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RÉSUMÉ

Cette thèse traite trois questions en finance internationale : (i) les effets du défaut partiel relatif à la dette souveraine sur le partage du risque international, (ii) la coordination imparfaite avec des créanciers multiples asymétriquement informés et ses implications politiques pour résoudre le problème de coordination lié à la dette souveraine, et (iii) le rendement empirique d'un modèle pour une petite économie ouverte avec l'indexation de salaire partiel lorsque les coûts de ressources entraînés par les dispersions des prix et des salaires sont considérés.

Le chapitre I introduit le défaut partiel dans un modèle de deux pays afin de résoudre l'anomalie relative aux corrélations croisées de la consommation. Principalement, les modèles standard de cycles réels internationaux génèrent des corrélations croisées de la consommation qui sont plus élevées que celles de l'output, tandis que dans les données l'opposé est vrai. Le défaut partiel est introduit en supposant qu'un pays défaillant est exclu temporairement à l'accès aux marchés financiers internationaux, et a la possibilité de renégocier ses dettes. Le modèle généralise la sanction d'exclusion permanente au cas de défaut utilisée dans les modèles de cycles réels internationaux avec des marchés incomplets endogènes. Dans ces modèles, bien que le partage du risque international soit davantage réduit, l'anomalie liée aux corrélations croisées de la consommation reste irrésolue. De plus, la menace d'exclusion n'est ni crédible dans un contexte de finance globalisée, ni cohérente avec les faits observés que le défaut est plutôt partiel que complet, et que les emprunteurs souverains peuvent emprunter après les défauts. Les résultats démontrent que : (i) la différence entre la corrélation croisée de la consommation et celle de l'output est décroissante par rapport au nombre de périodes d'exclusion ; (ii) le modèle avec défaut partiel peut générer la différence qui concorde bien avec celle dans les données.

Le chapitre II analyse la coordination imparfaite avec des créanciers multiples dans un jeu global et ses implications politiques pour résoudre le problème de coordination lié à la dette souveraine. Des créanciers plus informés ou plus optimistes réduisent-ils la vulnérabilité d'un projet à la ruée des créanciers ? Pour répondre à cette question, nous développons un modèle dans lequel un grand créancier et un continuum de petits créanciers indépendamment décident de reéchelonner la dette ou de liquider à la base de leurs informations privées sur la liquidité de l'emprunteur et la rentabilité du projet jusqu'à sa maturité. Nos résultats montrent qu'une amélioration de la qualité des informations ou une perception plus optimiste sur la rentabilité du projet de la part du grand créancier augmente la volonté de reéchelonner leurs dettes des petits créanciers, et donc réduit la probabilité de défaut du projet. Au niveau national, le problème de coordination est souvent résolu par la cour sur la faillite, tandis que dans un contexte international, telle institution supranationale n'existe pas. Le modèle développé dans ce chapitre permet d'évaluer les mécanismes conçus pour résoudre le problème de coordination relatif à la dette souveraine. Nous proposons également un mécanisme qui génère le même résultat qu'une cour nationale sur la faillite.

Le chapitre III développe un modèle d'une petite économie ouverte dans lequel l'indexation partielle est permise, et l'estime avec les données canadiennes utilisant les techniques d'estimation Bayésienne. Les travaux empiriques avec les modèles d'équilibre général stochastique et dynamique tels que Christiano, Eichenbaum et Evans (2005) imposent que les salaires et les prix non-ajustés sont complètement indexés à l'inflation retardée. Cette spécification a l'avantage de concorder avec l'inflation positive tendancielle observée dans les données sans nécessiter de considérer les coûts de ressources entraînés par les dispersions des prix et des salaires. Pourtant, l'évidence empirique pour appuyer cette spécification est manquante. Par contre, la base de données sur les conventions collectives au Canada montre que seulement une petite proportion de contrats de travail contient des clauses d'indexation aux coûts de la vie. Les résultats montrent que : (i) les salaires sont partiellement indexés à l'inflation retardée ; (ii) le rendement empirique décroit avec une version du modèle dans lequel les salaires sont complètement indexés à l'inflation retardée ; (iii) le modèle avec indexation partielle capte bien la dynamique de l'économie canadienne.

Mots clés : Cycles réels internationaux ; Défaut partiel ; Coordination ; Dette souveraine ; Petite économie ouverte ; Inférence Bayésienne.

ABSTRACT

This dissertation studies three issues in international finance: (i) the effects of partial default relative to sovereign debt on international risk sharing, (ii) the coordination problem with multiple creditors and its policy implications for solving the sovereign debt issues, and (iii) the empirical performance of small open economy model with partial wage indexation when resource costs induced by price and wage dispersions are considered.

Chapter I introduces partial default into a two-country model to solve the cross-country consumption correlation puzzle. Principally, standard international real business cycle models generate the cross-country correlations of consumption that are higher than those of output, while in the data the opposite is true. Partial default is introduced by allowing a defaulting country to be temporarily excluded from international financial markets, and the sovereign debt contract to be renegotiated. The model generalizes the sanction of permanent exclusion in case of default used in the international real business cycle models with endogenous incomplete market. In these models, though the international risk sharing is further reduced, the cross-country consumption correlation puzzle remains. Most importantly, the threat of permanent exclusion is neither credible in a context of financial globalization, nor consistent with the observed facts that default is rather partial than complete, and sovereign debtors can borrow again after defaults. The results show: (i) the gap between the cross-country correlations of output and consumption is decreasing in the number of exclusion periods; (ii) the model with partial default can generate the gap that is close to the data.

Chapter II analyzes imperfect coordination with multiple creditors in a global game, and its policy implications for solving the coordination problem of sovereign debt. Do better informed or more optimistic large creditors decrease the vulnerability of a project to creditor runs? To address this issue, we build a model in which a continuum of small creditors and a single large creditor independently decide whether to roll over their loans or foreclose based on their private information about the liquidity of the debtor and the future return of the project if it remains in operation. Our results show that an increase in the precision of the information or in the estimate on the value of continuation by the large creditor raises the willingness of small creditors to roll over their loans, and reduces the probability of project failure. At a national level, the coordination problem is often solved by bankruptcy filings. In an international context, there is no supranational bankruptcy court to handle sovereign default. We evaluate the existing mechanisms for solving the coordination problem of sovereign debt, and propose a mechanism that is shown to be able to generate the same outcome as a national bankruptcy court.

Chapter III develops a small open economy model in which partial wage indexation is allowed, and estimates it on Canadian data using Bayesian estimation techniques. Empirical work with dynamic stochastic general equilibrium models such as Christiano, Eichenbaum and Evans (2005) imposes full price and wage indexation to lagged inflation. This specification has the advantage of matching the positive trend inflation observed in the data without considering the resource costs induced by inefficient price and wage dispersions. However, there is no empirical evidence to support this specification. Instead, the Canadian database of wage bargaining contracts shows only a small proportion of wage contracts includes cost-of-living clauses. The results show: (i) wages are indexed only partially to lagged inflation; (ii) the empirical performance drops with a version of the model in which wages are fully indexed to lagged inflation; (iii) the model with partial indexation accounts well for the dynamics of the Canadian economy.

Key words: International real business cycles; Partial default; Coordination; Sovereign debt; Small open economy; Bayesian inference.

INTRODUCTION

Market imperfections have been at the center of analysis in international finance. This thesis aims to: (i) analyze international credit market imperfections and their theoretic and policy implications; (ii) provide an estimated small open economy model with imperfections in goods, labor and credit markets using Canadian data.

In the first essay, Chapter I, partial default relative to sovereign debt is introduced into a two-country model to solve the cross-country consumption correlation puzzle in international real business cycle (IRBC) literature. In the second essay, Chapter II, we analyze imperfect coordination with multiple creditors and its policy implications for solving the coordination problem of sovereign debt. In the third essay, Chapter III, a small open economy with partial wage indexation is estimated when resource costs induced by price and wage dispersions are considered.

The motivation of the first essay relies on the facts that: (i) output is less volatile in countries with high level of contract enforcement than those with low level of contract enforcement (Demirgüç-Kunt and Levine, 2001); (ii) the imperfect enforcement of loan contracts has aggregate consequences in a closed economy (Cooley, Marimon and Quadrini, 2003); (iii) at a national level, the enforcement of loan contracts is at least guaranteed by bankruptcy laws, while at an international level, there is no such supranational legal system to enforce sovereign debt contracts. This suggests the enforcement of sovereign loan contracts could contribute to explain international real business cycles, and in particular to resolve the cross-country consumption correlation puzzle documented by Backus, Kehoe and Kydland (1995). Principally, standard international real business cycle models generate cross-country consumption correlations that are higher than those of output, while in the data the opposite is true.

This cross-country consumption correlation puzzle reflects the assumption in the standard IRBC models that asset markets are complete and thereby consumers engage in perfect international risk sharing. Intuitively, the cross-country consumption is expected to be less correlated when asset markets are incomplete. Baxter and Crucini (1995), Kollmann (1996), and Heathcote and Perri (2002) show the cross-country consumption correlations decrease when only risk-free bonds are allowed to be traded in international financial markets. However, the cross-country consumption correlations generated by these models are still higher than those of output. In addition, debt contracts in these models are perfectly enforceable. One expects that the risk sharing will be even smaller when the enforcement of debt contracts is limited. Kehoe and Perri (2004) confirm this intuition by showing that the cross-country consumption is less correlated when countries are by immediate and permanent exclusion from international financial markets in case of repudiation. Though the international risk sharing is further reduced, the cross-country consumption correlation puzzle remains. Furthermore, the threat of permanent exclusion in case of repudiation is neither credible in a context of financial globalization, nor consistent with the facts, reported by Eichengreen and Portes (1986) and Sachs (1982), that default is rather partial than complete, and sovereign debtors can borrow again after defaults. Two questions arise: (i) what kind of theoretical framework should be with sanctions against sovereign defaulting that are consistent with stylized facts of sovereign defaults; (ii) whether the model with these sanctions is able to solve the cross-country consumption correlation puzzle.

To answer these questions, we develop a two-country model in which partial default is allowed with two principal contributions to international real business cycle literature. First, it provides a theoretic framework with partial default that a defaulting country is excluded only temporarily from international financial markets and can renegotiate its debt. Second, the model with partial default is able to generate cross-country consumption correlations that are lower than those of output.

In the model, a country subject to negative technological shocks defaults if and only if the expected value of serving its debt is strictly smaller than the expected value of partial default. With expectation in the incentive constraint, the standard Bellman equation is not satisfied. A co-state variable is introduced to generalize the Bellman equation. This co-state variable, capturing the defaulting history of the country, transforms the problem into a solution of a Nash bargaining problem in which the outside option value is the value of partial default. Renegotiation power is no more a parameter but a variable that depends on the default history of the debtor. This implicitly suggests a system of evaluation, based on which the social planner distributes the weights to countries in his problem of optimization.

As the incentive constraint is occasionally binding, the model is solved numerically using value function iteration method. The convergence is obtained by verifying three possible ways that the incentive constraint is bounded: (i) the incentive constraints of both countries are not binding; (ii) the incentive constraint of one country is binding, while the incentive constraint of the other country is not; (iii) *vise versa.* The simulated results show that the gap between the cross-country correlations of consumption and those for output is decreasing in the number of exclusion periods. If the defaulting debtor is excluded during one trimester from external finance, the cross-country correlation of consumption is 2.76 time higher than that of output. With an exclusion period of eight years, the model is capable of generating the the gap between the cross-country correlations of consumption and those of output that is close to the data.

In a two-country model, perfect coordination of creditors is assumed. However, in a competitive context, perfect coordination of rolling over or foreclosing loans is less probable for asymmetrically informed creditors that are reluctant to share information on the fundamentals of their common debtor in financial difficulty. This coordination failure could trigger or worsen financial distresses inducing important economic and financial costs. The coordination problem of domestic creditors is often solved by bankruptcy filings. However, in an international context, there is no such bankruptcy procedure for dealing with sovereign default. This contrast leads to the following questions: (*i*) how to characterize the equilibrium when there are two types of creditors (a small one and a large one) and the return of the project is uncertain if it remains in operation; (*ii*) whether small creditors rely more on the information of the large creditor as the large creditor's information becomes more accurate; (*iii*) whether the existing mechanisms designed to solve the coordination problem of sovereign debt are as effective as a national bankruptcy court; (*iv*) if they are not, whether there is a mechanism able to generate the same outcome as a national bankruptcy court.

The second essay, Chapter II, addresses these issues by developing a model in which a continuum of small creditors and a single large creditor independently decide whether to roll over their loans or foreclose based on their private information about the liquidity of the debtor and the value of continuation with three principal contributions. First, this chapter characterizes the equilibrium of a global game with two types of creditors that are uncertain about the value of continuation. Second, it provides general conclusions on the equilibrium effects when the large creditor's information and perception of continuation become more precise and optimistic, respectively. Third, it contributes to the literature of international finance by evaluating the actual and some proposed mechanisms for solving the coordination problem of sovereign debt, and proposes a mechanism for solving sovereign default that can generate the same outcome as a national bankruptcy court.

In such a game of incomplete information, creditors anticipate the information of others. A creditor's payoff depends on the unknown economic fundamentals (liquidity and return of continuation) of his debtor, his own action, and the actions of others. The actions of other creditors are in turn determined by their beliefs. Thus, creditors make their investment decisions by taking into account the beliefs of others. Though, intuitively appealing, it is a challenging task to keep tracking higher order beliefs of others with a large number of players. Global game provides a simple procedure to generate the same equilibrium outcome. Each creditor chooses the best action to a uniform belief over the proportion of other creditors choosing a certain action. The equilibrium is constructed by assuming that each player adopts a switching strategy in which a creditor rolls over whenever his estimate of the underlying fundamentals is higher than some given threshold. The unique switching equilibrium is shown to be the only equilibrium strategy to survive the iterative elimination of strictly dominated strategies.

Our results show that as a general rule, a higher precision of the large creditor's information relative to small creditors increases the willingness of small creditors to roll over their loans, and reduces the probability of project failure. This result can be explained by the behavior of relying on the information of others. Knowing that there are two types of creditors, a small creditor relies on the precise information of the large creditor in order to minimize the error of foreclosing too much, while the large creditor takes into account the risk of an overwhelming foreclosure by small creditors in order to minimize the the error of rolling over too much while others foreclose such that the project is failed. However, the degree of relying on the information of the large creditor dominates that of the small creditors. Thus, increasing the accuracy of the large creditor's information makes small creditors more willing to roll over. This behavior allows an ill-informed creditor to gamble against the beliefs of others, and a well-informed creditor to avoid being alone in standing against the overwhelming foreclosure even though the truth is in his hand.

Furthermore, a more optimistic estimate about the value of continuation by the large creditor makes small creditors more willing to roll over their loans. This channel of strategic interaction has not been explored in previous studies. Interestingly, if the large creditor is optimistic about the continuation of the project, this makes small creditors more confident to roll over their loans. Therefore, small creditors rely not only on the more precise information of the large creditor, but also on his estimate about the continuation of the project. The individually rational behavior of relying on the information of others aggregates the information, but only partially because the anticipation of one type of creditors about the information of others coincides with the true value at a very weak probability. Therefore, inefficient liquidation or rolling over may occur because of the imperfect coordination among creditors.

