the direct link gain is higher than the S-R link gain. In this case, the above mutual information reduces down to the capacity of the direct transmission between the S and D. The maximum mutual information can then be obtained by (2.27) :

$$I_R^I = \max\left(\log\left(1 + P_S\gamma_{SD}\right), I_R^I(1)\right). \tag{2.27}$$

In the rest of this paper, we denote the first term of the max function in (2.27) by $I_R^I(2) = \log\left(1 + P_S\gamma_{SD}\right)$.

2.3 Performance of FD Decode-and-Forward Relay Channel with Rayleigh Fading Channels

In this section, we focus on analyzing the maximum mutual information for the important case when the channel matrix $\mathbf{H}$ has independent Gaussian entries. The channel coefficients between R and D follow the Rayleigh distribution $\sim \mathcal{N}_{\mathbb{C}}(0, \overline{\gamma}_{RD})$. The transmit antennas of the relay are close to the receiver relay antenna and hence the direct path is important. Consequently, we consider that channels coefficient between transmit and received antennas of the relay follow Rician distribution $\sim \mathcal{N}_{\mathbb{C}}(a, \overline{\gamma}_R)$.

Lemma 1. *Let* $\boldsymbol{H} = \begin{pmatrix} \boldsymbol{h}_R \\ \boldsymbol{h}_{RD} \end{pmatrix}$ *a* $\mathbb{C}^{2\times 2}$ *channel matrix that have independent Gaussian entries where* $\boldsymbol{h}_R \sim \mathcal{N}_{\mathbb{C}}^{1\times 2}(a, \overline{\gamma}_R)$ *and* $\boldsymbol{h}_{RD} \sim \mathcal{N}_{\mathbb{C}}^{1\times 2}(0, \overline{\gamma}_{RD})$. *Let* G_{QR} *be the second diagonal element of* $\boldsymbol{G}$ *in the QR decomposition* $\boldsymbol{H} = \boldsymbol{GQ}$. *Then, the random variables* $\mid G_{QR} \mid^2$ *follows a Gamma distribution with the parameters* $(\overline{\gamma}_{RD}, 1)$.

Démonstration. See Appendix A. □

HD decode-and-forward relay systems are widely studied. Considering HD relay channel when the relay has one received antenna, the maximum mutual information is

$$I_{HD}^{I} = \frac{1}{2}\min(\log(1 + P_S\gamma_{SR_r}), \log(1 + P_S\gamma_{SD} + P_R\gamma_{RD}))). \tag{2.28}$$

In the case of Rayleigh channel distribution, γ_{RD} follows a Gamma distribution with parameters $(\overline{\gamma}_{RD}, 1)$. According to Lemma 1, $|G_{QR}|^2$ follows the same distribution as γ_{RD}. Thus, from (2.24) and (2.28), we can conclude that full-duplex relaying provides the twice as much maximum mutual information as HD relaying,i.e., $I_R^I(1) = 2 \times I_{HD}^I$.

2.4 Conclusion

In this chapter, we provided an explicit formula of the channel capacity for FD degraded relay channel under the constraint of null self-interference power. Moreover, we provided the exact form for maximum mutual information of FD decode-and-forward relay channel under the constraint of null received self-interference power constraint. We found that a full-duplex relay channel with three-antenna relay doubles the maximum mutual information compared to a half-duplex relay channel with single-antenna relay in the case where the channel follows Rayleigh distribution. However, in practice, the signal transmitted by the source interferes with the signal transmitted by the relay and thus it is very challenging to give the transmission scheme that achieves the maximum mutual information.

CHAPTER III

DISTRIBUTED ALAMOUTI FULL-DUPLEX RELAYING SCHEME WITH DIRECT LINK

In this chapter, we consider FD relay channel with decode-and-forward strategy and instantaneous CSI at the relay. We propose two transmission schemes based on distributed Alamouti encoding that provide a near maximum mutual information performance. These schemes mitigate the problem of direct link interference by appropriately combining at the destination the signal received via the direct link and its delayed copy received via the FD relay link. The first scheme is denoted by FD with distributed Alamouti encoding (FDAE) (Chraiti et al., 2013a). The performance of FDAE depends on the processing delay and FDAE give near optimal performance at low relay processing delay. The second scheme is denoted by modified FD distributed Alamouti encoding (MFDAE). Unlike FDAE, the performance of the second scheme does not depend on the processing delay and it achieves near-optimal performance at both low and high SIR between the relay and direct link.

The major part of this chapter have been published in the proceedings of IEEE Globecom 2013 (Chraiti et al., 2013a). The MFDAE scheme is not included in (Chraiti et al., 2013a) and it is a part of our paper submitted to IEEE TWC (Chraiti et al., 2014a).

3.1 System Model

We consider the system model described in 2.1. Especially, we consider FD relay channel with decode-and-forward strategy and instantaneous CSI at the relay. The relay decodes and forwards the received signal from the source with a processing delay $\tau > 0$. The relay uses the QR decomposition in order to completely eliminate the self-interference. The signals received by D and R at the k^{th} time slot can be respectively written as

$$y_D[k] = \sqrt{P_S}h_{SD}s[k] + \sqrt{P_R}G_{QR}s[k-1] + n_D[k]. \tag{3.1}$$

$$y_R[k] = \sqrt{P_S}h_{SR}s[k] + n_R[k]. \tag{3.2}$$

It can be observed that the relayed codeword ($s[k-1]$) interferes, at the destination, with the new codeword ($s[k]$) transmitted by the source. The channel capacity of the link between the source and the relay is $\log(1+P_S\gamma_{SR_r})$. Thus, the source can transmit, error-free signal, to the relay with a rate equal to $\log(1+P_S\gamma_{SR_r})$ and hence the end-to-end achievable data rate of the relayed signal is as follows (Laneman et al., 2004)

$$\min\left(\log(1+P_S\gamma_{SR_r}), \log\left(1+\frac{P|G_{QR}|^2}{1+P_S\gamma_{SD}}\right)\right). \tag{3.3}$$

When the direct link (S–D) is better than the link S–R i.e.,$\gamma_{SR_r} < \gamma_{SD}$, relaying signal does not enhance the end-to-end achievable data rate. The relay does not relay signals when $\gamma_{SR_r} < \gamma_{SD}$ and hence the end-to-end achievable data rate becomes

$$R^{I}_{R(FDI)} = \max\left\{\log\left(1+P_S\gamma_{SD}\right), \min\left(\log(1+P_S\gamma_{SR_r}), \log\left(1+\frac{P|G_{QR}|^2}{1+P_S\gamma_{SD}}\right)\right)\right\}. \tag{3.4}$$

3.2 Full-Duplex Relaying with Alamouti Encoding (FDAE)

The proposed FDAE scheme is based on the Alamouti coding technique to eliminate the direct link interference and to efficiently combine the received copies of each signal at the destination. Without loss of generality, we consider that the TS duration (denoted by d_{TS}) is an integer multiple of τ. Each TS is divided in $L+1$ sub-slots with equal durations. The k^{th} message, transmitted during the k^{th} TS by the source, is divided into L equal size codewords denoted by $s[k,l]$ where $l \in \{1, 2, ..., L\}$. During one time slot, the source can transmit $L = \frac{d_{TS}}{\tau} - 1$ codewords.

In this section, we start by explaining this transmission scheme for a processing delay equal to $\tau = \frac{d_{TS}}{3}$. Afterwards, we generalise the FDAE transmission process to any value of τ. Considering that $\tau = \frac{d_{TS}}{3}$, the source then transmits $L = 2$ codewords ($s[k,1]$ and $s[k,2]$) at the k^{th} TS. The transmission of the proposed FDAE scheme occurs in three phases.

