How the Photon Emerged Through the Prism of Formal Analogies

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It is well known that Einstein considered his 1905 paper on light quanta as a "very revolutionary" contribution to physics, as he wrote in May of that year to his best friend Conrad Habicht[1]. Although one might think that everything has been said about Einstein's conception of light quanta[2], I would like here to analyze the evolution of Einstein's thought on the structure of radiation from the point of view of the formal analogies he used to "see", so to speak, through the "black box" of Planck's blackbody radiation law. These changes of formal points of view are most of the time taken for granted or passed over in silence as if they had no special significance. As we will see, approaching the question from the angle of the specific mathematical tools, namely entropy calculations, used by Einstein in his paper suggests an answer to an intriguing question that has never really been raised: Why did Einstein first limit himself to Wien's approximation, instead of working directly with Planck's equation for the full spectrum of blackbody radiation[3]? As he was trying to get a hand on the structure of radiation, he knew perfectly in 1905 that the results he obtained using Wien's law for the distribution of radiation density ρ in terms of frequency ν and temperature T :

$$(1) \qquad \rho = \frac{8\pi h\nu^3}{c^3} e^{-h\nu/kT}$$

could only be an approximation and thus served only as a "heuristic point of view", "not strictly valid" but "fully confirmed by experiment for large values of [ν/T]".[4] But then, again, why not work instead directly with Planck's law:

$$(2) \qquad \rho = \frac{8\pi h\nu^3}{c^3} \frac{1}{e^{h\nu/kT} - 1}$$

and obtain more than a "heuristic" view of the structure of radiation?

It has been suggested that in 1905, Einstein did not trust Planck's equation because its foundations were not secure[5]. This is no doubt true but we must distinguish between the empirical validity of the law, relatively well established in 1905, and the understanding of its theoretical foundation. Einstein's problem with Planck's law concerned its foundations, not its empirical validity[6]. As much as he knew that Wien's law did not apply generally, there is no reason to think he could not accept the empirical validity of Planck's law and use it to make calculations as he did with Wien's law. Thus, the lack of trust in Planck's equation is probably not the main reason that led Einstein to limit himself to Wien's approximation in his 1905 paper.

I think the reason for choosing Wien's law lies in the formal tools he was using at the time, namely entropy calculations based on Boltzmann's equation, which were not well suited for "seeing" the hidden structure of Planck's equation. Thus one should search the answer to our question in the nature of the "prism" through which Einstein was trying to "probe" the structure of blackbody radiation. As we will see, the major tools he used was the entropy equation stemming from his knowledge of thermodynamics and statistical mechanics, a branch of physics to which he contributed major papers between 1901 and 1904. As the historian Martin Klein noted many years ago, all of Einstein's most original ideas are "intimately related to his understanding of thermodynamics" [7].

Einstein's work thus provides a nice example of the fact that the very choice of mathematical formalisms and the formal analogies they may suggest can play a creative role in physical thinking. By defining an angle of vision they make visible otherwise invisible physical structures.

Entropy as a probe into the nature of radiation

In his 1905 paper on the light quantum, Einstein used entropy calculations based on Boltzmann's equation:

$$(3) \qquad S = k \ln W.$$

Although he left no scraps of paper on which he wrote the calculations that led to this famous paper in which

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[1] A. Einstein to Conrad Habitch, 18 or 25 may 1905, *The Collected Papers of Albert Einstein*, vol. 5, *The Swiss Years: Correspondence, 1902-1914*, Anna Beck, Translator, Princeton University Press, (1995), document 27, p. 20.

[2] For examples, see Martin Klein, "No Firm Foundation: Einstein and the Early Quantum Theory", in Harry Woolf (Ed) *Some Strangeness in the Proportion. A Centennial Symposium to Celebrate the Achievements of Albert Einstein*, Addison-Wiley, (1980), pp. 161-185; S. Bergia and L. Navaro, "Recurrences and Continuity in Einstein's Early Research on Radiation between 1905 and 1916", *Archives for the History of Exact Sciences*, vol. 38, No 1, (1988), pp. 79-99.

[3] Note that the same question has been raised by John Stachel but in relation to Einstein's 1924-1925 papers on atomic gas and quantum statistics whereas I here raise the question in relation to his 1905 and 1909 papers, the second of which made visible for the first time what became known as "wave-particle duality"; see John Stachel, "Einstein's Light-Quantum Hypothesis, or Why Didn't Einstein Propose a Quantum Gas a Decade-and-a-half Earlier?", in John Stachel, *Einstein from 'B' to 'Z'*, (Einstein Studies Volume 9), Boston, Birkhäuser, (2002), pp. 427-444.