Existing mechanisms for solving the sovereign default are evaluated. Our results suggest that the provision of liquidity by international institutions such as the International Monetary Fund (IMF) to a distressed sovereign debtor with ex post conditionalities confirms the beliefs of creditors that the fundamentals of the debtor are not sound enough to get loans rolled over. This conclusion is valid for governmental bailouts at a national level as well. For instance, the US auto industry and financial institution bailouts[1] in 2009. Next, the model also suggests that collective action clauses can solve the coordination problem of one bond issue under the condition that the number of creditors reversing their investment decisions is large enough. Last, an international bankruptcy procedure, proposed by Krueger (2001) and White (2002) among others to deal with sovereign default, can solve the collective-action problem. However, it is difficult to implement due to the fact that there is no existing supranational bankruptcy court and an international treaty would require legal changes by a large number of countries.

Examining closely the above mechanisms suggests that a new proposal should be able to: (i) solve the coordination problem using limited financial resources; (ii) encourage the sovereign debtor to improve its creditworthiness; (iii) be implemented without large scale statutory changes. We propose a new mechanism, which satisfies the above criteria. This mechanism suggests that an international institution such as the IMF could: (i) impose *ex ante* the conditionalities to each sovereign debtor that accepts to participate in the program by specifying policy adjustments specific to the country; (ii) evaluate credibly the fundamentals of the sovereign debtor and the degree of fulfillment of policy adjustments specified by the conditionalities; (iii) decide the loan of last resort based on the evaluation. The loan of IMF serves as a positive signal to private creditors to roll over their loans in the sense that its loan or even its decision to lend corrects the negative beliefs of private creditors.

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[1]The objective of recapitalizing banks is to allow them to deleverage by avoiding massive assets selling and credit crunch, which both deteriorate further the economic situation. However, develeraging is a long process, and credit access can not be eased without imposing a condition that a proportion of public capital must be used to maintain the supply of credit.

The third essay, Chapter III, is motivated by the gap between the empirical evidence and the specification of full wage indexation used in some dynamic stochastic general equilibrium models. On the one hand, empirical work with dynamic stochastic general equilibrium models such as Christiano, Eichbaum and Evens (2005) imposes that each period when wages are unadjusted, they are fully indexed to the inflation rate of the previous period in their estimation. On the other hand, there is no empirical evidence to support this specification. Instead, the Canadian database of wage bargaining contracts shows only a small proportion of wage contracts includes cost-of-living clauses. Since these models have become a workhorse for monetary and fiscal policy analysis, it is important ask whether this specification is empirically robust, and if it is not, whether there is a specification with better empirical performance.

This essay addresses these concerns with two principal contributions to empirical work with DSGE models. First, given that the microdata supports partial wage indexation, the wage indexation parameter is estimated along with other structural parameters in a small open economy model on Canadian data. Second, the empirical performance of the model with partial wage indexation is assessed to be better in comparison to the version with full wage indexation.

In an economy with trend inflation, some important issues arise when moving from a specification with full indexation to a specification with partial indexation. To match the positive trend inflation in data, the model features a positive steady-date inflation rate. The positive trend inflation means by definition there are relative price movements in the steady state. For firms, the only thing that is moving is the relative average price through time. If unadjusted prices are indexed completely to lagged inflation, the automatic updating rule coincides with the optimal price setting rule. In this case, there is only one price in the steady state, and thereby there are no dead weight costs induced by price dispersions. However, when unadjusted prices are indexed partially to lagged inflation, the nominal prices charged by firms are dispersed around the average price prevailing in the economy. Though their nominal prices are fixed, trend inflation leads to different relative prices according to which firms produce different levels of output. In aggregate, some firms produce too much in comparison to the optimal level that would prevail in the absence of price dispersions, while others produce too little. Consequently the deadweight costs, sharing the same rational of those induced by a distortionary tax on final goods and services, occur in the steady state.

The same argument can be applied to wages. When unadjusted wages are fully indexed to lagged inflation, only one optimal salary is charged in the steady state. Consequently, there are no wage dispersions. In contrast, with partial indexation households offer different levels of labor to differentiated labor markets according to their relative wages. This generates an aggregate labor loss, which is similar to the deadweight loss induced by a distortionary labor income tax.

The model considers these inefficient resource costs that matter for both the steady state and the dynamics of the economy. Together, the model features four nominal frictions: (i) sticky prices, (ii) sticky wages, (iii) cash-in-advance constraints for households, and (iv) cash-in-advance on the wage bill of firms; and four real frictions: (i) variable capital utilization, (ii) capital adjustment costs, (iii) habit persistence, and (iv) imperfect competition in goods and factor markets. The empirical role of these frictions is assessed in the estimation procedure.

The model is estimated using Bayesian estimation techniques on quarterly data from Canada for the small open economy and U.S. data to approximate the rest of the world. Together, eleven time series from 1981Q3 to 2006Q4 are used to estimate the model. Since nine structural shocks are included in the model, two error terms are added to the observation equations to avoid the problem of stochastic singularity.

The results show: (i) data contain information about the value of the wage indexation parameter, and wages are indexed only partially to lagged inflation; (ii) the empirical performance drops with a version of the model in which wages are fully indexed to lagged inflation; (iii) the model with partial indexation accounts well for the dynamics of the Canadian economy.

CHAPITRE I

CYCLES RÉELS INTERNATIONAUX ET DÉFAUT PARTIEL

Résumé

Ce chapitre introduit le défaut partiel dans un modèle à deux pays pour résoudre l'anomalie relative aux corrélations croisées de la consommation. Les modèles standard de cycles réels internationaux génèrent les corrélations croisées de la consommation qui sont beaucoup plus élevées que celles de l'output, tandis que dans les données l'opposé est vrai. Par défaut partiel, l'emprunteur défaillant n'est exclu que temporairement, et a la possibilité de renégocier ses dettes. Les résultats montrent que : (i) la différence entre la corrélation croisée de la consommation et celle de l'output est décroissante par rapport au nombre de périodes d'exclusion ; (ii) le modèle avec défaut partiel génère la différence de corrélations croisées qui concorde bien avec les données.

Mots-clés : Cycles réels internationaux ; Défaut partiel.

1.1 Introduction

Les modèles standard de cycles réels internationaux (CRI), initiés notamment par Backus, Kehoe et Kydland (1995), génèrent les corrélations croisées de la consommation qui sont plus élevées que celles de l'output, tandis que dans les données[1] l'opposé est

[1] Ces corrélations croisées sont calculées de variables des Etats-Unis et d'un agrégat de 15 pays européens entre 1970Q1 et 1998Q4. Backus, Kehoe et Kydland (1995) utilisent un panel de 10 pays entre 1970Q1 et 1990Q2, et calculent les corrélations croisées de l'output et de consommation : 0,66 et 0,51. Ambler, Cardia et Zimmermann (2004) élargissent le panel avec 20 pays entre 1960Q1 et 2000Q4, et trouvent que ces corrélations sont moins élevées que celles de Backus, Kehoe et Kydland (1995) avec 0,22 et 0,14. Ceci peut être expliqué par le changement de système financier international et les taillles différentes des panels utilisés.

vrai. Cette anomalie relative aux corrélations croisées de la consommation résulte principalement de l'hypothèse de ces modèles que les marchés des actifs internationaux sont complets de sorte que les consommateurs des pays partagent parfaitement les risques.

L'introduction de frictions financières pour réduire le partage du risque fait l'objet de recherche de plusieurs études. Baxter et Crucini (1995), Kollmann (1996) et Heathcote et Perri (2002) montrent que l'introduction des marchés incomplets dans un modèle de CRI, en supposant de façon exogène que seulement un type d'obligation peut être échangé sur les marchés financiers, réduit le partage du risque. Les corrélations croisées de la consommation dans ces modèles sont toujours plus élevées que celles de l'output. D'ailleurs, ces modèles implicitement supposent que les emprunteurs ne font pas défaut. Kehoe et Perri (2002b) introduisent le problème de contrat de dettes souveraines dans un modèle de CRI avec une contrainte d'incitation basée sur l'hypothèse d'exclusion permanente au financement extérieur au cas de la répudiation. Leur modèle réduit la différence entre la corrélation croisée de la consommation et celle de l'output. Néanmoins, l'anomalie reste irrésolue.

Ce chapitre introduit le défaut partiel[2] relative à la dette souveraine dans un modèle de deux pays pour résoudre cette anomalie avec deux contributions principales. En premier lieu, nous construisons un modèle théorique avec des sanctions qui captent bien les faits stylisés des défauts relatifs aux dettes souveraines, reportés dans Eichengreen et Portes (1986) et Sachs (1982). Notamment, selon Miller, Tomz et Wright (1997), la moyenne de priodes d'exclusion entre 1870 et 1914 est de 9 ans. Gelos, Sahay et Sandleris (2004) trouvent que la moyenne de priode d'exclusion entre 1980 et 1990 est de 4.7 ans. Basé sur ces faits, dans le modèle l'emprunteur défaillant est exclu temporairement

[2] Une précision s'impose : le défaut est, par définition, partiel ou, pour le moins, provisoire. S'il est total et définitif, comme dans les premiers modèles de dette souveraine (Eaton et Gersowitz (1981), Atkeson (1991), Cole, Dow et English (1995), Grossman et Van Huyck (1989), Manuelli (1986), Eaton et Fernandez (1995), et Kletz et Wright (2000)), le défaut devient répudiation. Car, un emprunteur qui ne peut assurer la totalité du service de sa dette n'a aucun intérêt à l'assurer partiellement. L'utilisation du terme défaut dans ces modèles est plutôt au sens pour l'analyse économique qui est défini comme le fait qu' un pays ne soit pas en mesure d'honorer la totalité de ses engagements contractuels en temps prévu.

au financement extérieur, et a la possibilité de renégocier ses dettes. la renégociation, les prêteurs et l'emprunteur se retrouvent dans un contrat imparfait dans le sens où le respect des termes du contrat est limité dû au fait qu'il n'y a pas un système légale supranational pour faire exécuter les contrats de dette souveraine en cas de défaut. En second lieu, le modèle avec défaut partiel génèrent la corrélation croisée de la consommation moins élevée que celle de l'output.

Le choix de se focaliser sur le défaut de dette souveraine est motivé par les constats suivants. Demirgüç-Kunt et Levine (2001) ont démontré que l'output est moins variable dans les pays avec un degré de respect du contrat élevé que dans les pays avec un degré de respect du contrat faible. Cooley, Marimon et Quadrini (2003) ont également montré que le problème de contrat de dettes a des conséquences agrées dans une économie fermée. Au niveau national, le respect du contrat de dettes est au moins garanti par des dispositifs judiciaires comme les lois sur la faillite. Par contre, au niveau international, il n'y a pas un système légal et supranational pour faire respecter les engagements contractuels de dettes souveraines. En même temps, la part de la dette souveraine dans l'économie est importante, par exemple, la moyenne du ratio dette souveraine/PIB des pays émergents en 2004 est de 39 pourcents selon le Rapport de la Stabilité Financière du FMI et la moyenne du ratio dette souveraine/PIB des pays industrialisés en 2004 est de 51 pourcents. Ceci suggère que l'imperfection contractuel de dette souveraine limite le partage du risque international.

Dans le modèle, l'output est stochastique, et le problème de défaut se pose. Il est optimal pour l'emprunteur d'assurer le service de la dette en un certain temps, si et seulement si la valeur espérée avec remboursement est strictement supérieure à celle avec défaut. Lorsque l'output est déterministe, sous hypothèse que les prêteurs connaissent toutes les caractéristiques de l'emprunteur, le défaut ne se produit pas, en raison des anticipations rationnelles des agents. Par contre, lorsque l'output est stochastique, le problème de défaut se pose. Car, dans certains états du monde, l'utilité résultant d'un service de la dette assuré dans son intégralité et d'un accès garanti au financement extérieur pourrait être inférieure à l'utilité du défaut.

Le défaut est partiel dans le sens où si l'emprunteur ne peut servir ses dettes, il est exclu temporairement de marchés de crédit internationaux et a la possibilité de renégocier ses dettes. Même si le nombre de périodes d'exclusion peut varier d'une période à l'infini, l'accent est mis sur un nombre de périodes d'exclusion fini. L'exclusion permanente utilisée dans Kehoe and Perri (2002b) est un cas particulier dans le modèle. Cette menace d'exclusion permanente, avancée dans les modèles de dette souveraine de première génération dont le papier fondateur est celui d'Eaton et Gersowitz (1981) pour justifier l'existence de dette souveraine, n'est au fait plus crédible dans un contexte de finance globalisée. Il s'agit d'une sanction qui est fondée sur la réputation : le défaillant est exclu, immédiatement et définitivement, de l'accès aux marchés de crédits privés internationaux.

Cette hypothèse d'exclusion permanente est valide lorsque : (i) le financement extérieur de l'emprunteur est dominé par des prêts bancaires ; d'autre part ; (ii) une coordination parfaite entre les créanciers se présente pour sanctionner leur emprunteur commun ; (iii) la renégociation n'est pas une amélioration au sens de Paréto. L'histoire sur les défauts de dettes souveraines plutôt suggère que, comme reportée par Eichengreen et Portes (1986) et Sachs (1982), les conditions pour que cette hypothèse soit valide ne sont pas présentes. D'un côté, les sources de financement extérieur sont diversifiées ; d'un autre, la mise en place des plans comme Baker et Brady constituent une illustration de l'absence de coordination des banques entre elles. Par ailleurs, dans un jeu dynamique, la renégociation constitue une amélioration au sens de Paréto. Le raisonnement est le suivant : de même que l'emprunteur ne peut s'engager de manière crédible à rembourser les montants empruntés, il peut être impossible au prêteur de s'engager de façon crédible à appliquer la pénalité dès que le service de la dette n'est pas intégralement assuré, particulièrement si le montant sur lequel porte le défaut est faible. En fait, il peut être coûteux pour le créancier d'appliquer la pénalité dans la mesure où l'emprunteur peut assurer un paiement partiel. Cette application est susceptible de se révéler sous optimale par rapport à une renégociation.

Les faits que dans notre modèle le nombre de périodes d'exclusion peut varier d'une période à un nombre fini et le contrat conclu après la renégociation est toujours imparfait nous permettent, d'une part, d'endogénéiser les gains de renégociation en définissant des valeurs d'option externe plus flexibles ; d'autre part, de reformuler le problème comme un jeu coopératif de Nash en introduisant une variable de coétat qui capte le pouvoir de négociation relatif de deux parties. Le pouvoir de négociation relatif n'est plus un paramètre mais une variable qui dépend de l'histoire de défauts du pays, ce qui implicitement inclut un système de notations selon lequel le planificateur social distribue les poids de bien-être dans son problème d'optimisation.

Ce modèle est résolu numériquement par la méthode d'itérations sur les fonctions de valeur en vérifiant les trois possibilités dont les contraintes d'incitations soient saturées jusqu'à la convergence : les contraintes de tous les deux pays ne sont pas saturées ; la contrainte d'incitation du pays 1 est saturée, alors que celle de pays ne l'est pas ; et *vice versa*.