- **Transmission phase 1**

At the first sub-slot, the source transmits the codeword $s[1]$, while the relay transmit antennas stay silent. The received signal by R and D are written as :

$$y_R[k,1] = \sqrt{P_S} h_{SR} s[k,1] + n_R[k,1]. \quad (3.5)$$

$$y_D[k,1] = \sqrt{P_S} h_{SD} s[k,1] + n_D[k,1]. \quad (3.6)$$

- **Transmission phase 2**

At the first sub-slot of the $(k)^{th}$ TS, the relay receives the codeword $s[k,1]$ from the source. The relay processes $y_R[k,1]$ to decode $s[k,1]$ which is then retransmitted by the relay during the second sub-slot of the k^{th} TS. Simultaneously, the source transmits a new codeword $s[k,2]$, which is also received and decoded by the relay. We denote by P the transmit power assigned by the relay to transmit $s[k,1]$. The signal received by the relay during the second sub-slot of the k^{th} TS is written as :

$$y_R[k,2] = \sqrt{P_S} h_{SR} s[k,2] + n_R[k,2]. \tag{3.7}$$

and the received signal by the destination is written as :

$$y_D[k,2] = \sqrt{P} G_{QR} s[k,1] + \sqrt{P_S} h_{SD} s[k,2] + n_D[k,2]. \tag{3.8}$$

– **Transmission phase 3**

At the third sub-slot, the relay encodes and transmits weighted versions of $s^*[k,1]$ and $-s^*[k,2]$ in order to perform, with $y_D[k,2]$, an orthogonal matrix of signals at the destination (Alamouti code). Let us define α_1 and α_2 as the weight components assigned to $s^*[1,k]$ and $-s^*[k,2]$ respectively. The vector of signals received by the destination at the second and third sub-slots is then :

$$\begin{pmatrix} y_D[k,2] \\ y_D^*[k,3] \end{pmatrix} = \begin{pmatrix} \sqrt{P} G_{QR} & \sqrt{P_S} h_{S,D} \\ \alpha_1^* G_{QR} & \alpha_2^* G_{QR} \end{pmatrix} \begin{pmatrix} s[k,1] \\ s[k,2] \end{pmatrix} + \begin{pmatrix} n_D[k,2] \\ n_D^*[k,3] \end{pmatrix}. \tag{3.9}$$

The relay suitably adjusts the weight components (α_1 and α_2) to get an orthogonal equivalent channel matrix as follows. We start by adjusting the arguments of the weight components by solving the following system :

$$\begin{cases} \arg(\alpha_1) = \arg(h_{S,D}) \\ \arg(\alpha_2) = 0 \end{cases}$$

$$\Rightarrow \begin{cases} \alpha_1 = \dfrac{h_{S,D}}{|h_{S,D}|} \\ \alpha_2 = 1 \end{cases}. \tag{3.10}$$

The total energy used by the relay during two sub-slots is equal to $2P_R$. The relay adjusts the transmit powers ($|\alpha_1|^2$,$|\alpha_2|^2$ and P), with respect to this energy constraint, by solving the following system :

$$\begin{cases} |\alpha_1|^2 |G_{QR}|^2 = P_S |h_{S,D}|^2 \\ |\alpha_2|^2 |G_{QR}|^2 = P |G_{QR}|^2 \\ P + |\alpha_1|^2 + |\alpha_2|^2 = 2P_R \end{cases}$$

$$\Rightarrow \begin{cases} P = \frac{1}{2}\left(2P_R - P_S\frac{|h_{S,D}|^2}{|G_{QR}|^2}\right) \\ \mid \alpha_1 \mid^2 = P_S\frac{|h_{S,D}|^2}{|G_{QR}|^2} \\ \mid \alpha_2 \mid^2 = P \end{cases} . \tag{3.11}$$

Since P_R is the same order of magnitude than P_S and usually $\frac{|h_{S,D}|^2}{|G_{QR}|^2} \ll 1$, then P is most of the time greater than zero. Otherwise, if the direct link is better than the relay link (i.e., $\frac{P_S|h_{S,D}|^2}{P_R|G_{QR}|^2} \geq 1$), the relay does not retransmit the signal ($P = 0$), as explained previously, and the achievable data rate becomes equal to the capacity of the direct link. The received vector of signals during one TS can be obtained by combining (3.6), (3.9), (3.10) and (3.11) as follows :

$$\begin{pmatrix} y_D[k,1] \\ y_D[k,2] \\ y_D^*[k,3] \end{pmatrix} = \begin{pmatrix} \sqrt{P_S}h_{SD} & 0 \\ \sqrt{P_1}G_{QR} & \sqrt{P_S}h_{S,D} \\ \sqrt{P_S}h_{SD}^* & -\sqrt{P}G_{QR}^* \end{pmatrix} \begin{pmatrix} s[k,1] \\ s[k,2] \end{pmatrix} + \begin{pmatrix} n_D[k,1] \\ n_D[k,2] \\ n_D^*[k,3] \end{pmatrix} . \tag{3.12}$$

We denote by $\mathbf{H}$ the equivalent channel matrix defined in (3.12). It can be observed that, as desired, $\mathbf{H}$ is orthogonal. The destination then uses the Alamouti detector to decode the codewords. That is, the destination multiplies the vector of received signals by $\mathbf{H}^*$. The achievable end-to-end data rate of the considered relay channel is the sum of the achievable date rate associated to the two relayed signals ((s[k,1], s[k,2]) at the kth TS)

$$\frac{1}{3}\min\left(2\log(1+P_S\gamma_{SR_r}), \log(1+P|G_{QR}|^2+2P_S\gamma_{SD}) + \log(1+P|G_{QR}|^2+P_S\gamma_{SD})\right). \tag{3.13}$$

The achievable pre-log factor is

$$\lim_{P_S+P_R\to\infty} \frac{\frac{1}{3}\min\Big(2\log(1+P_S\gamma_{SR_r}), \log(1+P|G_{QR}|^2+2P_S\gamma_{SD})+\log(1+P|G_{QR}|^2+P_S\gamma_{SD})\Big)}{\log(P_S+P_R)}$$
$$= \frac{1}{3}\min(2,2) = \frac{2}{3} \tag{3.14}$$

Moreover, if the direct link is better than the relay link (i.e., $\frac{P_S|h_{S,D}|^2}{P_R|G_{QR}|^2} \geq 1$), the relay does not retransmit the signal ($P = 0$) and the achievable data rate becomes equal to the capacity of the direct link. Hence, the achievable end-to-end data rate can be given by :

$$R_R^I(FDAE)_{/(\tau=\frac{d_{TS}}{3})} = \max\Bigg\{ \log(1+P_S\gamma_{SD}), \frac{1}{3}\min\Big(2\log(1+P_S\gamma_{SR_r}),$$
$$\log(1+P|G_{QR}|^2+2P_S\gamma_{SD})+\log(1+P|G_{QR}|^2+P_S\gamma_{SD})\Big)\Bigg\} \tag{3.15}$$

It is important to note that the system performance remains the same as when a pseudo-inverse channel matrix is applied at D since $\mathbf{H}$ is orthogonal.

This transmission scheme can be extended to the general case as follows. Without loss of generality, we assume that the number of transmit codewords L is even (if L is odd, the relay just retransmits at the last sub-slot the last received codewords). The transmitted message during one TS is divided into blocks of two codewords. The transmission process of each block of codewords follows the three transmission phases described above. However, at the third transmission phase of the b^{th} ($b = 1, \ldots, \frac{L}{2}-1$) block, the source simultaneously starts the first transmission phase for the $(b+1)^{th}$ block which is treated as interference by the destination.

Accordingly, the relay adjusts the transmit powers and the weight components exactly as in (3.10) and (3.11). At the end of each TS, the relay multiplies the received signals vector with a pseudo-inverse matrix $(\mathbf{H}^*\mathbf{H})^{-1}\mathbf{H}^*$ to get free-interference codewords. The end-to-end achievable data rate of the systems is

$$R^{I}_{R(FDAE)/\tau=\frac{d_{TS}}{L+1}} = \max\left\{ \log(1+P_S\gamma_{SD}), \frac{1}{L+1}\min\Big(L\log\left(1+P_S\gamma_{SR_r}\right), \log\left(\det(I_{L\times L}+\mathbf{H}^*\mathbf{H})\right)\Big)\right\}. \tag{3.16}$$

3.3 Modified Full-Duplex Relaying with Alamouti Encoding (MF-DAE)

Considering an error-free decode-and-forward FD relay channel, the received signal by D at the k^{th} time slot is given by 3.1. It can be observed that the relayed codeword ($s[k-1]$) interferes with the new codeword ($s[k]$) transmitted by the source. In this section, we propose a modified FD relaying with distributed Alamouti encoder (MFDAE) scheme that efficiently combines the signal received via the direct link and its delayed copy received via the relay link. We show that the MFDAE performance is independent of the processing delay. We show using numerical results section that this scheme achieves near-optimal performances.