[4] A. Einstein, "On a Heuristic Point of View Concerning the Production and Transformation of Light", in *The Collected Papers of Albert Einstein*, vol. 2, *The Swiss Years: Writings, 1900-1909*, Anna Beck, Translator, Princeton University Press, (1989), Document 14, p. 93. For convenience we use modern notations.

[5] John Stachel, *op. cit*. p. 236-239.

he explained the photoelectric effect, it is likely that he first played with Planck's equation but got no insight into it using his entropy approach, while he found that in Wien's approximation, the calculation of entropy led to a nice formula:

$$(4) \qquad S - S_0 = k \ln(V/V_0)^{E/h\nu}$$

for the variation of entropy of radiation contained in a sub-volume V of the total volume V_0. Having obtained this form, he immediately remarked that "this equation shows that the entropy of a monochromatic radiation of sufficiently low density varies with the volume according to the same law as the entropy for an ideal gas or that of a dilute solution[8]". He derived this equation for a collection of independent particles submitted to the same change in volume and obtained:

$$(5) \qquad S - S_0 = k \ln(V/V_0)^n.$$

It is the formal analogy that makes *visible* the *structure* of radiation, which Einstein was looking for. Equations (4) and (5), combined with the fact that the argument of their logarithm is the expression of the probability of finding all radiation (or particles) in volume V, led him to the conclusion that "monochromatic radiation of low density (within the range of validity of Wien's radiation formula) behaves thermodynamically as if it consisted of mutually independent energy quanta of magnitude [$h\nu$][9]". Based on this formal analogy, he thus made the bold step of equating the exponents of both equations to get the famous equation:

$$(6) \qquad E = nh\nu.$$

Far from being a valid deduction, this equation could only be *suggested* by the formal identity of the entropy equations.

The problem with equation (6) and the "particle" interpretation of radiation is of course that it does not apply generally since the true equation empirically tested was Planck's and not Wien's. Einstein was conscious of this limitation and explained his views in a letter to Lorentz written in May 1909[10]:

> "As far as the light quanta are concerned, it seems that I did not express myself clearly. For I am not at all of the opinion that light has to be thought of as being composed of mutually independent quanta localized in relatively small spaces. To be sure, this would be the most convenient way to explain the Wien end of the radiation formula. But the splitting of light rays on the surface of refracting media makes already this approach absolutely inadmissible. A light ray splits, but a quantum cannot split without a change of frequency".

Interestingly, he wrote these lines just a few months after having published a paper "On the Present Status of the Radiation Problem" in which he had in fact analyzed Planck's formula but from a new angle, namely energy fluctuations instead of Boltzmann's equation. And much later, in 1925, he would again use calculations of energy fluctuation to confirm the dual nature of matter (particle and wave) first proposed by Louis de Broglie and predict what would become known at Bose-Einstein condensation. But we have no place here to develop these questions.

In all the cases mentioned above, we see in action one aspect of Einstein's way of doing physics which is not often enough emphasized: his unique capacity to take seriously formal analogies between different systems. Had Einstein not been the expert he was in statistical mechanics and thermodynamics geared as it were to see the world through these particular lenses, he would not have suggested the idea of light particle neither that of the dual nature of light and particles These structures could only be made vivid by using the particular mathematical representation of the phenomena that these disciplines provide and taking seriously the analogies they suggest.

[6] His reflections on those foundations led to his 1906 paper in which he clarified Planck's derivation, concluding that "Mr. Planck introduced into physics a new hypothetical element: the hypothesis of light quanta", thus confirming that his own views on the quantum of light and those of Planck were not incompatible as he first thought in 1905; A. Einstein, "On the Theory of Light Production and Light Absorption", *The Collected Papers of Albert Einstein*, vol. 2, op. cit., p. 192 and 196.

[7] Martin Klein, "Thermodynamics in Einstein's Thought", *Science*, vol. 157, (1967), pp. 509-516; for a more recent analysis see also Angelo Baracca, "Einstein's Statistical Mechanics", *Revista Mexicana de Fisica*, vol. 31, No 3, (1985), pp. 695-722; Seiya Abiko, "Einstein's Theories of Fluctuation and the Thermal Radiation: The First Quantum Theory Through Statistical Thermodynamics", *Historia Scientiarum*, vol. 10, No 2, (2000), pp. 130-147.

[8] A. Einstein, "On a Heuristic Point of View Concerning the Production and Transformation oif Light", *op. cit.*, p. 94.

[9] *Ibid.*, p. 97.

[10] A. Einstein to Henrick A. Lorentz, 23 May 1909, *The Collected Papers of Albert Einstein*, vol. 5, *The Swiss Years, Correspondence, 1902-1914*, Anna Beck, Translator, Princeton University Press, (1995), document 163, p. 123.