Les résultats du modèle démontrent que la différence entre la corrélation croisée de la consommation et celle de l'output est décroissante au nombre de périodes d'exclusion. Lorsque le pays n'est exclu qu'une période, les marchés financiers sont presque complets, la corrélation croisée de consommation est 2.76 fois plus forte que celle de l'output ; lorsque le pays est exclu de 4 périodes ou d'un an, la corrélation croisée de consommation est 2.08 fois plus forte que celle de l'output ; lorsque le pays est exclu pendant 6 ans, la corrélation croisée de consommation est 1.08 fois plus forte que celle de l'output ; enfin, lorsque le pays est exclu pendant 8 ans, la corrélation croisée de consommation est moins élevée que celle de l'output, ce qui correspond aux données.

Le modèle avec défaut partiel est présenté avec une revue de la littérature de dette souveraine sur le défaut partiel dans Section 1.2. Dans Section 1.3, la résolution et les résultats du modèle sont présentés. Enfin, dans Section 1.4, nous concluons.

1.2 Modèle avec défaut partiel

Dans cette section, premièrement le défaut partiel est comparé avec les autres alternatives ayant pour objectif de lever l'hypothèse d'exclusion permanente. Ensuite, l'économie avec défaut partiel est présentée. Enfin, le modèle est résolu en introduisant une variable de co-état pour le rendre récursif.

1.2.1 Défaut partiel

Dans la littérature relative à la dette souveraine, l'exclusion permanente est remise en question par Bulow et Rogoff (1989b), qui suggèrent une sanction directe. Ils argumentent que si un emprunteur souverain a la possibilité d'avoir l'accès à un contrat du type "cash-in-advance" indicé aux mêmes états de la nature que le contrat implicite de réputation, une contrainte d'incitation fondée sur des mécanismes de réputation ne peut être suffisante pour que le service de la dette soit assuré. Cet alternative disponible détruit tout équilibre avec un niveau de dette positif, puisqu'il existera toujours un état dans lequel un pays emprunteur fera mieux en faisant défaut. Ces modèles à sanction directe proposent une sanction qui consiste à saisir les actifs appartenant au défaillant. Dans ce cadre, on aboutit en effet au même résultat clef que dans les modèles de réputation : la nécessité d'introduire une contrainte d'incitation interdit la réalisation de l'optimum du premier rang. Par conséquent, le choix d'une sanction de type réputation, ou de type directe, repose généralement sur des raisons techniques, davantage que sur les conditions de l'hypothèse de sanction.

Deux autres types de modèles existent afin de lever l'hypothèse de défaut complet : modèles d'assurance et modèles de renégociation. Pour les modèles d'assurance, le principe est celui de contrats spécifiant un service de la dette en fonction de la conjoncture de l'emprunteur. Obstfeld et Rogoff (1995) présentent un modèle d'assurance basé sur la sanction du défaut directe. L'assureur accepte de signer tout contrat pour lequel le pays souverain s'engage de manière crédible à un paiement, positif lorsque la conjoncture est favorable et négatif sinon, satisfaisant la condition de profit zéro. Le résultat est que l'assurance intégrale n'est pas compatible avec la contrainte d'incitation pour des valeurs de chocs technologiques suffisamment élevées. Ce modèle ne permet pas de dépasser le problème que pose la contrainte d'incitation dans les modèles de dette standard : le rationnement du crédit persiste.

Deux types de modèles de renégociation visent à rendre compte de cette réalité : la première approche, synthétisée par Eaton et Gersowitz (1986), est la plus proche aux modèles de dette ; la seconde approche est celle des modèles de renégociation à la Rubinstein, et ces modèles qui, à la suite de Bulow et Rogoff (1989a), ont pour objectif de constituer un cadre d'analyse du marché secondaire de la dette, procèdent donc *a priori* de mécanismes fondamentalement différents[3] de ceux qui prévalent dans les modèles de dette.

Pour la première approche, l'objectif reste le même : il s'agit d'identifier les déterminants du rationnement du crédit auquel sont confrontés les pays en développement. Pour cela, une série de variants correspondant à différents types de sanctions possibles sont comparées à une situation de référence. La sanction peut être limitée dans le temps. Elle n'est pas forcément spécifiée en tant que sanction directe, ou par la réputation. Enfin, elle peut être, ou non, aléatoire, continue par rapport aux montants sur lesquels porte le défaut, et de montant fini.

Pour les modèles de renégociation à la Rubinstein, notamment dans celui de Bulow et Rogoff (1989a), dans un horizon infini, l'emprunteur est un petit pays. A chaque période, l'output peut être consumé domestiquement, échangé contre des biens étrangers ou stocké avec un taux de dépréciation. Le pays peut emprunter avec un taux d'intérêt mondial, et s'il fait défaut, les prêteurs peuvent saisir une fraction d'exportations du pays, tandis que le pays ne peut obtenir qu'une fraction de ce qu'il aurait pu obtenir

[3]Une exception est le modèle de Fernandez er Rosenthal (1990), dans lequel l'incitation à assurer le service de la dette repose sur un "bonus" obtenu par le pays quand le remboursement est total, lequel est également endogène et dépend positivement du stock de capital accumulé. Ce bonus est présenté comme une amélioration de la réputation, au sens large, de l'emprunteur. D'ailleurs, ils permettent aux joueurs de prendre des actions qui stratégiquement déterminent le rapport de force de deux joueurs, au lieu d'introduire la forme de l'offre-contreoffre dans le marchandage.

en respectant ses engagements. Par ailleurs, l'emprunteur et les prêteurs sont supposés neutres aux risques, et il y a une coordination parfaite entre les prêteurs.

Le marchandage à la Rubinstein (1982) pourrait être une possibilité pour modéliser la renégociation dans ce modèle. Dans cet environnement, le montant à prêter se décide par le minimum de deux options disponibles au pays : la répudiation et le gain de l'échange si le pays peut s'engager de façon crédible à servir la dette ultérieurement. Ceci suppose implicitement que le prêteur dispose totalement du pouvoir de marchandage. En introduisant la possibilité de la renégociation, il nous permet de passer au processus de négociation à la Rubinstein (1982) dans lequel un joueur fait une offre en temps t, l'autre décide de la prendre ou non — sinon, de son tour en temps $t + 1$, il fait une autre contre-offre, et le jeu s'arrête lorsque l'un accepte l'offre de l'autre. Enfin, outre le fait que les modèles de renégociation sont nécessaires si le défaut partiel et provisoire est la règle, leur intérêt réside dans la prise en compte de situations plus complexes. Néanmoins, l'hypothèse que les deux acteurs sont neutres aux risques limite l'application dans notre modèle. Pour cela, nous supposons que la relation contractuelle est bénéfique aux pays en question telle qu'il sera avantageux pour les deux pays de recommencer une relation contractuelle après certaines périodes d'exclusion. Le nombre de périodes d'exclusion peut varier d'une période à un nombre de périodes infinies, ce qui signifie que la sanction devient de plus en plus sévère jusqu'à l'extrême de l'exclusion permanente.

Le défaut partiel est introduit dans le modèle par l'exclusion temporaire après laquelle l'emprunteur et le prêteur se retrouvent dans un contrat de dettes imparfait. En ajoutant une variable de coétat, le problème reformulé a l'allure d'un jeu de renégociation. La différence entre ce modèle et ceux dans la littérature concerne non seulement les caractéristiques de la sanction qui garantit la condition d'incitation mais également les résultats, d'autant plus que le choix d'une sanction temporaire repose sur le réalisme des hypothèses sous-jacentes.

Avec défaut partiel, ce modèle se différentie de celui d'Atkeson (1991) qui inclut l'hypothèse d'un horizon infini et un emprunt destiné au lissage de la consommation et à l'investissement. La nouveauté de ce modèle repose sur la combinaison d'un risque moral au risque de répudiation. Ceci se traduit, au niveau technique, par quatre contraintes : contrainte de ressources, rationalité individuelle, contrainte d'incitation, et contrainte de répudiation. Néanmoins, comme nous avons mentionné ci-dessus, la menace d'exclusion définitive et immédiate n'est plus crédible. L'hypothèse de défaut partiel ou provisoire nous apparaît incontournable pour rendre compte les processus de traitement de la dette.

1.2.2 Économie de deux pays

L'économie inclut deux pays, $i = 1, 2$, et chaque pays est habité par un consommateur représentatif qui vit infiniment. Les deux pays sont dotés du même type de bien ; les préférences des consommateurs représentatifs et les technologies ont les mêmes structures et les même valeurs de paramètres ; la production est sujette aux chocs technologiques spécifiques au pays. A chaque période, le pays reçoit une dotation exogène, dénotée par $y_t \in Y$, qui est un bien de consommation non stockable et stochastique. Les chocs suivent une chaîne markovienne. A chaque période, l'économie mondiale éprouve un des événements s_t avec probabilité $\pi(s_t)$, dont l'histoire est dénotée par $s^t = (s_0, ..., s_t)$, et $\pi_0 = 1$. Les hypothèses cruciales de ce modèle sont : la répudiation ne provoque pas l'exclusion immédiate et définitive au financement extérieur ; mais plutôt une exclusion temporaire du défaillant au financement extérieur. Implicitement, la renégociation est possible dans ce modèle, et le pays défaillant sera exclu des marchés financiers internationaux pendant certaines périodes, et pourra renégocier avec son prêteur de sorte qu'un contrat de dettes incomplet soit élaboré.

Le problème de cette économie se présente comme suit :

$$\max_{\{c_1(s^t),c_2(s^t)\}} \left[\lambda_1 \sum_{t=0}^{\infty}\sum_{s^t} \beta^t \pi(s^t) U\left(c_1(s^t)\right) + \lambda_2 \sum_{t=0}^{\infty}\sum_{s^t} \beta^t \pi(s^t) U\left(c_2(s^t)\right)\right]$$

$$s.c. \quad \sum_{i=1,2} c_i(s^t) = \sum_{i=1,2} A_i(s^t) y_i(s^t) \tag{1.1}$$

$$\sum_{t=r}^{\infty}\sum_{s^r} \beta^{t-r}\pi(s^t|s^r)U\left(c_i(s^t)\right) \geq \sum_{t=r}^{K+r}\sum_{s^r}\beta^{t-r}\pi(s^t|s^r)U\left(\tilde{c}_i(s^t)\right) + \tag{1.2}$$

$$\sum_{t=K+r+1}^{\infty}\sum_{s^r}\beta^{t-(K+r+1)}\pi(s^t|s^{K+r+1})U\left(c_i(\hat{\lambda}_i(s^{t-1}), s^t)\right)$$

où λ_1 et λ_2 représentent respectivement les poids distribués au pays 1 et au pays 2 ; β le facteur d'escompte, le même pour les deux pays ; c_i, $i = 1, 2$, les consommations du pays 1 et du pays 2 ; $U(\cdot)$ fonction d'utilité du consommateur représentatif ; (1.1) est la contrainte de ressources dans laquelle $A_i(s^t)$ représente le choc technologique spécifique au pays ; (1.2) représente la contrainte d'incitation qui stipule qu'il est optimal pour l'emprunteur d'assurer le service de la dette en temps t, si et seulement si la valeur espérée du remboursement est strictement supérieure à celle du défaut partiel qui est la somme de la valeur en autarcie pendant un nombre de périodes et de la valeur d'un contrat imparfait. Dans cette contrainte, $K \in [0, \infty)$ est le nombre de périodes d'exclusion pendant lequel le pays qui fait défaut en temps r ne peut consommer que la consommation en autarcie $\tilde{c}$: lorsque $K = 0$, on se trouve dans un modèle sans exclusion dans lequel l'équilibre sera sans contrat de dettes selon l'induction à rebours. En temps $t-1$, la stratégie de l'emprunteur est de faire défaut en sachant que le prêteur n'imposera pas de sanction en temps t, alors que la stratégie du prêteur en $t-2$ est de ne pas prêter en sachant que l'emprunteur fera défaut en $t-1$. En continuant cette induction jusqu'au temps $t = 0$, il n'existera pas de prêts positifs dans ce jeu. Par contre, lorsque $K = \infty$, on se trouve dans un modèle avec l'hypothèse d'exclusion permanente, et la valeur du défaut devient celle en autarcie ; pour exclure le cas sans contrat de dettes, on se limite aux cas où $K \in (0, \infty)$. Après K périodes d'exclusion, les pays renégocient un contrat incomplet avec les poids distribués aux pays $\hat{\lambda}_i$.

Les deux termes du côté droit de la contrainte d'incitation représentent la valeur du défaut partiel qui se définit comme suit :

$$\begin{aligned} D_i(s^t) &= \sum_{t=r}^{K+r} \sum_{s^r} \beta^{t-r} \pi(s^t|s^r) U\left(\tilde{c}_i(s^t)\right) + \\ & \sum_{t=K+r+1}^{\infty} \sum_{s^r} \beta^{t-(K+r+1)} \pi(s^t|s^{K+r+1}) U\left(c_i(\hat{\lambda}_i(s^{t-1}), s^t)\right) \end{aligned}$$

où le premier terme représente la valeur en autarcie pendant K périodes résultant d'une exclusion temporaire si le pays fait défaut en temps r ; le deuxième terme la valeur après la renégociation avec un contrat imparfait. Cette stratégie de modélisation permet au fait d'endogénéiser les gains comme les coûts du défaut en définissant cette valeur du défaut en fonction de la valeur en autarcie comme suit :

$$D_i(s^t) = [1 + \kappa(\hat{\lambda}_i(s^{t-1})) - \epsilon(K)] V_i(s^t)$$

où $V_i(s^t)$ représente la fonction de valeur en autarcie, définie ci-cessus ; $\kappa V_i(s^t)$ le gain de la renégociation, et $\epsilon V_i(s^t)$ les coûts de la répudiation, avec $\kappa, \epsilon \geq 0$ et $\kappa \geq \epsilon$, et si $\kappa = \epsilon$ et/ou $\kappa = \epsilon = 0$ on se trouve dans l'hypothèse d'exclusion permanente. Dans ce modèle, nous supposons que $\kappa > \epsilon$. Il faut noter que κ, ϵ sont endogénéisés dans le sens où ils sont en fonctions respectivement de $\hat{z}_i$ et K, et que $\kappa'(\hat{\lambda}_i(s^{t-1})), \epsilon'(K) > 0$. La contrainte d'incitation (1.2) diffère de celle d'exclusion permanente par la possibilité du défaut partiel, plus précisément, par augmenter la valeur du défaut décomposée pour tout $K \neq \infty$. Par conséquent, l'incitation de faire défaut est plus forte que dans le cas d'exclusion permanente.

Dans ce problème, l'action de l'emprunteur en temps t affecte non seulement son paiement en temps t, mais aussi l'espérance de paiements futures. Selon Kydland et Prescott (1977), en présence de l'opérateur d'espérance dans la contrainte d'incitation, l'équation de Bellman habituelle n'est pas satisfaite. Ce modèle peut être résolu par la méthode de contrat récursif qui consiste à transformer un problème avec une contrainte d'incitation intertemporelle à un problème de point de selle récursif en ajoutant une variable de coétat qui synthétise l'évolution de multiplicateurs lagrangiens sur les contraintes d'incitations. Cette méthode est développée notamment par Marcet et Marimon (1999).