We consider that each TS is divided into two sub-slots with equal duration. The transmitted messages during the k^{th} TS, is divided into two equal size codewords denoted by $s[k,1]$ and $s[k,2]$. Without loss of generality, we describe the MFDAE scheme process during the k^{th} TS. The transmission occurs in two phases.

– **Transmission phase 1**

At the second sub-slot of the $(k-1)^{th}$ TS, the relay receives the codeword $s[k-1,2]$ from the source. The relay processes $y_R[k-1,2]$ to decode $s[k-1,2]$ which is then forwarded by the relay during the first sub-slot of the k^{th} TS. Simultaneously, the source transmits a new codeword $s[k,1]$, which is also received and decoded by the relay. We denote by P the transmit power assigned by the relay to transmit $s[k-1,2]$. The signal received by the relay during the first sub-slot of the k^{th} TS is written as

$$y_R[k,1] = \sqrt{P_S} h_{SR_r} s[k,1] + n_R[k,1] \tag{3.17}$$

and the signal received by the destination is written as :

$$y_D[k,1] = \sqrt{P_S} h_{SD} s[k,1] + \sqrt{P} G_{QR} s[k-1,2] + n_D[k,1]. \tag{3.18}$$

– **Transmission phase 2**

At the second sub-slot, the relay encodes and transmits weighted versions of $s^*[k-1,2]$ and $-s^*[k,1]$ in order to perform, with $y_D[k,1]$, an orthogonal matrix of signals at the destination (Alamouti code). Let us define α_1 and α_2 as the weight components assigned to $s^*[k-1,2]$ and $-s^*[k,1]$ respectively. During the second sub-slot, the source transmits a codeword $s[k,2]$. The vector of signals received by the destination, at the first and second sub-slots is then written as :

$$\begin{pmatrix} y_D[k,1] \\ y_D^*[k,2] \end{pmatrix} = \begin{pmatrix} \sqrt{P} G_{QR} & \sqrt{P_S} h_{SD} \\ \alpha_1^* G_{QR} & \alpha_2^* G_{QR} \end{pmatrix} \begin{pmatrix} s[k-1,2] \\ s[k,1] \end{pmatrix} + \begin{pmatrix} n_D[k,1] \\ \sqrt{P_S} h_{SD}^* s^*[k,2] + n_D^*[k,2] \end{pmatrix}. \tag{3.19}$$

The MFDAE process is depicted in Fig. 3.1. It describes the signals transmitted by the source and the relay during two TSs. It shows the interference and desired signals that will be received by the relay.

	[k,1]	[k,2]	[k+1,1]	[k+1,2]
Source	s[k,1] (Desired)	s[k,2] (Interference)	s[k+1,1] (Desired)	s[k+1,2] (Interference)
Relay	s[k-1,2] (Desired)	{-s*[k,1],s*[k-1,2]} (Desired)	s[k,2] (Desired)	{-s*[k+1,1],s*[k,2]} (Desired)

Interference signal   Desired signal

Figure 3.1: MFDAE transmission process during two TSs

The relay suitably adjusts the weight components (α_1 and α_2) and the transmit power P to get an orthogonal equivalent channel matrix as follows. We start by adjusting the transmitted signals arguments by solving the following system :

$$\begin{cases} \arg(\alpha_1) = \arg(h_{S,D}) \\ \arg(\alpha_2) = 0 \end{cases} . \tag{3.20}$$

The total energy used by the relay during two sub-slots is equal to $2P_R$. The relay thus adjusts the transmit powers ($|\alpha_1|^2$,$|\alpha_2|^2$ and P), with respect to this energy constraint, by solving the following system :

$$\begin{cases} |\alpha_1|^2 |G_{QR}|^2 = P_S|h_{S,D}|^2 \\ |\alpha_2|^2 |G_{QR}|^2 = P|G_{QR}|^2 \\ P + |\alpha_1|^2 + |\alpha_2|^2 = 2P_R \end{cases}$$

$$\Rightarrow \begin{cases} P = \frac{1}{2}\left(2P_R - P_S\frac{|h_{S,D}|^2}{|G_{QR}|^2}\right) \\ |\alpha_1|^2 = P_S\frac{|h_{S,D}|^2}{|G_{QR}|^2} \\ |\alpha_2|^2 = P \end{cases} . \tag{3.21}$$

Since P_R is the same order of magnitude than P_S and usually $\frac{|h_{S,D}|^2}{|G_{QR}|^2} \ll 1$, then P is most of the time greater than zero. Otherwise, if the direct link is better than the relay link (i.e., $\frac{P_S|h_{S,D}|^2}{P_R|G_{QR}|^2} \geq 1$), the relay does not retransmit the signal ($P = 0$), as explained previously, and the achievable data rate becomes equal to the capacity of the direct link. Considering that the relay forwarded the signals (i.e., $P > 0$), the received vector of signals during one TS, can be obtained by combining (3.19), (3.20) and (3.21) as follows :

$$\begin{pmatrix} y_D^*[k,1] \\ y_D^*[k,2] \end{pmatrix} = \begin{pmatrix} \sqrt{P}G_{QR} & \sqrt{P_S}h_{S,D} \\ -\sqrt{P_S}h_{S,D}^* & \sqrt{P}G_{QR}^* \end{pmatrix} \begin{pmatrix} s[k-1,2] \\ s[k,1] \end{pmatrix} + \begin{pmatrix} n_D[k,1] \\ \sqrt{P_S}h_{SD}^* s^*[k,2] + n_D^*[k,2] \end{pmatrix} . \tag{3.22}$$

It can be observed that the equivalent channel matrix in (3.22) is orthogonal, as desired. The destination then uses the Alamouti detector to decode codewords. That is, the destination multiplies the vector of received signals by $\mathbf{H}^*$. The channel capacity of the link S-R is $\log(1 + P_S\gamma_{SR_r})$. Thus, the source can transmit an error-free signal to the relay with a rate equal to $\log(1 + P_S\gamma_{SR_r})$. The achievable end-to-end data rate of the considered relay channel is the sum of the achievable date rate associated to the two relayed signals $((s[k-1,2], s[k,1])$ at the k^{th} TS)

$$\min\left(\log(1+P_S\gamma_{SR_r}), \frac{1}{2}\log\left(1+\frac{\left(P|G_{QR}|^2+P_S\gamma_{SD}\right)^2}{P|G_{QR}|^2+P_S\gamma_{SD}+P_S^2\gamma_{SD}^2}\right)\right. \\ \left.+\frac{1}{2}\log\left(1+\frac{\left(P|G_{QR}|^2+P_S\gamma_{SD}\right)^2}{P|G_{QR}|^2+P_S\gamma_{SD}+P|G_{QR}|^2P_S\gamma_{SD}}\right)\right)\Bigg\}. \tag{3.23}$$

Assuming that the relay can decide to forward the signal according to the channel state knowledge, the relay does not relay signals when $\gamma_{SR_r} < \gamma_{SD}$. The achievable end-to-end data rate, of the MFDAE scheme, then becomes as (3.24).

$$R_R^I \\ = \max\left\{\log\left(1+P_S\gamma_{SD}\right), \min\left(\log(1+P_S\gamma_{SR_r}), \frac{1}{2}\log\left(1+\right.\right.\right. \\ \left.\frac{\left(P|G_{QR}|^2+P_S\gamma_{SD}\right)^2}{P|G_{QR}|^2+P_S\gamma_{SD}+P_S^2\gamma_{SD}^2}\right)+\frac{1}{2}\log\left(1+\frac{\left(P|G_{QR}|^2+P_S\gamma_{SD}\right)^2}{P|G_{QR}|^2+P_S\gamma_{SD}+P|G_{QR}|^2P_S\gamma_{SD}}\right)\Bigg)\Bigg\}. \tag{3.24}$$

In section 3.4, we compare the achievable data rate given in (3.16)and(3.24) to the maximum mutual information with instantaneous CSI at the relay given in (2.27).

