

is an amount  $M_z^{(0)}$  of angular momentum of the static electromagnetic field stored between the two cylinders. A mechanical torque is applied to the outer cylinder so as to stop it completely. Thereby the angular momentum of the electromagnetic field becomes zero. The inner cylinder acquires a mechanical angular momentum equal to  $M_z^{(0)}$ .

At a slightly higher pedagogical level, one might wish to take into account the displacement current. From the symmetry of the model it is easily seen, without solving Maxwell's equations, that these can be satisfied by taking  $E_r = \lambda/(2\pi\epsilon_0 r)$  (static value),  $E_z = 0$ ,  $H_r = 0$ ,  $H_\phi = 0$ ,  $E_\phi$ , and  $H_z$  functions of  $r$  and  $t$ . With this Ansatz, the equation for Faraday's law reads

$$\frac{1}{r} \frac{\partial}{\partial r} (r E_\phi) = - \frac{\partial B_z}{\partial t}. \quad (7)$$

Then the expression for the electromagnetic torques can be transformed as follows:

$$\begin{aligned} \tau_{\text{elec}} + \tau'_{\text{elec}} &= R \lambda E_\phi(R, t) - R' \lambda E_\phi(R', t) \\ &= \lambda \int_{R'}^R \frac{\partial}{\partial r} (r E_\phi) dr \\ &= -\lambda \int_{R'}^R \frac{\partial}{\partial t} (r B_z) dr \\ &= \epsilon_0 \frac{d}{dt} \int_r^{R'} E_r B_z 2\pi r dr \\ &= - \frac{dM_z}{dt}. \end{aligned} \quad (8)$$

This accounts for the simplicity of the model.

This note is cordially dedicated to Professor John Gordon Stipe on the occasion of his retirement from Boston University.

<sup>1</sup>R. P. Feynman, R. B. Leighton, and M. Sands, *The Feynman Lectures on Physics* (Addison-Wesley, Reading, MA, 1964), Vol. II, p. 17-5.

<sup>2</sup>M. Phillips, in *Encyclopedia of Physics* (Springer, NY, 1962), Vol. IV, pp. 30 and 31.

## Comment on "What the electromagnetic vector potential describes"

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In a previous article<sup>1</sup> published in this journal, Konopinski discussed the physical meaning of the electromagnetic vector potential  $\mathbf{A}$ . The point is interesting and quite appropriate as most textbooks emphasize that the vector potential is only a convenient tool and that "physics" lies in the electromagnetic fields  $\mathbf{E}$  and  $\mathbf{B}$ . That this is indeed the "orthodox" viewpoint is also shown by the cautious introduction presented by Konopinski in his paper.

The purpose of this note is to point out that the physical interpretation of  $\mathbf{A}$  as some kind of field momentum was explicitly stated by Maxwell himself and that the "modern" viewpoint stated in most textbooks originated with Heaviside and Hertz. For a detailed historical account, the reader is referred to the excellent papers by Bork<sup>2</sup> and Doran.<sup>3</sup>

In his 1865 paper entitled "A dynamical theory of the electromagnetic field," Maxwell refers to the vector potential (which he then calls "electromagnetic momentum") and makes the following statement<sup>4</sup>:

What I have called the electromagnetic momentum is the same quantity which is called by Faraday the electrotonic state of the circuit, every change of which involves the action of an electromotive force, just as a change of momentum involves the action of mechanical force.

The interest of this viewpoint lies in the analogy that Maxwell draws between the equation  $\mathbf{F} = d\mathbf{p}/dt$  of mechanics and his relation  $\mathbf{E} = -\partial\mathbf{A}/\partial t$ , thereby suggesting the identification of  $\mathbf{A}$  as a field momentum. It is worthwhile stressing further the importance Maxwell gives to the concept of vector potential by referring to the third edition of his treatise where, on Sec. 540, he says about  $\mathbf{A}$  that it "may even be called the fundamental quantity in the theory of electromagnetism."<sup>5</sup>

As to the modern attitude towards the vector potential mentioned by Konopinski it was first stated by Heaviside<sup>6</sup>

and can be found in the following quotation from Hertz's 1890 paper<sup>7</sup>

... it does not appear to me that any such advantage is attained by the introduction of the vector potential in the fundamental equations; furthermore, one would expect to find in these equations relations between the physical magnitudes which are actually observed and not between magnitudes which serve for calculation only.

It is interesting to notice the emphasis set on the observable quantities in the formulation of the theory; an attitude still in vogue in our textbooks.

In the last decade of the 19th century Maxwell's interpretation fell into oblivion while the idea put forward by Heaviside and Hertz gained general acceptance and is currently used in most textbooks on electrodynamics.

One wonders if the advent of the Aharonov-Bohm effect<sup>8</sup> which stimulated interest in the quantum interpretation of the vector potential as well as in the classical one,<sup>9,10</sup> will not bring us back to Maxwell's viewpoint or, at least, closer to its spirit!

<sup>1</sup>E. J. Konopinski, *Am. J. Phys.* **46**, 499 (1978); see also M. G. Calkin, *ibid.* **47**, 118 (1979).

<sup>2</sup>A. M. Bork, *Isis* **58**, 219 (1967).

<sup>3</sup>B. G. Doran, *Hist. Stud. Phys. Sci.* **6**, 226 (1975).

<sup>4</sup>See Ref. 2, p. 215.

<sup>5</sup>J. C. Maxwell, *A Treatise on Electricity and Magnetism*, 3rd ed. (Dover, NY, 1954), Vol. 2, p. 187.

<sup>6</sup>See Ref. 3, p. 228.

<sup>7</sup>H. Hertz, *Electric Waves* (Dover, NY, 1962), p. 196.

<sup>8</sup>Y. Aharonov and D. Bohm, *Phys. Rev.* **115**, 485 (1959); see also H. Herlichson, *Am. J. Phys.* **38**, 162 (1970) and P. Bocchieri and A. Loinger, *Nuovo Cimento A* **47**, 475 (1978).

<sup>9</sup>G. T. Trammel, *Phys. Rev. B* **134**, 1183 (1964).

<sup>10</sup>M. G. Calkin, *Am. J. Phys.* **34**, 921 (1966).