1.2.3 Introduire la variable de co-état

Selon la méthode de contrat récursif, la contrainte (1.2) peut être reécrite comme suit :

$$U\left(c_i(s^r)\right) - D_i(s^t) + \sum_{t=r+1}^{\infty} \sum_{s^r} \beta^{t-r-1} \pi(s^t|s^{r+1}) U\left(c_i(s^t)\right) \geq 0$$

Soit $\beta^t \pi(s^t)\gamma_i(s^t)$ le multiplicateur relatif à la contrainte d'incitation, le Lagrangien relatif à la contrainte (1.2) s'écrit comme suit :

$$L \equiv \sum_{t=0}^{\infty} \sum_{s^t} \sum_i \left\{ \begin{array}{l} \lambda_i \beta^t \pi(s^t) U\left(c_i(s^t)\right) + \beta^t \pi(s^t)\gamma_i(s^t) \left[U\left(c_i(s^t)\right) - \right. \\ \left. D_i(s^t) + \sum_{t=r+1}^{\infty} \sum_{s^r} \beta^{t-r-1} \pi(s^t|s^{r+1}) U\left(c_i(s^t)\right) \right] \end{array} \right\}$$

Soit $\mu_i(s^t) = \lambda_i + \gamma_i(s^0) + \ldots + \gamma_i(s^t)$ ou $\mu_i(s^t) = \mu_i(s^{t-1}) + \gamma_i(s^t)$ avec $\mu_i(s^{-1}) = \lambda_i$, il s'agit alors les poids distribués aux pays plus la somme de multiplicateurs relatifs aux contraintes d'incitation, et selon $\pi(s^r) = \pi(s^r|s^t)\pi(s^t)$, le Lagrangien relatif à la contrainte d'incitation sera :

$$L \equiv \sum_{t=0}^{\infty} \sum_{s^t} \sum_i \beta^t \pi(s^t) \left[\mu_i(s^t) U\left(c_i(s^t)\right) - \gamma_i(s^t) D_i(s^t)\right]$$

Cette transformation nous donne un problème qui a l'allure d'un jeu coopératif de Nash dans lequel $\gamma_i(s^t) D_i(s^t)$ représente la valeur de l'option externe et la variable de co-état représente le pouvoir de négociation.

En ajoutant la contrainte de ressources, le Lagrangien sera :

$$L \equiv \sum_{t=0}^{\infty} \sum_{s^t} \sum_{i} \beta^t \pi(s^t) \left\{ \begin{array}{c} \mu_i(s^{t-1}) U\left(c_i(s^t)\right) + \gamma_i(s^t)\left[U\left(c_i(s^t)\right) - D_i(s^t)\right] + \\ \phi(s^t)\left[A_i(s^t) y_i(s^t) - c_i(s^t)\right] \end{array} \right\}$$

Les variables de choix sont $c_i(s^t)$, $\gamma_i(s^t)$, et $\phi(s^t)$. Les conditions de premier ordre se présentent comme suit :

$$\begin{aligned} c_i(s^t) : &\qquad U_{ci}\left(c_i(s^t)\right)\left[\mu_i(s^{t-1}) + \gamma_i(s^t)\right] = \phi(s^t) \\ \gamma_i(s^t) : &\qquad U\left(c_i(s^t)\right) = D_i(s^t) \\ \phi(s^t) : &\qquad A_i(s^t) y_i(s^t) = c_i(s^t) \end{aligned} \tag{1.3}$$

En substituant la nouvelle loi du mouvement relative à $\mu_i(s^t)$, et les multiplicateurs $\phi(s^t)$ et $\phi(s^{t+1})$ dans ces contraintes, nous avons un système d'équations comme suit :

$$\frac{U_{c1}\left(c_1(s^t)\right)}{U_{c2}\left(c_2(s^t)\right)} = \frac{\mu_2(s^{t-1}) + \gamma_2(s^t)}{\mu_1(s^{t-1}) + \gamma_1(s^t)}$$

$$\begin{aligned} \gamma_i(s^t) &\quad : \quad U\left(c_i(s^t)\right) = D_i(s^t) \\ \phi(s^t) &\quad : \quad A_i(s^t) y_i(s^t) = c_i(s^t) \end{aligned}$$

pour $i = 1, 2$ et tout s^t. Par simplification, nous normalisons les multiplicateurs en définissant $v_i(s^t) = \gamma_i(s^t)/\mu_i(s^t)$ et $z(s^t) = \mu_2(s^t)/\mu_1(s^t)$. Ceci nous permet d'utiliser seulement les poids relatifs au lieu de poids absolus. La loi de mouvement de $\mu_i(s^t)$ peut être écrite comme suit :

$$\mu_i(s^{t-1}) = (1 - v_i(s^t))\mu_i(s^t)$$

La loi du mouvement de $z(s^t)$ s'écrit comme suit :

$$z(s^t) = \frac{1 - v_1(s^t)}{1 - v_2(s^t)} z(s^{t-1}) \tag{1.4}$$

En substituant ces deux multiplicateurs normalisés dans la condition relative à la consommation, nous avons :

$$\frac{U_{c1}\left(c_1(s^t)\right)}{U_{c2}\left(c_2(s^t)\right)} = z(s^t) \tag{1.5}$$

Avec un processus markovien, $\pi(s^t|s^{t-1})$ peut être écrit comme $\pi(s_t|s_{t-1})$. De façon alternative, ce problème peut être résolu en utilisant la forme du problème de point de selle. La solution optimale de ce problème satisfait :

$$c_i(x_t), \gamma_i(x_t) = \psi\left(z(s^{t-1}), s_t\right)$$

où $x_t = \left(z(s^{t-1}), s_t\right)$, et $\psi(\cdot)$ les fonctions de politique qui satisfont aux conditions (1.3), (1.4) et (1.5).

Analytiquement nous pouvons illustrer les différences entre ce modèle et celui de Kehoe et Perri (2000b) à travers les conditions du premier ordre. Pour faciliter l'induction, nous écrivons les conditions du premier ordre par rapport aux $c_i(s^t)$ et $c_i(s^{t+1})$,

$$\begin{aligned} c_i(s^t): \qquad & \phi(s^t) = U_{ci}\left(c_i(s^t)\right)\left[\mu_i(s^{t-1}) + \gamma_i(s^t)\right] \\ c_i(s^{t+1}): \qquad & \phi(s^{t+1}) = \beta \sum \pi(s^{t+1}|s^t) U_{ci}\left(c_i(s^{t+1})\right)\left[\mu_i(s^t) + \gamma_i(s^{t+1})\right] \end{aligned}$$

et nous avons le taux marginal de substitution intertemporelle,

$$\frac{\phi(s^{t+1})}{\phi(s^t)} = \frac{\beta \sum \pi(s^{t+1}|s^t) U_{ci}\left(c_i(s^{t+1})\right)\left[\mu_i(s^t) + \gamma_i(s^{t+1})\right]}{U_{ci}\left(c_i(s^t)\right)\left[\mu_i(s^{t-1}) + \gamma_i(s^t)\right]}$$

qui peut être recrit comme suit :

$$\begin{aligned} \frac{\phi(s^{t+1})}{\phi(s^t)} \quad = \quad & \frac{\beta \sum \pi(s^{t+1}|s^t) U_{ci}\left(c_i(s^{t+1})\right)}{U_{ci}\left(c_i(s^t)\right)} \\ & + \frac{\beta \sum \pi(s^{t+1}|s^t) U_{ci}\left(c_i(s^{t+1})\right)}{U_{ci}\left(c_i(s^t)\right)} \frac{\gamma_i(s^{t+1})}{\mu_i(s^t)} \end{aligned}$$

où le premier terme du côté droit est standard, et le deuxième terme capte l'effet d'incitation qui, par définition de $\mu_i(s^t)$ et de $\gamma_i(s^{t+1})$, signifie que la baisse de la consommation en période t renforce la contrainte d'incitation de la période 0 jusqu'à la période t, et la hausse de la consommation en période $t+1$ relaxe la contrainte d'incitation de la période 0 jusqu'à la période $t+1$. Alors, l'effet net sera de relaxer la contrainte d'incitation en période $t+1$. Étant donné que ce terme est positif, le taux marginal de substitution intertemporelle est plus élevé que celui dans les modèles avec marchés complets ou avec marchés incomplets exogènes. Par rapport au modèle de Kehoe et Perri, le défaut partiel est plus intéressant que la répudiation par la valeur de réserve plus élevée, ce qui augmente le taux marginal de substitution intertemprelle.

1.3 Résultats

Ce modèle est résolu numériquement par la méthode d'itération sur les fonctions de valeur. Il contient deux variables d'état, z et A. Nous discrétisons la première et approximions la deuxième par une chaîne markovienne selon la méthode proposée par Tauchen (1986). Soit $x = (z, A)$, les fonctions de politique satisfont les conditions du premier ordre (1.3), (1.4) et (1.5), et les fonctions de valeur définies comme suit :

$$W_i(x) = U(c_i(x)) + \beta \sum_{s^r} \pi(s'|s) W_i(x')$$

Les valeurs sur un marché complet sont utilisées comme des valeurs initiales pour calculer les valeurs de la prochaine période. Si la valeur du pays 1 est inférieure à la valeur du défaut, il fait défaut et la contrainte d'incitation de ce pays est en égalité, ce qui nous permet de calculer la consommation et une nouvelle fonction de valeur ; si la valeur du pays 2 est inférieure à la valeur du défaut, il fait défaut et la contrainte d'incitation de ce pays est en égalité ; sinon, les deux contraintes ne sont pas saturées ; renouveler fonctions de valeurs jusqu'à la convergence. Les détails se trouvent dans Appendice A.

La fonction d'utilité est $U(c) = c^{1-\sigma}/(1-\sigma)$. Les valeurs des paramètres s'illustrent dans Tableau 1.1.

Tab. 1.1 Valeurs des paramètres

	Paramètres
Préférences	$\beta = 0.99$, $\sigma = 2$
Chocs technologiques	$a_1 = 0.95$, $a_2 = 0$
	$\text{var}(\varepsilon_1) = \text{var}(\varepsilon_2) = 0.007^2$
	$\text{corr}(\varepsilon_1, \varepsilon_2) = 0.25$
Périodes d'exclusion	$K = \{1, 4, 8, 12, 16, 20, 24, 28, 32\}$
Poids après la regénociation	$\hat{\lambda}_1 = \hat{\lambda}_2$

Les chocs technologiques de deux pays suivent un VAR sous forme comme suit :

$$\begin{bmatrix} \log A_{1t} \\ \log A_{2t} \end{bmatrix} = \begin{bmatrix} a_1 & a_2 \\ a_2 & a_1 \end{bmatrix} \begin{bmatrix} \log A_{1t-1} \\ \log A_{2t-1} \end{bmatrix} + \begin{bmatrix} \varepsilon_{1t} \\ \varepsilon_{2t} \end{bmatrix}$$

où les innovations $(\varepsilon_{1t}, \varepsilon_{2t})$ sont temporellement indépendantes, mais corrélées entre les pays. Le coefficient a_1 capte le degré de persistance, tandis que a_2 capte l'effet de débordement. Baster et Crucini (1995) et Kollman (1996) montrent, en utilisant les séries temporelles de A_{it} entre certains pays européens et les Etats-Unis, que l'effet d'externalités est proche de zéro, et que la persistance est forte. Nous mettons $a_1 = 0.95$, $a_2 = 0$. La matrice de covariance, selon la littérature, comprend $\text{var}(\varepsilon_1) = \text{var}(\varepsilon_2) = 0.007^2$ et $\text{corr}(\varepsilon_1, \varepsilon_2) = 0.25$.

La valeur du défaut dépend de deux variables clés K et $\hat{\lambda}_i$: K varie de façon discrète d'une période jusqu'à l'infini ; les poids du bien-être distribués aux pays par le planificateur après la renégociation sont égaux et constants au cours du temps, ce qui signifie que les pouvoirs de renégociation de pays après une répudiation s'égalisent et que peu importe la période à laquelle les deux pays renégocient un contrat de dette ils parviennent à aboutir au même résultat[4].

[4]Les résultats principaux maintiennent avec une version du modèle dans lequel les pouvoirs de renégociation de pays se diffèrent.

Une fois que les fonctions de politique optimales sont calculées, nous simulons le modèle sur 100, 000 périodes, et nous calculons 1000 répétitions en enlevant les 200 premières observations pour calculer les moments. A part les exportations nettes, le filtre d'HP est employé. Les résultats obtenus sont illustrés dans Tableau 1.2.

Tab. 1.2 Corrélations croisées

Corrélations/K	$K = +\infty$	$K = 1$	$K = 4$	$K = 8$	$K = 12$
corr(y_1, y_2)	0.25	0.25	0.25	0.25	0.25
corr(c_1, c_2)	0.21	0.69	0.52	0.50	0.47
corr$(x_1/y_1, y_1)$	0.39	0.09	0.10	0.12	0.19
Corrélations/K	$K = 16$	$K = 20$	$K = 24$	$K = 28$	$K = 32$
corr(y_1, y_2)	0.25	0.25	0.25	0.25	0.25
corr(c_1, c_2)	0.40	0.32	0.27	0.26	0.24
corr$(x_1/y_1, y_1)$	0.22	0.29	0.35	0.37	0.38

Etant donné qu'il s'agit d'une économie de dotation, les corrélations croisées de l'output ne varient pas avec le nombre de périodes d'exclusion. On constate que : 1) lorsque $K = +\infty$, qui signifie une exclusion permanente, la corrélation croisée de la consommation est plus faible que celle de l'output, néanmoins la corrélation entre les exportations nettes relatives au revenu et le revenu est positive, tandis que celle dans les données est négative ; 2) lorsque $K = 1$, le pays est menacé d'être exclu d'une période, les marchés financiers sont presque complets, la corrélation croisée de la consommation est 2.76 fois plus forte que celle de l'output, et la corrélation entre les exportations nets relatives à l'output et l'output est toujours positive ; 3) lorsque $K = 4$, le pays est menacé d'être exclu de 4 périodes, la corrélation croisée de la consommation est 2.08 fois plus forte que celle de l'output, et la corrélation entre les exportations nets et l'output est toujours postive ; 4) jusqu'à $K = 24$, la corrélation croisée de la consommation est 1.08 fois plus forte que celle de l'output, et la corrélation entre les exportations nets et l'output est toujours positive ; 5) lorsque $K = 28$, les corrélations de la consommation et de l'output sont proches ; 6) lorsque $K = 32$, qui signifie une exclusion de 8 ans, la corrélation de la consommation est moins élevée que celle de l'output.