3.4 Numerical Results

In this subsection, we evaluate the performance of the FDAE and MFDAE schemes, and we compare them to the maximum mutual information and to the achievable data rate when the signal received via direct link is considered as interference (FD relaying with interference denoted as FDI). Moreover, we compare the performance to the transmission scheme proposed in (Krikidis and Suraweera, 2013) denoted by FD relaying with Alamouti code. MATLAB software tool is used for numerical results. The noise variance is set to $N_0 = 1$. We consider Rayleigh channel. The channel gain of the links S–R and R–D follow an exponential distribution with average given by $\overline{\gamma}_{SR} = \overline{\gamma}_{RD} = 0$ dB. We assume that S–D link is poor but it is not negligible. Unless mentioned otherwise, the average channel gain of the S–D link is set to $\overline{\gamma}_{SD} = -10dB$. We analyze the performance of the FDAE scheme for two values of the processing delay τ; a low processing delay $\tau = \frac{d_{TS}}{11}$ and high processing delay $\tau = \frac{d_{TS}}{3}$.

Fig. 3.2 shows the end-to-end achievable data rate versus P_R for a fixed value of $P_S = 10$dB. Fig. 3.3 shows the end-to-end achievable data rate as function of $\overline{\gamma}_{SD}$, when the transmit powers are set to $P_R = 10$dB and $P_S = 10$dB. FDI results show the harmful effect of the interference signal received via direct link on the end-to-end achievable data rate. Especially, at low and medium P_R to $P_S\overline{\gamma}_{SD}$ ratio, the interference at the destination becomes higher and hence the difference between the maximum mutual information and FDI end-to-end achievable data rate becomes more important. For a relay processing delay $\tau = \frac{d_{TS}}{3}$. The transmission schemes FDAE and FD relaying with Alamouti encoding efficiently combine each transmitted signal and its delayed copy. However, it achieves a pre-log factor equal to $\frac{2}{3}$ and hence it does not achieve near optimal performance. The performance of MFDAE is independent of the processing delay and outperforms FDAE at higher processing delay. Even if FD with Alamouti encoding schemes outperform FDI scheme at low and medium P_R to $P_S\overline{\gamma}_{SD}$ ratio, it is not optimal when this ratio becomes high and it provides lower performance, as shown in Fig. 3.2 and 3.3, due to the low achieved pre-log factor.

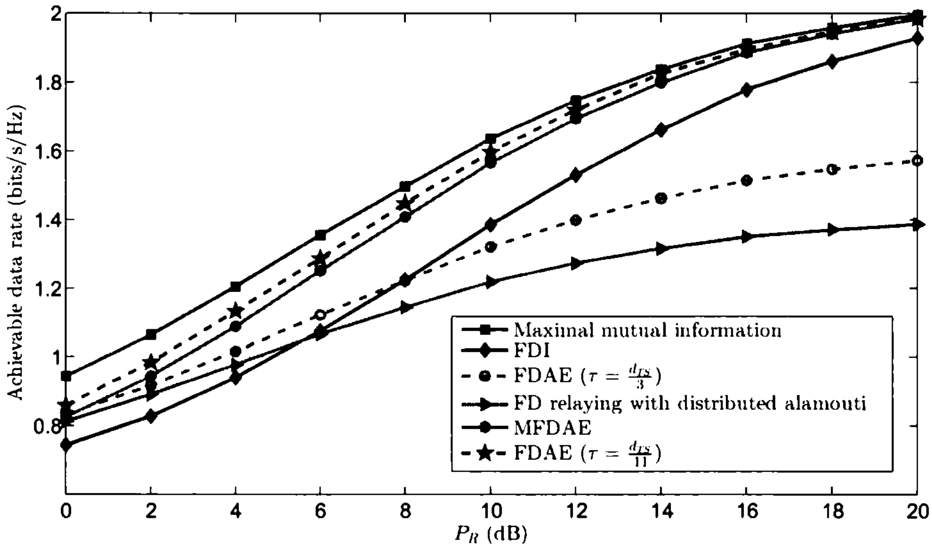


Figure 3.2: End-to-end achievable data rate versus P_R (P_S = 20dB)

At lower processing delay such as $\tau = \frac{d_{TS}}{11}$, the transmission scheme FDAE provides a pre-log factor near to 1 and outperforms the other schemes. Meanwhile, the proposed MFDAE scheme outperforms the FDI and FD with Alamouti encoding schemes and achieves near optimal performance for all cases independently to the processing delay as shown in Fig. 3.2 and Fig. 3.3.

3.5 Conclusion

In full duplex relaying systems, the destination suffers from interference caused by the received signal via the direct link when the processing delay is not negligible.

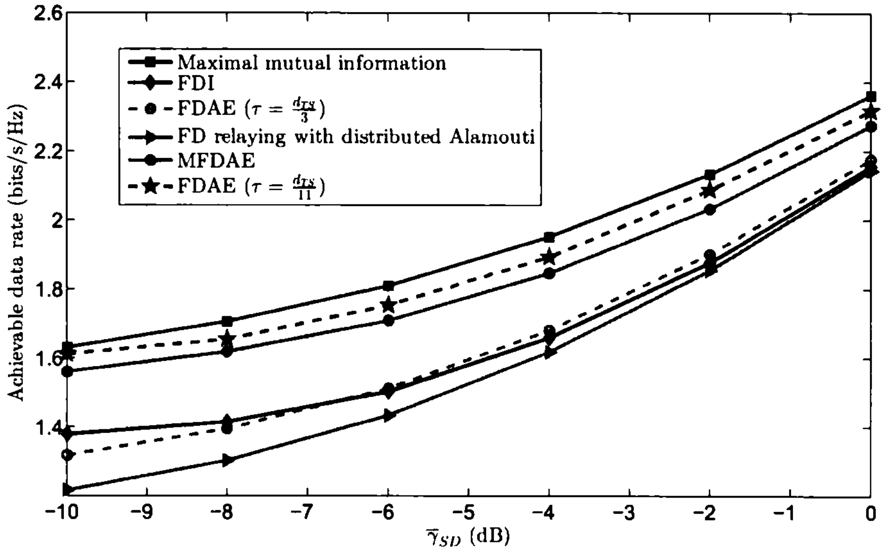


Figure 3.3: End-to-end achievable data rate versus $\overline{\gamma}_{SD}$ ($P_S = P_R = 10$dB)

In order to enhance the system end-to-end achievable data rate, we proposed in this chapter two novel FD relaying schemes based-on distributed Alamouti encoding which efficiently combine each signal and its delayed copy. Numerical results show that the proposed FDAE scheme ensures near-optimal performance and outperforms the MFDAE scheme at low processing delay. Meanwhile, the proposed MFDAE scheme achieves near optimal performance for all cases independently of the processing delay and outperform the FDAE scheme at high processing delay.

CHAPTER IV

OPTIMAL LONG-TERM POWER ADAPTATION FOR FD DECODE-AND-FORWARD RELAY CHANNEL

In this chapter, we consider that the relay has full CSI knowledge (i.e., channel state and channel gain PDF for all links are available at the relay). Hence, (2.27) does not give anymore the maximum mutual information. The relay should adapt its transmit power in time in order to achieve the maximum mutual information. We formulate the problem of long-term relay power adaptation and we derive the optimal relay power distribution over time. In addition, we propose an algorithm that provides the exact optimal solution for the important case of a discrete channel distribution. The performance of our proposed scheme is compared to the DF relay channel with fixed relay transmit power.

The contents of this chapter have been published in IEEE Wireless Communications Letters (WCL) (Chraiti et al., 2014b).

4.1 Capacity of Fading Channel with Channel Side Information

The capacity of point to point wireless communication system with a time-varying channel was introduced by Jacob (Jacob, 1964) assuming that the channel side information is known at both transmitter and receiver. Let γ refers to a channel gain to noise ratio which takes values in the set Γ. The probability that the channel gain to noise ratio be in the state γ is denoted by $f(\gamma)$. If we consider a fixed transmit power in time, the capacity of the considered system, is then obtained by [(Jacob, 1964), Theorem 4.6.1] :

$$C = \mathbb{E}_\gamma [C_\gamma] = \int_\Gamma C_\gamma f(\gamma) \, \mathrm{d}\gamma., \tag{4.1}$$

where C_γ is the instantaneous system capacity when the channel gain to noise ratio is in state γ.

If we consider a long-term average power constraint, we cannot apply (4.1) to obtain the system capacity. The transmit power should be optimally distributed in time in order to achieve the fading channel capacity. Given an average power constraint, the optimal power distribution $\mathcal{P}(\gamma)$ that gives the fading channel capacity with channel side information, is obtained by solving the following system

$$\begin{aligned} \underset{\mathcal{P}(\gamma)}{\text{maximize}} \quad & \int_\Gamma \log\left(1 + \mathcal{P}(\gamma)\gamma\right) f(\gamma) \, \mathrm{d}\gamma \\ \text{s.t} \quad & \int_\Gamma \mathcal{P}(\gamma) f(\gamma) \, \mathrm{d}\gamma \leq \overline{\mathcal{P}} \\ & \mathcal{P}(\gamma) \geq 0. \end{aligned} \tag{4.2}$$

where $\overline{\mathcal{P}}$ is the average transmit power.