Ces résultats sont assez intuitifs, puisque l'augmentation de la valeur de défaut partiel serve accroît le partage du risque, ce qui fait que la corrélation de consommations devient plus forte. Il faudrait noter que lorsque le pays défaillant est menacé d'exclu de 8 ans, la valeur de défaut est plus élevée que celle avec la menace d'exclusion permamente utilisée dans Kehoe et Perri (2002b). Intuitivement, ceci deverait augmenter le partage du risque. Pourtant, le fait que dans une économie de dotation le pays défaillant ne peut épargner diminue le partage du risque, ce qui implicitement implique une sorte de sanction directe. Selon ces résultats, nous pouvons conclure que : en premier lieu, les corrélations croisées de la consommation décroissent lorsque les sanctions deviennent de plus en plus sévères ; en second lieu, la corrélation entre les exportations nets et l'output est toujours positive.

1.4 Conclusion

Ce chapitre démontre que lorsque l'emprunteur défaillant est sanctionné par l'exclusion temporaire aux marchés de crédit internationaux et peut renégocier ses dettes en concluant des contrats de dette toujours imparfaits, la corrélation croisée de la consommation est négativement liée au nombre de périodes d'exclusion, et qu'elle est moins élevée que celle de l'output lorsque le nombre de périodes d'exclusion est plus de 8 ans. Cette modélisation capte bien les faits observés de défauts : le nombre de période d'exclusion varie d'un pays à un autre, les défauts sont plutôt partiels que complets, et le mécanisme de renégociation est largement utilisé par les emprunteurs souverains. Cette modélisation fait abstraire au processus de renégociation. Une possibilité de modéliser la renégociation est d'introduire les taux d'intérêt dans une petite économie ouverte en introduisant les intermédiaires financiers internationaux. La renégociation pourrait être modélisée par la solution de Nash sur la prime de risque du pays. Les coûts de répudiation se traduisent d'une part par la sanction directe et indirecte ; d'autre part, par la dégradation de notations distribuées par une organisation privée ou publique.

APPENDICE A

ALGORITHME NUMÉRIQUE

Les étapes à suivre de l'algorithme numérique se présentent comme suit :

Etape 1 : Approximer les chocs technologiques, A_i et discrétiser la variable de co-état, z.

Les chocs technologiques sont approximés par une chaîne markovienne selon la méthode proposée par Tauchen (1986). Pour un nombre d'état donné, 3 dans le modèle, cette méthode permet d'appoximer un VAR(1) par une chaîne markovienne en produisant les chocs et une matrice de transition.

Une fois que les chocs sont produits, nous pouvons avoir trois dotations, $A_i^1 y_i^1$, $A_i^2 y_i^2$, $A_i^3 y_i^3$, ce qui nous permet de continuer à discrétiser la variable de coétat. En premier lieu, on calcule les bornes de cette grille, $z_{\min}$ et $z_{\max}$, selon la condition du premier ordre par rapport à la consommation, ce qui nous donne que

$$z_{\min} = \frac{A_i^1 y_i^1}{(A_i^3 y_i^3)^{-\sigma}}$$

et

$$z_{\max} = \frac{1}{z_{\min}}$$

En second lieu, selon

$$z(s^t) = \frac{1 - v_1(s^t)}{1 - v_2(s^t)} z(s^{t-1})$$

pour un nombre de points choisi, 500 dans le modèle, les grilles entre ces bornes sont calculés de façon suivante :

$$z(i) = z(i-1)(\frac{z_{\max}}{z_{\min}})^{\frac{1}{499}}$$

où $i = 2, ..., 499$.

Nous avons des grilles qui satisfont $z_{\min} < \bar{z} < z_{\max}$, et $\bar{z} = 1$.

Etape 2 : Calculer les valeurs du défaut

La valeur du défaut est décomposée en valeur d'exclusion pendant K périodes et en valeur après la renégociation. En sachant la matrice de transition M, la valeur du défaut est calculée comme suit :

$$D_i = \frac{1-\beta^K}{1-\beta} M \frac{(A_i(s)yi)^{1-\sigma}}{1-\sigma} + \frac{1-\beta^K}{1-\beta} M \frac{(c(\hat{\lambda}_i))^{1-\sigma}}{1-\sigma}$$

Nous avons deux vecteur de 3 par 1 contenant les valeurs du défaut.

Une autre façon de calculer ces valeurs est de calculer d'abord les valeurs en autarcie, et puis compare avec les valeurs du défaut, le rapport de ces deux valeurs donne les variations en pourcentage résultant de K.

Etape 3 : Calculer les valeurs initiales résultant des marchés complets

Étant donné que les contraintes d'incitation ne sont pas saturées, les multiplicateurs relatifs à ces deux contraintes sont nuls, alors $z' = z$, selon la condition du premier ordre relative à la consommation, nous pouvons avoir les valeurs de consommations comme suit :

$$\begin{aligned} c_1(x) &= \frac{A_1(s)y_1 + A_2(s)y_2}{1 + z(s)^{\frac{1}{\sigma}}} \\ c_2(x) &= (A_1(s)y_1 + A_2(s)y_2)\left(1 - \frac{1}{1 + z(s)^{\frac{1}{\sigma}}}\right) \end{aligned}$$

En sachant les valeurs de consommations, on utilise la méthode d'itération sur les fonctions de valeur jusqu'à la convergence, ce qui nous permet de trouver les valeurs initiales que nous allons utiliser par la suite.

Etape 4 : Commencer les itérations

L'espace d'état a deux dimensions et commençons avec le premier point dans l'espace. Supposons qu'initialement les contraintes d'incitations ne sont pas saturées, les valeurs, $W_i^0(x)$ sont calculées simplement en faisant la sommation entre les états, autrement dit,

$$W_i^0(x) = U(c_i^0(x)) + \beta \sum_{s'} \pi(s'|s) W_i^0(x')$$

Etape 5 Vérifier les trois possibilités que les contraintes d'incitations soient saturées

1) Si $W_1^0(x)$ est inférieure à la valeur du défaut, la contrainte d'incitation du pays 1 est en égalité. Il faut résoudre un système d'équations nonlinéaires avec deux inconnus et deux équations. Etant donné que les solutions risquent ne pas être sur les grids, on utilise l'interpolation linéaire. Les fonctions sont susceptibles de produire des solutions complexes, alors nous utilisons la méthode de quasi-Newton pour résoudre ce système d'équations.

2) Par contre, si $W_2^0(x)$ est inférieure à la valeur du défaut, la contrainte d'incitation du pays 2 est en égalité. On résoud les nouvelles consommations et les poids

relatifs comme pour le pays 1 ;

3) Sinon, les deux contraintes ne sont pas saturées, alors, le point initial reste le même pour calculer les consommations et les poids relatifs.

Etape 6 : Calculer les nouvelles fonctions de valeur

Cette étape est cruciale et on se trompe facilement dans cette étape. Il s'agit de renouveler les fonctions de valeur. En premier lieu, nous pouvons calculer les consommations selon les solutions issues de l'étape 4. En suite, puisque $z' \neq z$, il faut encore une fois utiliser l'interpolation pour calculer les nouvelles fonctions de valeurs,

$$W_i^1(x) = U(c_i^1(x)) + \beta \sum_{s^r} \pi(s'|s) W_i^0(x')$$

De même façon, nous faisons les étapes 4 à 6 pour chaque point dans l'espace d'état, et c'est pour cette raison là que cette méthode consomme beaucoup de temps, puisqu'il s'agit de résoudre au plus 1500 fois les systèmes d'équations pour une itération.

Une fois que tous les points sont vérifiés, nous avons des fonctions de valeurs pour tous les points

$$W_i^1(X) = U(c_i^1(X)) + \beta \sum_{s^r} \pi(s'|s) W_i^0(X')$$

Etape 7 : Convergence

Répéter l'étape 4 à l'étape 6, jusqu'à ce que

$$W_i^n(X) \approx W_i^{n-1}(X)$$

où n est le nombre d'itérations.

CHAPTER II

MULTIPLE CREDITORS, INFORMATION AND COORDINATION

Abstract[1]

Do better informed or more optimistic large creditors decrease the vulnerability of a project to creditor runs? To address this issue, we build a model in which a continuum of small creditors and a single large creditor independently decide whether to roll over their loans or foreclose based on their private information about the liquidity of the debtor and the project's return until its maturity. Our results show that an increase in the precision of the information or in the estimate on the value of continuation by the large creditor raises the willingness of small creditors to roll over their loans, and reduces the probability of project failure. The model has implications for solving the coordination problem of sovereign debt: (i) the existing mechanisms for handling sovereign default are evaluated; (ii) we propose a mechanism that can generate the same outcome as a national bankruptcy court.

Key words: Coordination; Sovereign default.

2.1 Introduction

In a competitive setting, creditors are reluctant to share information about the fundamentals of their debtor in financial distress. Each creditor has information about other creditors such as the quality of information in their possession, their number and their investment decisions. However, this set of information is often soft in the sense that it is not verifiable. These creditors cannot coordinate perfectly when making their

[1] This chapter is co-authored with Claude Fluet.

investment decisions, such as rolling over the loans or foreclosing. This coordination failure could trigger or worsen financial distress inducing important economic and financial costs. For instance, Andrade and Kaplan (1998) report that the *ex post* financial costs of distress are between 10 and 23 percent of firm value, and at bankruptcy, these costs can go up to more than 30 percent of firm value; for countries, Eichengreen (2004) estimates that over the last quarter century, financial crises have reduced incomes of developing countries by around 25 percent.

At a national level, the coordination problem is often solved by bankruptcy filings. However, in an international context, there is no such bankruptcy procedure for dealing with sovereign default. In addition, sovereign bonds are hold by a small number of large creditors and a large number of small creditors. For instance, Sgard (2005) reports that in 2001 56.5 percent of Argentine debt is hold by a small number of large creditors, and 43.5 percent by more than 1/4 million of small creditors. This leads to the following questions: (*i*) how to characterize the equilibrium when there are two types of creditors (a small one and a large one), and the return of the project is uncertain if it remains in operation; (*ii*) whether small creditors rely more on the information of the large creditor as the large creditor's information becomes more accurate; (*iii*) whether the existing mechanisms designed to solve the coordination problem of sovereign debt are as effective as a national bankruptcy court; (*iv*) if they are not, whether there is a mechanism able to generate the same outcome as a national bankruptcy court.

This chapter addresses these issues by developing a model in which a continuum of small creditors and a single large creditor independently decide whether to roll over their loans or foreclose based on their private information about the liquidity of the debtor and the value of continuation with three principal contributions. First, this chapter characterizes the equilibrium of a global game with two types of creditors that are uncertain about the value of continuation. Second, it provides general conclusions on the equilibrium effects when the large creditor's information and perception of continuation become more precise and optimistic (*i.e.* the continuation of the project is profitable, or the business model of the debtor is sustainable.), respectively. Third, it contributes to the international finance literature by evaluating the actual and some proposed mechanisms for solving the coordination problem of sovereign debt, and proposes a mechanism for solving sovereign default that can generate the same outcome as a national bankruptcy court.

Concerning the first question, introducing uncertainty about the value of continuation makes the strategic interaction between small creditors and a large one even richer. Models with global games, developed by Calsson and van Damme (1993a, b) and Morris and Shin (2001), treat one type of player. Corsetti, Dasgupta, Morris and Shin (2004) introduce a large player into a global game to study the role of large traders in currency attacks. Corsetti, Guimaraes and Roubini (2006) model the IMF as a large lender to show that the lending of the IMF can be complementary to private loans if its lending is conditional on the fundamentals of the debtor. In these games, the value of continuation is known with certainty. However, there is a good reason for creditors to be concerned not only about the financial capacity of the debtor to meet his claims and the actions of others, but also about the value of continuation, since at the interim stage when they have to decide to roll over or foreclose this information is not perfectly known. We have a game of incomplete information that contains both a fundamental component and a collective-action component. A creditor's payoff depends on the unknown economic fundamentals (liquidity and future return of continuation) of his debtor, his own action, and the actions of others. Other creditors' actions are in turn determined by their beliefs. Thus, creditors make their investment decisions by taking into account the beliefs of others in the spirit of the beauty-contest game as described in Keynes (1936):

> "It is not a case of choosing those [faces] which, to the best of one's judgment, are really the prettiest, nor even those which average opinion genuinely thinks the prettiest. We have reached the third degree where we devote our intelligence to anticipating what average opinion expects the average opinion to be. And there are some, I believe, who practice the fourth, fifth and higher degree."

Though anticipating the beliefs of others is intuitively appealing, it is a challenging task to keep tracking higher-order beliefs of others with a large number of players. A global game provides a simple procedure able to generate the same equilibrium outcome. With this procedure, each creditor chooses the best action to a uniform belief over the proportion of other creditors choosing a certain action. The equilibrium is constructed by assuming that each player adopts a switching strategy in which a creditor rolls over whenever his estimate of the underlying fundamentals is higher than some given threshold. Otherwise, he forecloses. The equilibrium is characterized by two critical switching points respectively for the small creditors and the large creditor, and two critical failure points: one for the situation where the incidence of foreclosure from the small creditors is sufficient to make the project fail, and the other for the situation where additional pressure from the foreclosure of the large creditor is needed to make the project fail. The unique switching equilibrium is shown to be the only equilibrium strategy to survive the iterative elimination of strictly dominated strategies.

As regards the second question, our results show that as a general rule, a higher precision of the large creditor's information relative to small creditors increases the willingness of small creditors to roll over their loans, and reduces the probability of project failure. Intuitively, if the large creditor has arbitrarily more precise information on the fundamentals of the debtor, his own switching point is lowered. Since, the switching point of the small creditors is positively related to that of the large creditor, it is reduced as well. This result can be explained by the behavior of relying on the information of others. Knowing that there are two types of creditors, a small creditor relies on the precise information of the large creditor in order to minimize the error of foreclosing too much, while the large creditor takes into account the risk of an overwhelming foreclosure by small creditors in order to minimize the the error of rolling over too much while others foreclose such that the project is failed. However, the degree of relying on the information of the large creditor dominates that of the small creditors. Thus, increasing the accuracy of the large creditor's information makes small creditors more willing to roll over.

Furthermore, a more optimistic estimate about the value of continuation by the large creditor makes small creditors more willing to roll over their loans. This channel of strategic interaction has not been explored in previous studies. Interestingly, if the large creditor is optimistic about the continuation of the project, this makes small creditors more confident to roll over their loans. Therefore, small creditors rely not only on the more precise information of the large creditor, but also on his estimate about the value of continuation. Relying on the information of others aggregates the information, but only partially because the expectation of one type of creditors about the information of others coincides with the true value at a very weak probability. Therefore, inefficient liquidation or rolling over may occur because of the imperfect coordination among creditors[2]. Finally, though the size effect of the large creditor on the willingness of small creditors to roll over is not always positive, the total size effect is always negative. This suggests that a project financed by a larger number of creditors is more vulnerable to fail.

For the third question, our results suggest that the provision of liquidity by international institutions such as the International Monetary Fund (IMF) to a distressed sovereign debtor with *ex post* conditionalities confirms the beliefs of creditors that the fundamentals of the debtor are not sound enough to get loans rolled over. This result is supported by a number of empirical studies surveyed by Bird and Rowland (2000), Cottarelli and Giannini (2002). Next, the model also suggests that collective action clauses (CACs), proposed in particular[3] by Taylor (2002), can solve the coordination problem of one bond issue under the condition that the number of creditors reversing their investment decisions is large enough. However, this contractual approach does not suffice to solve the coordination problem of sovereign default with multiple bond issues because it does not specify how to aggregate the different preferences of credi-

[2]These results are in line with those reported by Fluet and Garella (2006) in a recontracting game with one debtor, and two creditors who receive private information that can differ in precision to address the issue that creditors may not fully make use of the information in their possession.

[3]Others who have argued in favor of CACs include Eichengreen and Portes (1995), Eichengreen and Ruehl (2000), Buchheit, Gulati and Mody (2002), and Bartholomew (2002).

tors. Kreuger (2001) and White (2002) among others[4] propose a bankruptcy procedure more or less based on the U.S. Chapter 11 (corporate reorganization) and Chapter 9 (municipalities reorganization) to deal with sovereign default. This approach can, as a national bankruptcy court, solve the collective-action problem. However, it is difficult to implement due to the fact that there is no existing supranational bankruptcy court and an international treaty would require legal changes by a large number of countries.

Finally, to address the last question, examining closely the above mechanisms suggests that a new proposal should be able to: (*i*) solve the coordination problem; (*ii*) encourage the sovereign debtor to improve its creditworthiness; (*iii*) be implemented without large scale statutory changes. We propose a new mechanism, which satisfies the above criteria. This mechanism suggests that an international institution such as the IMF could: (*i*) impose *ex ante* the conditionalities to each sovereign debtor that accepts to participate in the program by specifying policy adjustments specific to the country; (*ii*) evaluate credibly the fundamentals of the sovereign debtor and the degree of fulfillment of policy adjustments specified by the conditionalities; (*iii*) decide the loan of last resort based on the evaluation. The loan of IMF serves as a positive signal to private creditors to roll over their loans in the sense that its loan is complementary to private loans.

The chapter is organizes as follows. We present the model in Section 2.2, and solve the equilibrium in Section 2.3. Then, we present the equilibrium properties in Section 2.4. The policy implications of the model in an international context are presented in Section 2.5. Finally, Section 2.6 provides some conclusions.

2.2 The model

This section first describes the players, timing, and payoffs of the game. Then, it displays the information structure of two types of creditors. Last, it presents two special cases to set a benchmark for our main results. The first is when all creditors are

[4] See Kreuger (2004), Sachs (1999), Schwarcz (2000), Chun (1996) and Macmillan (1995).

small, and the second is the case when the project is financed solely by a large creditor.

2.2.1 Players, timing, payoffs, and perfect information

The game involves an entrepreneur, a continuum of small creditors indexed by the unit interval $[0, 1]$, and a large creditor. There are three event dates. At date 0, the entrepreneur seeks financing for a large project. The amount to be raised is normalized to 1. A continuum of creditors and a large creditor provide the funds by means of debt contracts. For simplicity, the opportunity cost of funds is zero. The fraction of loans financed by the large creditor is $p \in [0, 1]$, and $1 - p$ the proportion of loans financed by small creditors. With a certain probability, the project is completed by date 1 and yields a return of $V > 1$, where V is sufficiently large for the net expected value of the project to be positive and thereby creditors are willing to sign the contracts *ex ante*. We focus on the situation when the project runs into problems and is not completed by date 1. At this interim stage, creditors have to reconsider their investments by deciding to roll over their loans or to foreclose. Their decisions decide whether the project can remain in operation until date 2.

At the interim stage, each creditor has a choice of either rolling over the loan until date 2, or foreclosing. Let θ denote the liquidity of the debtor, which is defined as the financial capacity of the debtor to meet his claims. If the incidence of foreclosure, denoted by zl, where z represents the mass of creditors and l the mass of creditors who foreclose, is less than the liquidity of the debtor ($\theta \geq zl$), then the project remains in operation. Otherwise, the project has to be liquidated immediately, yielding a value $L < 1$ (*i.e.* liquidating the project induces a loss)[5]. If the project remains in operation, it delivers a random return $\tilde{V}$ at date 2. Uncertainty about continuation return, a realistic concern for creditors, allows us to illustrate the coordination problem under two circumstances. One is that continuation leads to the success of the project at date

[5]Liquidation values are assumed to be known with certainty. One could argue that these values are more likely random. However, using expected liquidation values in the model does not change the results.

2, and rolling over is a right decision. Then, coordination failure induces inefficient liquidation. The other is that the project fails even after continuation, and rolling over is a bad decision. Without further private information, the unconditional expected return of the project $\overline{V} = E(\tilde{V})$ is assumed to be smaller than the liquidation value. These values are ordered as follows.

Assumption 1 $E(\tilde{V}) < L < 1$.

The payoff of a creditor depends on his action and the state induced by the actions of other creditors. When a creditor rolls over the loan and the project remains in operation, he gets a random actualized payment χ, which follows a normal distribution with mean μ_{χ} and variance $1/\sigma_{\chi}$. However, if the project discontinues, he gets a a normalized value 0[6]. Foreclosing always brings a recovery of $\overline{L}$. This gives a matrix of gains for a creditor:

Table 2.1 Matrix of gains

State	Continuation	Discontinuation
Roll over	χ	0
Foreclose	$\overline{L}$	$\overline{L}$

The gains are ordered as follows.

Assumption 2 $0 < E(\chi) = \mu_{\chi} < \overline{L}$.

The sum of the unconditional expected payoff from continuation over all creditors equals $E(\tilde{V})$. Therefore, $E(\chi) = \mu_{\chi}$ satisfies $\mu_{\chi} < \overline{L}$. If no additional information becomes available, from a collective standpoint, it is best to liquidate at date 1. Therefore, foreclose is a dominant strategy.

[6]Creditors who decide to foreclose seize more valuable collateral to sell or some of them make a preemptive grab and force the firm to liquidate its assets at fire sale prices.

However, at the interim stage, creditors receive information on the fundamentals of the debtor θ and χ. If creditors observe perfectly the fundamentals, the optimal strategy for a creditor is to foreclose when: the financial capacity of the debtor is strictly negative, or/and the liquidation value is larger than the return of rolling over (*Condition 1*). It is optimal for a creditor to roll over if the debtor is liquid and the return of rolling over is strictly larger than the value of liquidation (*Condition 2*).

The decision to foreclose is socially optimal. However, the decision of rolling over is not completely socially optimal because of the coordination problem, which is illustrated as follows. The mass of creditors who foreclose becomes 1 under Conditions 1. It becomes 0 under Condition 2. However, for $\theta \in (0, z)$, and $\chi > \overline{L}$, the mass of creditors who foreclose or roll over their loans is between 0 and 1. Therefore, there is a problem of coordination. For a creditor, if all other creditors roll over their loans, then the payoff to rolling over the loan is χ. However, if everyone else recalls the loan, the payoff is 0. In any of the states, the payoff to foreclosing is $\overline{L}$.

2.2.2 Imperfect information

When the fundamentals are not perfectly observable, each creditor decides to roll over or foreclose under the interaction of triple uncertainty[7]: future return uncertainty, liquidity uncertainty, and strategic uncertainty. The random variable, representing debtor's liquidity, is assumed to follow a normal distribution with mean μ and variance $\frac{1}{\sigma}$. This can be considered as public information or creditors' common priors with precision σ. Before taking an action, creditors receive imperfect information on the debtor's financial capacity to meet his claims and the value of continuation. Small creditors receives noisy signals, x_i for financial capacity of the debtor and $x_{i\chi}$ for the

[7] In a standard model on coordination problem of creditors, such as in Morris and Shin (2004), creditors face double uncertainty: liquidity uncertainty and strategic uncertainty. Under this setup, if everyone rolls over, the project will succeed with certainty.

value of continuation, such that

$$x_i = \theta + \eta_i \tag{2.1}$$

$$x_{i\chi} = \chi + \eta_{i\chi}, \tag{2.2}$$

where η_i and $\eta_{i\chi}$ are normally distributed with mean 0, precision α_i, and $\alpha_i\chi$ respectively. Their cumulative distribution functions are denoted respectively by $G(.)$ and $G_\chi(.)$. By the same token, the large creditor receives noisy signals, y for financial capacity of the debtor and y_χ for the value of continuation, such that

$$y = \theta + \epsilon \tag{2.3}$$

$$y_\chi = \chi + \epsilon_\chi, \tag{2.4}$$

where ϵ and ϵ_χ are normally distributed with mean 0 and precision β and β_χ. Their cumulative distribution functions are denoted respectively by $H(\cdot)$ and $H_\chi(\cdot)$.

For a small creditor, the posterior of his belief on the fundamentals given the signals is obtained through a simple updating rule.

$$X = \frac{\sigma\mu + \alpha x_i}{\sigma + \alpha} \tag{2.5}$$

$$X_\chi = \frac{\sigma_\chi\mu_\chi + \alpha_\chi x_{i\chi}}{\sigma_\chi + \alpha_\chi} \tag{2.6}$$

In a similar way, the posterior of the large creditor is

$$Y = \frac{\sigma\mu + \beta y}{\sigma + \beta} \tag{2.7}$$

$$Y_\chi = \frac{\sigma_\chi\mu_\chi + \beta_\chi y_\chi}{\sigma_\chi + \beta_\chi} \tag{2.8}$$

Now, we consider the strategies of creditors. A strategy for a creditor of is a decision rule which maps each realization of the signal to an action that consists of foreclosing or rolling over the loan. It can be typified as naive and sophisticated, defined as follows.

Definition 1 *A naive strategy is a decision rule based only on the private information in the possession concerning the fundamentals without considering the beliefs of others. A sophisticated strategy is a decision rule based not only on the private information in the possession concerning the fundamentals but also by taking the beliefs of others into account.*

For competitive considerations, neither creditors of different types nor creditors of the same type share information such that the information is not aggregated. If a creditor adopts a naive strategy, then he will foreclose if his signals reveal that the fundamentals are not sound, otherwise he will roll over despite the actions of others. This makes the game as a single player's decision problem.

Since the payoff of a creditor depends on the actions of others, he will be better off adopting a sophisticated strategy. If a creditor forecloses, while the mass of creditors that foreclose is not large enough to make the project fail, then he can only recover $\overline{L}$ by losing the opportunity of getting fully repaid. If the mass of creditors that foreclose is large enough to make the project fail, then he sill gets $\overline{L}$ without opportunity cost. When he rolls over the loan and the project remains, he will be fully repaid. If other creditors foreclose and make the project fail, he will recover only 0 by losing the opportunity of getting a larger recovery $\overline{L}$. Hence, even a creditor has precise information about the fundamentals, it is not rational to act as if he were the sole player.

For a player, it is rational to take higher order beliefs of others into account, but to construct the equilibrium in a game with a continuum players, it is a challenging task to keep tracking each layer of a player's anticipation about the beliefs of others. Global game provides a simple procedure. As showed in Morris and Shin (2001), a simplistic strategy in which each creditor chooses the best action to a uniform belief over the proportion of other creditors choosing a certain action generates the same equilibrium outcome as a sophisticated strategy in which each creditor takes higher order beliefs of others into account. The equilibrium is constructed by assuming that each player adopts a switching strategy that is defined for the context with creditors as follows.

Definition 2 *A switching strategy is a strategy in which a creditor rolls over whenever his estimate of the underlying fundamentals is higher than some given threshold. Otherwise, he forecloses.*

A creditor forecloses when his estimate of the repayment χ is lower than the value of liquidation $\overline{L}$. When the estimate of $\chi > \overline{L}$, he will roll over whenever their estimate of the cash flow θ is higher than a some given threshold level, and foreclose if his estimate of θ is lower than this threshold. For a small creditor, these two critical values are

$$\begin{aligned} x_i &\equiv \frac{\sigma + \alpha}{\alpha} X - \frac{\sigma}{\alpha}\mu \\ x_{i\chi} &\equiv \frac{\sigma_\chi + \alpha_\chi}{\alpha_\chi} X_\chi - \frac{\sigma_\chi}{\alpha_\chi}\mu_\chi \end{aligned}$$

For the large creditor, the critical values are

$$\begin{aligned} y &\equiv \frac{\sigma + \beta}{\beta} Y - \frac{\sigma}{\beta}\mu \\ y_\chi &\equiv \frac{\sigma_\chi + \beta_\chi}{\beta_\chi} Y_\chi - \frac{\sigma_\chi}{\beta_\chi}\mu_\chi \end{aligned}$$

Before solving the game with two types of creditors, we present a brief discussion of two special cases to set a benchmark for our main results. The first is when all creditors are small ($p = 0$), and the second is the case with only the large creditor ($p = 1$).

2.2.3 Small creditors only

The case with small creditors alone leads to the symmetric game of Morris and Shin (2004) with one difference that here the value of continuation is a random variable. Since at the interim stage, the signal on the project's return at date 2 is independent of collective action, the critical value of $\tilde{V}$, at which the project is not worthy of continuing, can be expressed in terms of payoff such that the critical value of χ is $\overline{L}$. Thus the project fails if $\chi \leq \overline{L}$ or if $\theta < zl$ and $\chi > \overline{L}$; and succeeds if $\theta \geq zl$ and $\chi > \overline{L}$.

The critical value of the fundamentals θ at which the project is on the margin between failing and succeeding is $\theta = zl$. Each creditor of the same type possesses by the same way the information, and adopts the same switching strategy in which he forecloses if his updated signal falls below a critical value X^*. An equilibrium is defined as follows.