4.2 Problem Formulation

We start by formulating the optimization problem to derive the optimal power distribution that maximizes the average mutual information in time subject to a long-term average power constraint $\overline{\mathcal{P}_R}$. We use the following notations.

- Γ_{SD}, Γ_{SR} and Γ_{RD} are the set of γ_{SD}, γ_{SR_r} and $|G_{QR}|^2$ values respectively ;
- $\boldsymbol{\gamma}$ denotes the vector $(\gamma_{SD}, \gamma_{SR_r}, |G_{QR}|^2)$;
- $\boldsymbol{\Gamma}$ denotes the set $\Gamma_{SR} \times \Gamma_{RD} \times \Gamma_{RD}$;
- $\mathcal{P}_R(\boldsymbol{\gamma})$ denotes the instantaneous relay transmit power given a channel gains vector $\boldsymbol{\gamma}$.

Using the same approach as in (Jacob, 1964), the optimization problem can be written as

$$\underset{\mathcal{P}_R(\gamma)}{\text{maximize}} \quad \int_{\mathbf{\Gamma}} f(\gamma) \max\left(I_R^I(1), I_R^I(2)\right) \, \mathrm{d}\gamma \tag{4.3a}$$

$$\text{s.t} \quad \int_{\mathbf{\Gamma}} f(\gamma) \mathcal{P}_R(\gamma) \, \mathrm{d}\gamma \leq \overline{\mathcal{P}_R} \tag{4.3b}$$

$$\mathcal{P}_R(\gamma) \geq 0, \;\; \forall \gamma \in \mathbf{\Gamma}, \tag{4.3c}$$

where $f(\gamma)$ is the joint probability of γ_{SD}, γ_{SR} and γ_{RD}.

The min (in the definition of $I_R^I(1)$) and the max function in (4.3a) make the optimization problem difficult to solve. We thus start by reformulating the max function as follows. Notice that when $\gamma_{SR_r} \leq \gamma_{SD}$, the direct link can transmit more information than the relay link (i.e., $I_R^I(2) \geq I_R^I(1), \forall \mathcal{P}_R(\gamma) \geq 0$). In this case, the relay assigns a null power to relay the signal. Then, the max function can be reformulated as the instantaneous power constraint $0 \leq \mathcal{P}_R(\gamma) \leq C\left[\gamma_{SR_r} - \gamma_{SD}\right]^+$, where C is a positive real number which will be given later. To reformulate the min function, we use the definition of the channel capacity. A receiver can decode a signal with an error probability as small as desired if the transmission rate is less than the channel capacity. The relay can then decode correctly a signal transmitted with a rate less than $\log(1 + P_S\gamma_{SR_r})$. The mutual information of relay channel cannot be greater than the capacity of S–R link. The relay transmit power should thus be such that ; $\log(1 + P_S\gamma_{SD} + \mathcal{P}_R(\gamma)|G_{QR}|^2) \leq \log(1 + P_S\gamma_{SR_r})$. Then, the max and min function can be reformulated as an instantaneous power constraint as follows

$$0 \leq \mathcal{P}_R(\gamma) \leq \frac{P_S}{|G_{QR}|^2}\left[(\gamma_{SR_r} - \gamma_{SD})\right]^+. \tag{4.4}$$

The real number C is then equal to $C = \frac{P_S}{|G_{QR}|^2}$.

We can therefore reformulate the optimization problem in (4.3) as a maximization problem subject to an average and an instantaneous power constraints as follows

$$\underset{\mathcal{P}_R(\gamma)}{\text{maximize}} \int_{\mathbf{\Gamma}} f(\gamma) \log\left(1 + P_S\gamma_{SD} + \mathcal{P}_R(\gamma)|G_{QR}|^2\right) \mathrm{d}\gamma \tag{4.5a}$$

$$\text{s.t} \int_{\mathbf{\Gamma}} f(\gamma)\mathcal{P}_R(\gamma)\,\mathrm{d}\gamma \leq \overline{\mathcal{P}_R} \tag{4.5b}$$

$$0 \leq \mathcal{P}_R(\gamma) \leq \frac{P_S}{|G_{QR}|^2}\left[\gamma_{SR_r} - \gamma_{SD}\right]^+, \quad \forall\gamma \in \mathbf{\Gamma}. \tag{4.5c}$$

4.3 Optimal Power Distribution

In this section, we derive the optimal relay power distribution $\mathcal{P}_R^*(\gamma)$ that maximizes the average mutual information by solving the optimization problem (4.5). The objective function is clearly concave and the constraints are linear. Thus, the local maximum is the global maximum. We use the Kursh-Kuhn-Tucker (KKT) conditions to obtain the optimal solution by finding the minimum value of the Lagrangian over $\mathcal{P}_R(\gamma)$ in the feasible set. The Lagrangian of (4.5) is written as

$$\begin{aligned}\mathcal{L}(\mathbf{P}_R, \mu) = &\int_{\mathbf{\Gamma}} f(\gamma) \log\left(1 + P_S\gamma_{SD} + \mathcal{P}_R(\gamma)|G_{QR}|^2\right) \\ &-\mu\left(\int_{\gamma} f(\gamma)\mathcal{P}_R(\gamma)\,\mathrm{d}\gamma - \overline{\mathcal{P}_R}\right)\end{aligned} \tag{4.6}$$

where $\mathbf{P}_R = \left\{\mathcal{P}_R(\gamma); 0 \leq \mathcal{P}_R(\gamma) \leq \frac{P_S(\gamma_{SR_r} - \gamma_{SD})}{|G_{QR}|^2}, \gamma \in \mathbf{\Gamma}\right\}$ and μ is the KKT multiplier. The gradient of the Lagrangian in (4.6) where $(\gamma_{SD}, \gamma_{SR_r}, |G_{QR}|^2)$ have occurred is written as

$$\frac{\partial\mathcal{L}(\mathbf{P}, \mu)}{\partial\mathcal{P}_R(\gamma)} = f(\gamma)\left(\frac{|G_{QR}|^2}{1 + P_S\gamma_{SD} + \mathcal{P}_R(\gamma)|G_{QR}|^2} - \mu\right). \tag{4.7}$$

The instantaneous optimal transmit power $\mathcal{P}_R^*(\gamma)$ is such that the gradient function (4.7) equals to zero. The optimal solution must respect the instantaneous power constraint. Hence, the optimal transmit power for a given realization (γ_{SD}, γ_{SR} and $|G_{QR}|^2$) is written as (4.8).

$$\mathcal{P}_R^*(\gamma,\mu) = \begin{cases} \frac{1}{\mu} - \frac{1+P_S\gamma_{SD}}{|G_{QR}|^2} & \text{if } 0 < \frac{1}{\mu} - \frac{1+P_S\gamma_{SD}}{|G_{QR}|^2} < \frac{P_S(\gamma_{SR_r}-\gamma_{SD})}{|G_{QR}|^2} \text{ and } \gamma_{SD} < \gamma_{SR_r} \\ 0 & \text{if } \frac{1}{\mu} - \frac{1+P_S\gamma_{SD}}{|G_{QR}|^2} \leq 0 \text{ or } \gamma_{SD} \geq \gamma_{SR_r} \\ \frac{P_S(\gamma_{SR_r}-\gamma_{SD})}{|G_{QR}|^2} & \text{if } \frac{1}{\mu} - \frac{1+P_S\gamma_{SD}}{|G_{QR}|^2} \geq \frac{P_S(\gamma_{SR_r}-\gamma_{SD})}{|G_{QR}|^2} \text{ and } \gamma_{SD} < \gamma_{SR_r} \end{cases} \quad (4.8)$$