Definition 3 *An equilibrium is a profile of strategies such that the strategy of a creditor maximizes his expected payoff conditional on the information available, when all other creditors are following the strategies in the profile. Then, the equilibrium is characterized by a critical value θ^* below which the project will always fail, and a critical value of the individual signal x^* such that creditors receiving a signal below this value will always foreclose.*

The equilibrium is solved in two steps. The first step is to derive the critical mass condition. If the true state is θ combining with χ, creditors foreclose if they observe signals revealing that the expected return is below $\overline{L}$ or if the expected return is higher than $\overline{L}$ but the liquidity is below x^*. The probability that any particular creditor receives a signal below $\overline{L}$ is

$$\Pr(x_\chi \leq \overline{L}|\chi) = G_\chi(\overline{L} - \chi).$$

To ease notation, let G_χ denote $G_\chi(\overline{L} - \chi)$, then the probability that any particular creditor observes a signal higher than $\overline{L}$ and a signal on liquidity below x^* is

$$\Pr(x_\chi > \overline{L}, x_\theta \leq x^*|\chi, \theta) = (1 - G_\chi)G(x^* - \theta)$$

Since the noise terms are i.i.d, the proportion of creditors foreclosing l equals the sum of the two above probabilities. That is, it turns out that a creditor has a uniform belief[8] over l. Then, the failure point at which the project is liquidated θ^* is defined by the

[8] See Morris and Shin (2001) for more details.

following critical mass condition

$$\theta^* = z\left[G_\chi + (1 - G_\chi)\, G(x^* - \theta^*)\right]$$

Second, we derive the indifference condition between rolling over and foreclosing. Conditional on the updated signals, given θ^*, the creditor has the conditional probability of a successful continuation of

$$\Pr(\chi > \overline{L}, \theta > \theta^* | X_\chi, X) = \left(1 - G_\chi(\overline{L} - X_\chi)\right)(1 - G(\theta^* - X))$$

Let $\overline{G}_\chi$ denote $G_\chi(\overline{L} - X_\chi)$. The expected payoff of rolling over is $X_\chi\,(1 - G_\chi)\,(1 - G(\theta^* - X))$, while the payoff to foreclosure is $\bar{L}$. Hence, the indifference condition is

$$X_\chi\left(1 - \bar{G}_\chi\right)(1 - G(\theta^* - X^*)) = \overline{L} \tag{2.9}$$

which implies in terms of standard normal

$$\theta^* - X^* = \frac{\Phi_\theta^{-1}\left(1 - \frac{\overline{L}}{X_\chi(1-\bar{G}_\chi)}\right)}{\sqrt{\alpha + \sigma}}$$

Let $\lambda = \frac{\overline{L}}{X_\chi(1-\bar{G}_\chi)}$, then

$$\theta^* - X^* = \frac{\Phi^{-1}(1-\lambda)}{\sqrt{\alpha + \sigma}}$$

Together with the critical mass condition, we can solve for the two unknows X^* and θ^*. Since x^* is a function of X^*, the critical mass condition can be rewritten in terms of standard normal as

$$\begin{aligned}
\theta^* &= z\left[G_\chi + (1 - G_\chi)\,\Phi\left(\sqrt{\alpha}\left(\frac{\sigma + \alpha}{\alpha} X^* - \frac{\sigma}{\alpha}\mu - \theta^*\right)\right)\right] \\
&= z\left[G_\chi + (1 - G_\chi)\,\Phi\left(\frac{\sigma}{\sqrt{\alpha}}(\theta^* + \frac{\Phi^{-1}(\lambda)}{\sqrt{\alpha + \sigma}} - \mu) + \sqrt{\alpha}\frac{\Phi^{-1}(\lambda)}{\sqrt{\alpha + \sigma}}\right)\right] \\
&= z\left[G_\chi + (1 - G_\chi)\,\Phi\left(\frac{\sigma}{\sqrt{\alpha}}(\theta^* + \frac{\sqrt{\alpha + \sigma}\Phi^{-1}(\lambda)}{\sigma} - \mu\right)\right]
\end{aligned}$$

Solving for θ^*, we have

$$\theta^* - zG_X = z\left[(1 - G_X)\,\Phi\left(\frac{\sigma}{\sqrt{\alpha}}\left(\theta^* - \mu + \Phi^{-1}(\lambda)\,\frac{\sqrt{\alpha+\sigma}}{\sqrt{\sigma}}\right)\right)\right] \tag{2.10}$$

The failure point θ^* is obtained as the intersection between the line $\theta^* - zG_X$ and a scaled-up cumulative distribution with mean $\mu - \Phi^{-1}(\lambda)\sqrt{\alpha+\sigma}/\sqrt{\sigma}$ and precision σ^2/α. Modeling the value of continuation as a random variable brings insights to the coordination problem in a general context. If continuation of the project is profitable, the interval $[0, \theta^*]$ represents the incidence of inefficient liquidation. Otherwise, it represents the incidence of inefficient rolling over. The equation (2.10) has a unique solution if the term on the right hand side has a slope that is less than 1 everywhere. The slope is given by

$$z(1 - G_X)\frac{\sigma}{\sqrt{\alpha}}\phi(\cdot)$$

where $\phi(\cdot)$ is the density of the standard normal. When the value of θ is identical with the mean, the largest value of the density of the standard normal is

$$\phi(\cdot) \leq \frac{1}{\sqrt{2\pi}}$$

Thus a sufficient condition for a unique solution for θ^* is

$$\frac{\sigma}{\sqrt{\alpha}} \leq \frac{\sqrt{2\pi}}{z(1 - G_X)} \tag{2.11}$$

Recall that $(1 - G_X)$ represents the probability of observing a signal on the return of continuation higher than the liquidation value. When continuation is profitable with certainty, we have a case of Morris and Shin (2004). The condition (2.11) is satisfied whenever the private information is precise enough relative to the underlying uncertainty. If everyone observes a signal that the return of continuation is below the liquidation value, $G_X = 1$, everyone forecloses. However, for $(1 - G_X) \in (0, 1)$, introducing uncertainty about the return of continuation relaxes the condition for the unique equilibrium. Thus, the condition is still satisfied when the private information is less precise than the case of Morris and Shin (2004). The equilibrium is affected by the introduction of uncertainty on the return of continuation. The following proposition summarizes our result.

Proposition 1 *Both thresholds θ^* and X^* are decreasing in $(1 - G_X)$.*

Proof. See Appendix B.1. ■

Ceteris paribus, a higher probability that continuation is profitable increases the willingness by creditors to roll over their loans. The thresholds θ^* and X^* are higher in comparison to the case of Morris and Shin (2004). Uncertainty about the return of continuation raises the probability of project failure.

2.2.4 Large creditor only

The other opposite extreme case of $p = 1$, in which there is a large creditor, reduces the game into a single player's decision problem. The creditor will roll over if and only if the expected payoff of rolling over is strictly higher than the liquidation value, that is when

$$Y_X \left(1 - H_X(\overline{L} - Y_X)\right)\left(1 - H(z - Y)\right) > \overline{L}$$

where $z = \theta^*$ is used. Expressing this condition in terms of standard normal, the creditor will roll over if and only if his updated signal

$$Y > Y^* = z - \frac{1}{\sqrt{\sigma + \beta}}\Phi^{-1}\left(1 - \frac{\overline{L}}{Y_X\left(1 - H_X(\overline{L} - Y_X)\right)}\right) \tag{2.12}$$

Differentiating equation (2.12) with respect to β, we have

$$\frac{dY^*}{d\beta} = -\frac{1}{(\sigma + \beta)^{3/2}}(z - Y^*) < 0$$

Proposition 2 *The large creditor's critical switching point Y^* is decreasing in β.*

A higher precision of information by the large creditor decreases his switching point, and thereby reduces the probability of project failure. In addition, note that the trigger Y^* is smaller than z, but tends to z as the precision $\beta \to \infty$. In this case, the decision of rolling over or foreclosing tends to be socially optimal.

2.3 Equilibrium with 2 types of creditors

We now turn to the general case of $p \in (0,1)$, in which there are both small and large creditors. We prove that there is a unique switching equilibrium, and this unique switching equilibrium is the only equilibrium strategy to survive the iterative elimination of strictly dominated strategies. The equilibrium is solved by assuming that both types of creditors follow their respective trigger strategies around the switching points X^* and Y^*. With two types of creditors, we should consider two situations under which the project fails at the interim stage. This first is when foreclosure by small creditors alone is sufficient to make the project fail. The second is when additional foreclosure from the large creditor is needed to make the project fail. To ease notation, we will set $\sigma/\alpha \to 0$, and $\sigma/\beta \to 0$. This implies either public information $\to 0$ for α and β finite or α, $\beta \to \infty$ for σ finite,

$$\begin{aligned} \lim_{\sigma/\alpha \to 0} x &= X, \\ \lim_{\sigma/\beta \to 0} y &= Y, \end{aligned}$$

so that public information is disregarded in building our equilibrium.

For the first situation, conditional on a given θ, the proportion of creditors that foreclose is $G_X + (1 - G_X)\, G(X^* - \theta)$. The project fails at θ if

$$z(1-p)\left(G_X + (1 - G_X)\, G(X^* - \theta)\right) > \theta.$$

Let $\underline{\theta}$ be defined by:

$$\underline{\theta} = (1-p)z\left[G_{\chi} + (1-G_{\chi})\,G\left(X^{*}-\underline{\theta}\right)\right] \tag{2.13}$$

Whenever θ is below $\underline{\theta}$, the project fails irrespective of the action of the large creditor.

Next, we consider the value of θ, below which the project fails if additional foreclosure from the large creditor is needed. The incidence of foreclosing at θ attributable to the small creditors is $(1-p)z\left[G_{\chi} + (1-G_{\chi})\,G\left(X^{*}-\theta\right)\right]$. Then, the additional incidence brought by the large creditor is p. Therefore, the project fails at θ if

$$z\left[p + (1-p)\left(G_{\chi} + (1-G_{\chi})\,G\left(X^{*}-\theta\right)\right)\right] > \theta$$

The critical value at which the project fails if and only if both types of creditors foreclose is defined as

$$\overline{\theta} = z\left[p + (1-p)\left(G_{\chi} + (1-G_{\chi})\,G\left(X^{*}-\overline{\theta}\right)\right)\right] \tag{2.14}$$

Note that $\overline{\theta}$ lies above $\underline{\theta}$. Both $\underline{\theta}$ and $\overline{\theta}$ are functions of the switching point X^{*}. In turn, X^{*} will depend on the large creditor's switching point Y^{*}. The equilibrium is characterized by two critical failure points $\left\{\underline{\theta},\overline{\theta}\right\}$, and two switching points $\{X^{*}$, $Y^{*}\}$. To solve these unknows simultaneously, we need to derive two other indifference conditions. For the large trader, conditional on his posterior, he assigns probability $\left(1-H_{\chi}(\overline{L}-Y_{\chi})\right)\left(1-H(\overline{\theta}-Y)\right)$ to the joint events that $\chi > \overline{L}$ and $\theta \geq \overline{\theta}$. Since his conditional expected payoff of rolling over is Y_{χ}, his optimal strategy is to roll over if and only if $Y \geq Y^{*}$, where Y^{*} is defined by the indifference condition

$$Y_{\chi}\,(1-H_{\chi})\,(1-H(\underline{\theta}-Y^{*})) = \overline{L} \tag{2.15}$$

Now, consider a small creditor's problem. The posterior density over θ for this creditor is $g(\theta - X)$ with precision $\sigma + \beta$. When $\theta \leq \underline{\theta}$, foreclosure by small creditors alone is sufficient to make the project fail. When $\theta \in (\underline{\theta}, \overline{\theta}]$, the project fails if the large creditor forecloses, while if $\theta > \overline{\theta}$, the project remains regardless of the actions of the creditors. Thus, the expected payoff to roll over conditional on updated signals X and X_X is

$$X_X\left(1-\bar{G}_X\right)\left[1-\left(\int_{-\infty}^{\underline{\theta}} g(\theta-X)d\theta+\int_{\underline{\theta}}^{\overline{\theta}} g(\theta-X)H(Y-\theta)d\theta\right)\right]$$

In the region $(-\infty, \underline{\theta}]$, a small creditor receiving signal X will assign probability $\int_{-\infty}^{\underline{\theta}} g(\theta-X)d\theta$ to the event that the project fails regardless of the actions of the large creditor. In the region of $(\underline{\theta}, \overline{\theta}]$, the project fails if the large creditor forecloses. The probability that the large creditor forecloses at θ, given his trigger strategy around Y^*, is $H(Y-\theta)$. Then, the indifference condition is given by

$$G(\underline{\theta}-X^*)+\int_{\underline{\theta}}^{\overline{\theta}} g(\theta-X^*)H(Y^*-\theta)d\theta = 1-\frac{\overline{L}}{X_X\left(1-\bar{G}_X\right)} \tag{2.16}$$

Before proving that there is a unique X^* solving this equation, we present our result regarding to the equilibrium by the following proposition.

Proposition 3 *There is a unique dominance solvable equilibrium in the game in which the large creditor uses the switching strategy around Y^*, while the small creditors use the switching strategy around X^*.*

First, we will show that there is a unique X^* that solves equation (2.16). Second, we will show this unique switching equilibrium is dominance solvable (See the second part of our argument in Appendix B.2.). Let

$$\begin{aligned} k &\equiv \theta - X^* \\ \underline{\delta} &\equiv \underline{\theta} - X^* \\ \overline{\delta} &\equiv \overline{\theta} - X^*. \end{aligned}$$

Then, the left hand side of the equation (2.16) can be rewritten as

$$G(\underline{\delta}) + \int_{\underline{\delta}}^{\overline{\delta}} g(k) H\left(\underline{\delta} - k - H^{-1}\left(1 - \overline{\lambda}\right)\right) dk \tag{2.17}$$

where the switching point for the large creditor is replaced by

$$Y^* = \underline{\theta} - H^{-1}\left(1 - \frac{\overline{L}}{Y_X(1 - H_X)}\right)$$

Let $\overline{\lambda} = \frac{\overline{L}}{Y_X(1-H_X)}$, then

$$Y^* = \underline{\delta} + X^* - H^{-1}\left(1 - \overline{\lambda}\right)$$

Hence, the indifference condition of a small creditor (2.16) gives

$$G(\underline{\delta}) + \int_{\underline{\delta}}^{\overline{\delta}} g(k) H\left(\underline{\delta} - k - H^{-1}\left(1 - \overline{\lambda}\right)\right) dk - 1 + \frac{\overline{L}}{X_X\left(1 - \bar{G}_X\right)} = 0 \tag{2.18}$$

Equations (2.13) and (2.14) can be rewritten as

$$\begin{aligned} \underline{\delta} &= (1-p)z\left[G_X + (1 - G_X)\, G\left(-\underline{\delta}\right)\right] - X^* \\ \overline{\delta} &= z\left[p + (1-p)\left(G_X + (1 - G_X)\, G\left(-\overline{\delta}\right)\right)\right] - X^* \end{aligned}$$

both $\underline{\delta}$ and $\overline{\delta}$ are monotonically decreasing in X^*, since

$$\begin{aligned} \frac{d\underline{\delta}}{dX^*} &= -\frac{1}{1 + (1-p)z(1 - G_X)g(-\underline{\delta})} < 0 \\ \frac{d\overline{\delta}}{dX^*} &= -\frac{1}{1 + (1-p)z(1 - G_X)g(-\overline{\delta})} < 0 \end{aligned}$$

Note that (2.18) is strictly increasing in both $\underline{\delta}$ and $\overline{\delta}$, since

$$\begin{aligned} g(\underline{\delta})\overline{\lambda} + \int_{\underline{\delta}}^{\overline{\delta}} g(k) h\left(\underline{\delta} - k - H^{-1}\left(1 - \overline{\lambda}\right)\right) dk &> 0 \\ g(\overline{\delta}) H\left(\underline{\delta} - \overline{\delta} - H^{-1}\left(1 - \overline{\lambda}\right)\right) &> 0 \end{aligned}$$

Hence, (2.18) is strictly decreasing in X^*. For sufficiently small X^*, (2.18) is positive, while for sufficiently large X^*, it is negative. The left hand side of (2.18) is continuous in X^*. Thus there is a unique solution to (2.18). From (2.15), the large creditor's switching point Y^* is determined.

The conditions for uniqueness turn out to be sufficient for the dominance solvability. See Milgrom and Roberts (1990), Vives (1990), and Morris and Shin (2004) for more details.