The optimal power allocation solution is a function of the KKT multiplier. Specifically, when the inequality power constraint is inactive, $\int_{\Gamma} f(\gamma) \frac{P_S[\gamma_{SR_r}-\gamma_{SD}]^+}{|G_{QR}|^2} \mathrm{d}\gamma < \overline{\mathcal{P}_R}$, the instantaneous optimal transmit power is then the instantaneous upper bound $\mathcal{P}_R^*(\gamma) = \frac{P_S[\gamma_{SR_r}-\gamma_{SD}]^+}{|G_{QR}|^2}$ and the optimal value of μ, denoted by μ^*, is equal to $\mu^* = 0$. In this case, the system reaches the maximum mutual information without using all available average relay power $\overline{\mathcal{P}_R}$. Hence, the power adaptation technique reduces the average power consumption with a factor equal to $G = 1 - \frac{\int_{\Gamma} f(\gamma) \frac{P_S\left[\gamma_{SR_r}-\gamma_{SD}\right]^+}{|G_{QR}|^2} \mathrm{d}\gamma}{\overline{\mathcal{P}_R}}$. Otherwise, μ^* (where $\mu^* > 0$) that satisfies the average power constraint should be derived using :

$$\int_{\Gamma} f(\gamma)\, \mathcal{P}_R^*(\gamma,\mu^*)\, \mathrm{d}\gamma = \overline{\mathcal{P}_R}. \quad (4.9)$$

The bounds of integration must be precisely defined before solving (4.9). Let $\gamma^{(1)} = (\gamma_{SR_r}, \gamma_{SD})$ and

$$g(\mu^*,\gamma^{(1)}) = \int_{\mu^*(P_S\gamma_{SD}+1)}^{\mu^*(P_S\gamma_{SR_r}+1)} f(|G_{QR}|^2) \left(\frac{1}{\mu^*} - \frac{1+P_S\gamma_{SD}}{|G_{QR}|^2}\right) \mathrm{d}|\mathrm{G_{QR}}|^2, \quad (4.10)$$

$$h(\mu^*,\gamma^{(1)}) = \int_{\mu^*(P_S\gamma_{SR_r}+1)}^{+\infty} f(|G_{QR}|^2) \left(\frac{P_S(\gamma_{SR_r}-\gamma_{SD})}{|G_{QR}|^2}\right) \mathrm{d}|\mathrm{G_{QR}}|^2. \quad (4.11)$$

From (4.9), we then obtain

$$\int_0^{\infty} f(\gamma_{SD}) \int_{\gamma_{SD}}^{\infty} f(\gamma_{SR_r}) \left[g(\mu^*,\gamma^{(1)}) + h(\mu^*,\gamma^{(1)})\right] \mathrm{d}\gamma_{\mathrm{SR_r}}\, \mathrm{d}\gamma_{\mathrm{SD}} = \overline{\mathcal{P}_R} \quad (4.12)$$

The optimal KKT multiplier is derived by solving (4.12).

Next, we investigate two special channel cases. In the first one, we consider Rayleigh fading channels. In the second one, we consider a practical case where the channel gains are discretized.

4.3.1 Optimal power distribution for Rayleigh fading channels

From (Caire and Shamai, 2001, Lemma 4), $|G_{QR}|^2$ follows exponential distributed with an average $\overline{\gamma}_{RD}$. Moreover, the channel gains γ_{SD} and γ_{SR_r} follow exponential distributions with invariant averages $\overline{\gamma}_{SD}$ and $\overline{\gamma}_{SR_r}$ respectively. (4.10) hence becomes

$$\begin{aligned} g(\mu^*, \gamma^{(1)}) = \frac{1}{\mu^*}\left[e^{-\frac{\mu^*(P_S\gamma_{SD}+1)}{\overline{\gamma}_{RD}}} - e^{-\frac{\mu^*(P_S\gamma_{SR_r}+1)}{\overline{\gamma}_{RD}}}\right] - \frac{P_S\gamma_{SD}+1}{\overline{\gamma}_{RD}} \\ \left[\Gamma\left(0, \frac{\mu^*\left(P_S\gamma_{SD}+1\right)}{\overline{\gamma}_{RD}}\right) - \Gamma\left(0, \frac{\mu^*\left(P_S\gamma_{SR_r}+1\right)}{\overline{\gamma}_{RD}}\right)\right], \end{aligned} \tag{4.13}$$

where $\Gamma(p, \lambda)$ is the gamma distribution function with parameter (p, λ). Also, (4.11) becomes

$$h(\mu^*, \gamma^{(1)}) = \frac{P_S\left(\gamma_{SR_r} - \gamma_{SD}\right)}{\overline{\gamma}_{RD}}\Gamma\left(0, \frac{\mu^*\left(P_S\gamma_{SR_r}+1\right)}{\overline{\gamma}_{RD}}\right). \tag{4.14}$$

The analytical solution of (4.12) is intractable. A numerical solver can thus be used to numerically compute μ^*. As seen in (4.8), the function of the instantaneous transmit power is continuous and decreasing as a function of μ. Hence, a bisection algorithm can be used for the numerical computation of μ^*.

4.3.2 Optimal power distribution for discrete distribution channels

In this case, we proceed similarly to the continuous case. The optimization problem is also concave (Boyd and Vandenberghe, 2004, Example 9.4) and using the Lagrange condition we get the same transmit power function as in (4.8). Similarly, the solution is given as a function of μ and if the inequality power constraint is inactive i.e., $\sum_{\Gamma} f(\gamma) \frac{P_S[\gamma_{SR_r}-\gamma_{SD}]^+}{|G_{QR}|^2} < \overline{\mathcal{P}_R}$, then the optimal solution is $\mathcal{P}_R^*(\gamma) = \frac{P_S[\gamma_{SR_r}-\gamma_{SD}]^+}{|G_{QR}|^2}$. Hence, the power adaptation technique reduces the average power consumption with a factor equal to $G = 1 - \frac{\sum_{\Gamma} f(\gamma) \frac{P_S[\gamma_{SR_r}-\gamma_{SD}]^+}{|G_{QR}|^2}}{\overline{\mathcal{P}_R}}$. Otherwise, μ^*, the optimal value of the KKT multiplier, is such that

$$\sum_{\Gamma} f(\gamma)\, \mathcal{P}_R^*(\mu^*) = \overline{\mathcal{P}_R}. \tag{4.15}$$

μ^* can be obtained numerically using the bisection algorithm. However, we propose an efficient algorithm summarized in **Algorithm** 1 to derive its exact value. The algorithm starts by calculating the KKT multiplier $\nu(\gamma)$ that reaches the instantaneous upper bound in (4.5c) for each vector γ by solving $\frac{1}{\nu(\gamma)} - \frac{1+P_S\gamma_{SD}}{|G_{QR}|^2} = \frac{P_S[\gamma_{SR_r}-\gamma_{SD}]^+}{|G_{QR}|^2}$. In step 1, the algorithm then constructs the set $\mathcal{J}_2$ of channel gain vectors such that the instantaneous optimal power $\mathcal{P}_R^*(\gamma, \mu^*)$ reaches the instantaneous upper bound. It is obtained based on the KKT multiplier ($\mu^{(0)}$) calculated as follows

$$\mu^{(0)} = \underset{\nu(\gamma)\in\{\nu(\gamma')|\gamma'\in\Gamma, K(\nu(\gamma'))\geq\overline{\mathcal{P}_R}\}}{\arg\min} K(\nu(\gamma)) - \overline{\mathcal{P}_R} \tag{4.16}$$

where $K(\gamma)$ is the average transmit power over time when the KKT multiplier is equal to $\nu(\gamma)$. In step 2 the algorithm varies the KKT multiplier until it finds the optimal value μ^*.

4.4 Numerical Results

In this subsection, we study the performance of the optimal relay power adaptation technique and compare it with fixed relay instantaneous transmit power equal to $\overline{\mathcal{P}_R}$. MATLAB software tool is used for numerical results. We use a similar

Algorithm 1: Optimal instantaneous transmit power algorithm

Initialization:

$i \leftarrow 0;$

$\min \leftarrow null;$

Step 0:

foreach $\gamma \in \Gamma$ **do**

1 $\quad \nu(\gamma) \leftarrow \frac{|G_{QR}|^2}{P_S \gamma_{SR_r} + 1}$

Step 1:

foreach $\gamma' \in \Gamma$ **do**

$\quad K(\nu(\gamma')) \leftarrow \sum_\gamma f(\gamma) \mathcal{P}_R^*\left(\gamma, \nu\left(\gamma'\right)\right)$

$\quad$ **if** $K(\nu(\gamma')) - \overline{\mathcal{P}_R} \geq 0$ **then**

$\quad\quad$ **if** $\left(min = null \;\; or \;\; K(\nu(\gamma')) - \overline{\mathcal{P}_R} < min\right)$ **then**

$\quad\quad\quad \min \leftarrow K(\nu(\gamma')) - \overline{\mathcal{P}_R}$

$\quad\quad\quad \mu^{(0)} \leftarrow \nu\left(\gamma'\right)$

$$\mathcal{J}_2 = \left\{\gamma, \; \frac{1}{\mu^{(0)}} - \frac{1 + P_S \gamma_{SD}}{|G_{QR}|^2} > \frac{P_S(\gamma_{SR_r} - \gamma_{SD})}{|G_{QR}|^2} \;\&\; \gamma_{SD} < \gamma_{SR_r}\right\}$$

foreach $\gamma \in \mathcal{J}_2$ **do**

$\quad \mathcal{P}^*(\gamma, \mu^{(0)}) \leftarrow \frac{P_S(\gamma_{SR_r} - \gamma_{SD})}{|G_{QR}|^2}$

Step 2:

$$\mathcal{J}_0^{(i)} = \left\{\gamma, \; \frac{1}{\mu^{(i)}} - \frac{1 + P_S \gamma_{SD}}{|G_{QR}|^2} \leq 0 \,\text{or}\, \gamma_{SD} \geq \gamma_{SR_r}\right\}$$

$$\mathcal{J}_1^{(i)} = \left\{\gamma, \; 0 < \frac{1}{\mu^{(i)}} - \frac{1 + P_S \gamma_{SD}}{|G_{QR}|^2} \leq \frac{P_S(\gamma_{SR_r} - \gamma_{SD})}{|G_{QR}|^2} \;\&\; \gamma_{SD} < \gamma_{SR_r}\right\}$$

$$\overline{\mathcal{P}_R}^{(0)} \leftarrow K(\mu^{(0)})$$

while $\overline{\mathcal{P}_R}^{(i)} > \overline{\mathcal{P}_R}$ **do**

$\quad i \leftarrow i + 1;$

2 $\quad \mu^{(i)} \leftarrow \dfrac{\sum_{\gamma \in \mathcal{J}_1} f(\gamma)}{\overline{\mathcal{P}_R} + \sum_{\gamma \in \mathcal{J}_1} \frac{1 + P_S \gamma_{SD}}{|G_{QR}|^2} - \sum_{\gamma \in \mathcal{J}_2} \frac{P_S\left(\gamma_{SR_r} - \gamma_{SD}\right)}{|G_{QR}|^2}}$

$\quad$ Update $\mathcal{J}_0^{(i)}$ and $\mathcal{J}_1^{(i)}$

$\quad$ **foreach** $\gamma \in \mathcal{J}_0^{(i)}$ **do**

$\quad\quad \mathcal{P}^*(\gamma, \mu^{(i)}) \leftarrow 0$

$\quad$ **foreach** $\gamma \in \mathcal{J}_1^{(i)}$ **do**

$\quad\quad \mathcal{P}^*(\gamma, \mu^{(i)}) \leftarrow \frac{1}{\mu^{(i)}} - \frac{1 + P_S \gamma_{SD}}{|G_{QR}|^2}$

$\quad \overline{\mathcal{P}_R}^{(i)} \leftarrow \sum_\gamma f(\gamma) \mathcal{P}_R^*(\gamma, \mu^{(i)})$

$\mu^* \leftarrow \mu^{(i)}$

algorithm to the one proposed in (Chraiti et al., 2014b) but we consider the mutual information (2.27) instead of the spectrum efficiency formula. We consider that the channels gain follow an exponential distribution with average $\overline{\gamma}_{SD} = -10$ dB and $\overline{\gamma}_{SR} = \overline{\gamma}_{RD} = 0$ dB. The channel state is discretized with a precision equals to 0.05.

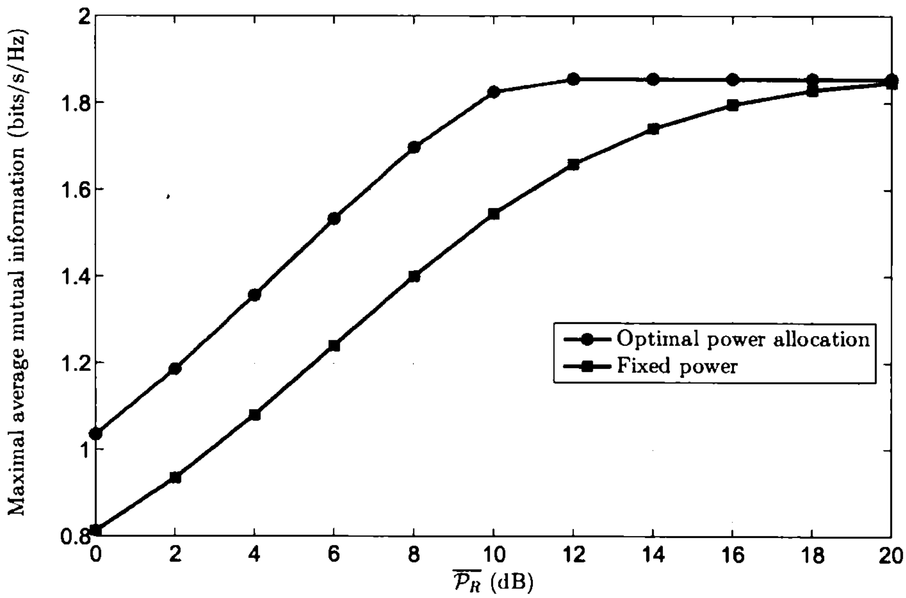


Figure 4.1: Mutual information versus $\overline{\mathcal{P}_R}$

Fig. 4.1 shows the average mutual information versus $\overline{\mathcal{P}_R}$ assuming $P_S = 10$ dB. We can see that for a low $\overline{\mathcal{P}_R}$, the power adaptation technique is able to efficiently allocate the power and to significantly improve the mutual information. Meanwhile, for higher $\overline{\mathcal{P}_R}$ the mutual information becomes bounded by the capacity of the S–R link (power constraint is inactive) but the power adaptation technique is able to reduce the actual relay power consumption as shown in Fig. 4.2.

Fig. 4.3 shows the average mutual information as a function of $\overline{\gamma}_{SD}$ assuming $P_S = \overline{\mathcal{P}_R} = 10$ dB. The results show that the direct link has an important impact on the

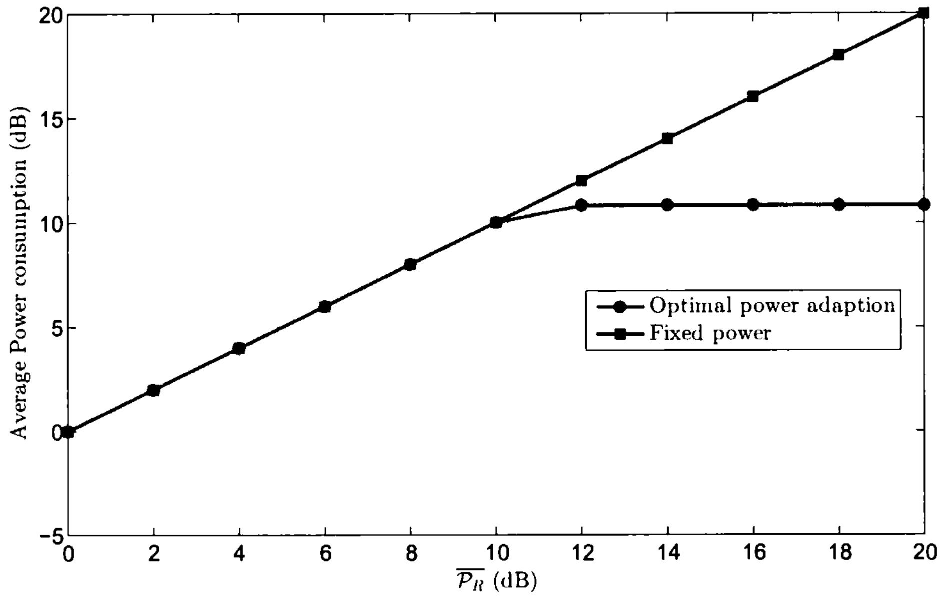


Figure 4.2: The relay power consumption versus $\overline{\mathcal{P}_R}$

system mutual information. We again notice that the optimal power adaptation technique significantly outperforms the fixed relay power allocation for weak direct links. Moreover, when the capacity of the direct link becomes important, the relay decreases its transmit power and the system reduces the power consumption as shown in Fig. 4.4.

4.5 Conclusion

In this chapter, we showed that the relay could adapt its transmit power in time, when it has full CSI (the instantaneous channel state and the gain PDF) knowledge, to maximize the mutual information. We have also proposed an algorithm to obtain the exact optimal transmit power for discrete channels distribution. Analytical results show the interesting gain provided by the optimal power adaption

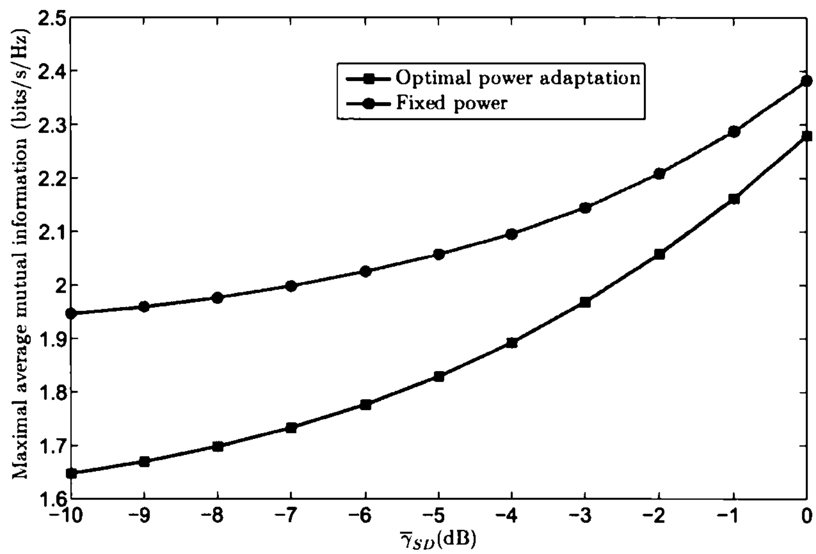


Figure 4.3: Mutual information versus $\overline{\gamma}_{SD}$

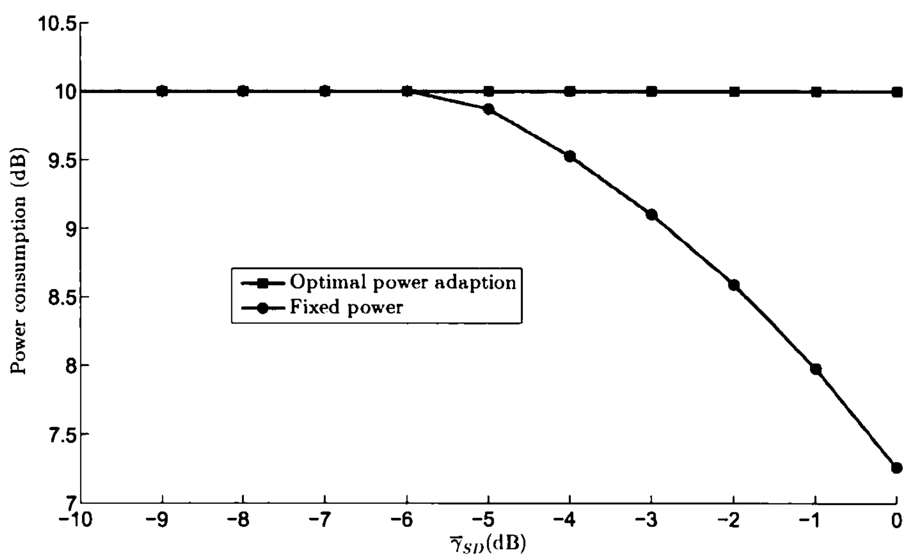


Figure 4.4: The relay power consumption versus $\overline{\gamma}_{SD}$

technique in terms of mutual information and relay power consumption.

CONCLUSION

In this thesis, we analyze the performance of full-duplex relay channel under the constraint of null self-interference power. In the first chapter, we introduce MIMO systems and some basic concepts of information theory. Then, we show how MIMO technology can be used to eliminate the self-interference at a full-duplex node and thus it makes possible full-duplex communication. We also presented the full-duplex relay channel as a promising technology that has the ability to assist the transmission of others nodes without spectrum efficiency loss.

We consider in this thesis full-duplex relay channel under the constraint of null self-interference power. We analyze the system performance with instantaneous CSI as our first contribution presented in the second chapter. We first provide the capacity of the full-duplex degraded relay channel with instantaneous CSI at the source and the relay. We found that the capacity can be achieved using the QR decomposition at the relay. We second derive the explicit form of the mutual information of full-duplex decode-and-forward relay channel under the constraint of null self-interference power. We found that the maximum mutual information of full-duplex relay channel with three-antenna relay is twice the one of half-duplex relay channel with single-antenna relay when the channel state follows Rayleigh distribution.

In the third chapter, we propose as our second contribution two transmission schemes based on Alamouti encoding that address the problem of the direct link interference. Numerical results show that the performance of the first proposed scheme, named FDAE, increases as the relay processing delay decreases.

It achieves near-optimal performance only at low processing delay. The second proposed scheme, named modified FDAE, achieves always near maximum mutual information independently of the processing delay. However, FDAE scheme outperforms MFDAE at low processing delay.

At the fourth chapter, we consider full-duplex relay system with full-channel side information at the relay. We show, as our third contribution, that the relay could adapt its transmit power in time, when it has full CSI, to achieve the maximum mutual information. We also propose an efficient algorithm to obtain the exact optimal solution for the important case of discrete channel states. Analytical results showed the significant gains provided by the optimal power adaptation technique in terms of mutual information and relay power consumption.

Future works

In this thesis, we analyzed the performance of full-duplex relay channel under the constraint of null self-interference power. However, we considered single-antenna source, single-antenna destination and three-antenna relay. The general case of multi-antenna source, multi-antenna destination and multi-antenna relay still undiscovered and our given results may be extended to this more general scenario. We also proposed transmission schemes that achieve near maximum mutual information. Thus, it is still interesting to give the transmission scheme that achieves the maximum mutual information. We have analyzed theoretically the performance of full-duplex relay channels which achieve approximately twice the maximum mutual information of half-duplex relay channels. The next step is to design full-duplex relay channel and to implement the proposed schemes on software defined radio such as Universal Software Radio Peripheral (USRP). Furthermore, in this thesis, we considered that full CSI is only available at the relay, but investigating the maximum mutual information and how to achieve it when full CSI is available at both the source and the relay is interesting. A joint optimization problem should be then reformulated and the joint optimal power distribution should be derived.

APPENDIX A

Let denote by $\mathbf{q}_1$ and $\mathbf{q}_2$ the column vectors of $\mathbf{Q} = \begin{pmatrix} \mathbf{q}_1 \\ \mathbf{q}_2 \end{pmatrix}$. $\mathbf{Q}$ is an unitary matrix and hence $\mathbf{q}_1$ and $\mathbf{q}_2$ are orthogonal and $\|\mathbf{q}_1\| = \|\mathbf{q}_2\| = 1$. The channel vector vector $\mathbf{h}_{RD}$ can be thus decomposed into a parallel component and a perpendicular component to $\mathbf{q}_2$ as follow :

$$\mathbf{h}_{RD} = h^{\|}_{RD}\mathbf{q}_2^{\dagger} + h^{\perp}_{RD}\mathbf{q}_1^{\dagger}. \tag{A.1}$$

Moreover, we have

$$\mathbf{G} = \mathbf{H}\mathbf{Q}^{*} = \begin{pmatrix} \mathbf{h}_R \\ \mathbf{h}_{RD} \end{pmatrix} \begin{pmatrix} \mathbf{q}_1^{*} & \mathbf{q}_2^{*} \end{pmatrix} \tag{A.2}$$

which gives $G_{QR} = \mathbf{h}_{RD}\mathbf{q}_2^{*} = h^{\|}_{RD}$.

We may thus use [Appendix A, (Chraiti et al., 2013b)] to provide that

$$|G_{QR}|^2 \sim \Gamma(\gamma_{RD}, 1). \tag{A.3}$$

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