2.4 Equilibrium properties

A distinctive feature of our model is that the return of continuation is uncertain at the interim stage. This feature allows the debt problem to have both a fundamental component and a collective-action component. Each creditor faces triple uncertainty on continuation, liquidity, and the actions of other creditors. The presence of a large creditor affects the strategy of the small creditors, and makes the equilibrium effects of these uncertainties even richer. In this section, we analyze the effects of the presence of the large creditor and of the increasing size of all creditors. More specifically, we can articulate our analysis addressing the following five questions:

1. Does a larger creditor increase the willingness of the small creditors to roll over?
2. Does a higher precision of the information about the financial capacity of the debtor by the large creditor relative to the small creditors matter? In other words, do small creditors rely more on the information of the large creditor as the large creditor's information becomes more accurate?
3. Does a more optimistic perception of the large creditor about the value of continuation increase the willingness of the small creditors to roll over?
4. Do small creditors rely more on the large creditor's information on the value of continuation as the latter's information becomes more precise?
5. What is the effect of a larger credit financing?

As regards question 1 above, we summarize our comparative static exercise by means of the following proposition.

Proposition 4 *All thresholds ($\underline{\theta}$, $\overline{\theta}$, X^*, and Y^*) are decreasing in p if and only if*

$$c_3 \frac{zG\left(X^* - \underline{\theta}\right)}{1 + z(1-p)g\left(X^* - \underline{\theta}\right)} > c_4 \frac{z\left(1 - G\left(X^* - \overline{\theta}\right)\right)}{1 + z(1-p)g\left(X^* - \overline{\theta}\right)}$$

, where $c_3 = g(\underline{\delta})\overline{\lambda} + \int_{\underline{\delta}}^{\overline{\delta}} g(k)h\left(\underline{\delta} - k - H^{-1}\left(1-\overline{\lambda}\right)\right)dk$*, and* $c_4 = g(\overline{\delta})H\left(\underline{\delta} - \overline{\delta} - H^{-1}\left(1 - \overline{\lambda}\right)\right)$.

Proof. See Appendix B.3. ■

Interestingly, it is not always possible to give a definitive answer to the question whether all thresholds are decreasing in the size of the large creditor. Under the above condition, all thresholds are decreasing in the size of the large creditor. A larger proportion of loans financed by the large creditor raises the proportion of small creditors who are willing to roll over their debt at any level of the future return of continuation and liquidity. Since the return of continuation and liquidity of the debtor are normally distributed, if $\underline{\theta}$, $\overline{\theta}$, X^*, and Y^* are all decreasing in p, the *ex ante* probability of project failure also falls with p. Then, the answer to question 1 is that under certain condition a larger size of the large creditor indeed lowers the likelihood of project failure.

Question 2 raises a central issue in our analysis regarding the equilibrium effect, if any, of improving the quality of the large creditor's information. What happens when the large creditor's information becomes more precise? The following proposition synthesizes our result.

Proposition 5 *An increase in the precision of the large creditor's information on the financial capacity of the debtor decreases all thresholds ($\underline{\theta}$, $\overline{\theta}$, X^*, and Y^*).*

Proof. See Appendix B.4. ■

Ceteris paribus, a higher precision of information concerning the financial capacity of the debtor by the large creditor increases the willingness by small creditors to roll over their loans, and thereby reduces the probability of project failure. Intuitively, if the large creditor has arbitrarily more precise information on the liquidity of the debtor, his switching point will be reduced. It in turn lowers the switching point of the small creditors, since it is a function of the switching point of the large creditor. In such a simultaneous game, the equilibrium is solved without imposing a higher ability for a creditor to trace higher order beliefs of others. However, the same equilibrium outcome can be reached by modeling the anticipation of each creditor about the actions of the others. Our result can be implied by the behavior of relying on the information of others. Knowing that there are two types of creditors. With a sophisticated strategy, an small creditor relies on the precise information of the large creditor in order to minimize the error of foreclosing too much by losing the opportunity of getting full repayment. The large creditor takes into account the risk of an overwhelming foreclosure by small creditors in order that he can minimize the error of rolling over too much while others foreclose to make project fail by getting only 0, which is smaller than $\overline{L}$. In equilibrium, the degree of relying on the information of the large creditor dominates that of the small creditors. Thus, increasing the accuracy of the large creditor's information makes small creditors more willing to roll over.

Since, the return of continuation is a random variable at the interim stage, question 3 raises an issue regarding the equilibrium effect of a more optimistic perception by the large creditor about continuation. The following proposition synthesizes our result.

Proposition 6 *An increase in the probability that the large creditor observes a signal on the return of continuation higher than the liquidation value $1 - H\chi$ decreases all thresholds.*

Proof. See Appendix B.5. ■

A higher probability that the large creditor observes a signal on the return of continuation higher than the liquidation value increases the willingness by small creditors to roll over their loans, and reduces the probability of project failure.

This effect is reciprocal, as showed by the following proposition.

Proposition 7 *All thresholds ($\underline{\theta}$, $\overline{\theta}$, X^*, and Y^*) are decreasing in* $1 - G_\chi$.

Proof. See Appendix B.6. ■

A higher probability that small creditors observe a signal on the return of continuation higher than the liquidation value has a positive effect on the willingness by the large creditor to roll over his loans.

Now, concerning the estimate of the future return of continuation, what happens if a large creditor has a higher estimate on the return of continuation? The following proposition gives the answer to this question.

Proposition 8 *An increase in the estimate of the return of continuation by the large creditor Y_χ decreases all thresholds.*

Proof. See Appendix B.7. ■

A more optimistic perception on the return of continuation by the large creditor makes small creditors more confident to roll over their loans, and reduces the probability of project failure. Therefore, small creditors rely not only on the precise information of the large creditor, but also on his estimate on the value of continuation.

Does the positive information effect persists for the quality of the information on the return of continuation? The answer is summarized by the following proposition.

Proposition 9 *An increase in the precision of the large creditor's information on the return of continuation β_χ decreases all thresholds.*

Proof. See Appendix B.8. ■

Thus, the information effect on the return of continuation is still positive. An increase in the precision of the large creditor' information on the project's return makes small creditors more willing to roll over their loans.

Though, the size effect of the large creditor is not unambiguously defined, there is a definitive answer to the total size effect. The following proposition summarizes this result.

Proposition 10 *All thresholds are increasing in z.*

Proof. See Appendix B.9. ■

This implies that a larger number of creditors increases the probability of project failure. The static nature of our model cannot distinguish between short- and long-term claims on the project, but this result can still suggest that the greater the proportion of short-term debt in the capital structure, the more fragile is the project to creditor runs.

Though, improving the quality of information or the estimate about the project's return until date 2 reduces all thresholds, the interval of inefficient liquidation or rolling over remains. In other words, relying on the information or estimate of others aggregates the information or the estimate but only partially, because the anticipation of one type of creditors on the information or the estimate of the other type of creditors corresponds to the true value with a weak probability. Even, if everyone has arbitrarily precise information and optimistic estimate on the project's return, the interval of inefficient liquidation or rolling over persists because of strategic uncertainty. Setting α and β $\rightarrow \infty$, in limit, $\underline{\theta} = \overline{\theta} = X^* = Y^*$, but these thresholds are still above 0. This implies that there is always inefficient liquidation.

At a national level, the coordination problem is often solved by bankruptcy filings[9]. In spite of the similar coordination problem that is present for domestic debt and sovereign debt, in an international context, there is no sovereign bankruptcy court to handle sovereign default. Since a government cannot be insolvent in the normal sense of the word, the model treated in this paper is applicable to the situation of a solvent but illiquid sovereign debtor with asymmetrically informed creditors. The following section evaluates the current and some proposed mechanisms for solving sovereign default, using a national bankruptcy court as a benchmark.

2.5 Policy implications for sovereign debt

Sovereign default differs from corporate default in the following respects. First, sovereign default often involves a currency crisis and a financial crisis[10]. This increases both economic and political costs of the sovereign default. Second, few assets are available to repay creditors holding sovereign debt because of the sovereign immunity. Finally, most sovereign debt is unsecured and has equal priority if default occurs with the exception of the loans from the IMF and World Bank that have priority over other debt. In this section, we evaluate the current approaches dealing with sovereign default by treating the IMF as a third party, and propose a new mechanism generating the same outcome as a national bankruptcy court.

2.5.1 Current mechanism

When the mass of creditors who foreclose is large enough to put the country into default, the sovereign debtor in trouble could ask for new funds from the IMF in exchange of respecting the relative conditionalities or/and have loans reorganized by

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[9]In general, *ex post* the process of reorganization permits to correct the incidence of inefficient liquidation and makes a viable firm emerge from bankruptcy. The process of liquidation permits to correct the incidence of inefficient rolling over and make a non-viable firm cease to exist.

[10]This results generally from the fragility of financial and banking systems partly occurring as a result of deteriorated fundamentals, which themselves result from the imperfect coordination of private creditors (Kaminsky and Reinhart, 1998; Corsetti, Pesenti and Roubini, 1999).

creditors. One of arguments to support the international last resort lending[11] is to consider the loans of the IMF as catalytic finance, which are complementary to the international private loans[12]. By imposing *ex post* conditionalities and an assistant program, the lending of the IMF can have catalytic effect if and only if creditors believe the sovereign debtor, particularly the government will respect the conditionalities and adopt adjustment policies to improve its fundamentals. Otherwise, the lending is just a limited additional loan to the sovereign debtor. The principal reason why the lending of last resort combining with *ex post* conditionalities is not strategically complementary to international private loans is that the lending of last resort confirms the belief of creditors that the fundamentals are not sound enough and the improvement is uncertain. This is reflected in our model by increasing slightly the proportion of loans financed by the large creditor, that is $p' > p$.

Why is the change only slight? Compared to the total private loans, the loan of the IMF can not be very important, because the IMF has only limited financial resources to lend and has not only one country to lend but more. At the new equilibrium, the incidence of inefficient liquidation reduces slightly and the mass of creditors who roll over the loan increases only a little bit. Hence, the lending of last resort with *ex post* conditionalities has only very little catalytic effects, or even has no effects at all in the sense that it serves as a substitute to private loans. This result is supported by a number of empirical studies about catalytic effects of the IMF's lending, surveyed by Bird and Rowland (2000) and Cottarelli and Giannini (2002). In addition, the expected bailout from the IMF induces both the sovereign debtor and the private creditors to take more excessive risks in their investment decisions.

This leads to two proposals to reform the current arrangements dealing with sovereign default. The first calls for a statutory approach that provides a legal frame-

[11]See Fischer (1999) for the role of the IMF as an international lender of last resort.

[12]Corsetti, Guimaraes, and Roubini (2006), modeling the IMF as a large lender in a global model, find that the loans of the IMF are complementary to the private loans. In their model, the lending of the IMF is conditional on the fundamentals of the country in financial distress. This modeling strategy is closer to our proposal of lending with *ex ante* conditionalities than the actual practice of the IMF.

work to deal with sovereign bankruptcy. The second is a contractual approach that calls for more widespread use of collective action clauses (CACs). The statutory approach adopts many of the characteristics of Chapter 11 and Chapter 9 such as automatic stay, priority to new loans, and voting procedure. Even though this proposal can solve the coordination problem and enhance information sharing, it is difficult to implement due to the fact that there is no existing bankruptcy court and an international treaty would require legal changes by a large number of countries.

2.5.2 Contractual approach

Collective action clauses indicate that a super majority of bondholders, usually 75 percent, can change the terms of the bonds such as debt reduction and restructuring. By including collective action clauses into each bond, if a super majority of bondholders accept changes in the financial terms of the issue, these new terms would become binding on all its holders. The model suggests that this approach alone can resolve the problem of coordination if the mass of creditors who after having the set of information aggregated roll over the loans is large enough to have the majority to vote for a restructuring plan.

This situation is quite similar to friendly reorganization in a domestic context, which aggregates the set of information. Aggregating information can be interpreted as an increase in the precision of the large lender's information. This will decrease all thresholds. The plan of reorganization will succeed if the fundamentals θ are greater than zl where l represents the mass of creditors who refuse to roll over. In other words, the restructuring will succeed if the mass of creditors who decided to foreclose but after the negotiation reverse their decisions to roll over, plus the mass of creditors who from the beginning decide to roll over, are large enough to make the restructuring plan pass. However, if θ is smaller than zl, then this approach will not help the sovereign debtor out. Furthermore, collective action clauses might not suffice to deal with sovereign default with multiple obligations because they provide provisions for a specific instrument but do not specify a way of aggregating the preferences of creditors holding different issues.

2.5.3 Paris Club and London Club

Some informal groups such as Paris Club for sovereign creditors and London Club for private creditors are formed to reduce coordination problem[13]. If a consensus among members is reached, then these clubs can replace the large creditor in the model. Through negotiations, this large creditor has more precise information on the fundamentals of their debtor. In the case that the rolling over by the large creditor alone is enough to help the debtor out, the failure point is defined by pz. In the case that additional rolling over by small creditors is needed, we have a new equilibrium at which a higher precision of information or a more optimistic estimate on the fundamentals of their debtor by the large creditor increases the willingness by small creditors to roll over their loans. Thus, the model suggests that as far as a consensus among creditors can be reached, these clubs reduce inefficient liquidation or rolling over.

2.5.4 Transparency

The model also suggests that even both private and public information become very precise the problem of coordination still remains because of strategic uncertainty, so does the incidence of inefficient liquidation. To illustrate this result in a simple way, we set $p = 0$, and the critical failure point is defined by

$$\theta^* - zG_\chi = z\left(1 - G_\chi\right)\Phi\left(\frac{\sigma}{\sqrt{\alpha}}\left(\theta^* - \left(\mu - \frac{\sqrt{\sigma+\alpha}}{\sigma}\Phi^{-1}\left(\lambda\right)\right)\right)\right)$$

And the unique equilibrium condition becomes

$$\frac{\sigma}{\sqrt{\alpha}} \leq \frac{\sqrt{2\pi}}{z\left(1 - G_\chi\right)}$$

---

[13]We are grateful to Pascal François for pointing out the implication of the model to both Paris Club and London Club.

To have a unique equilibrium, we should keep the ratio $\frac{\sigma}{\sqrt{\alpha}}$ constant as $\sigma \to \infty$ and $\alpha \to \infty$, that is $\frac{\sigma}{\sqrt{\alpha}} = c \leq \frac{\sqrt{2\pi}}{z(1-G_X)}$, where c is a constant. Then,

$$\frac{\sqrt{\frac{\sigma+\alpha}{\alpha}}}{\sigma/\sqrt{\alpha}} = \frac{\sqrt{\frac{c}{\sqrt{\alpha}}+1}}{c} \to 1/c$$

and the limit of θ^* is

$$\theta^* - zG_X = z\left(1 - G_X\right)\Phi\left(c\left(\theta^* - \left(\mu - \frac{1}{c}\Phi^{-1}\left(\lambda\right)\right)\right)\right)$$

In equilibrium, the interval $[0, \theta^*]$ represent the incidence of inefficient liquidation and the *ex ante* mean of θ still matters for equilibrium. This means the problem of coordination still remains even though each creditor has access to both very precise public and private information.

2.5.5 A new proposal

By treating the IMF as a third party in the model, we focus on the equilibrium effects of its signals through the interactions between the large creditor and small creditors. Therefore, the main question is to know how the IMF could correct credibly the beliefs of creditors. From the above analysis, a new proposal should satisfy the following criteria: (*i*) solve the coordination problems not just for one bond issue but also across bond issues; (*ii*) encourage the sovereign debtor to improve its creditworthiness; (*iii*) relatively easy to implement. Inspired by academic continual evaluation of students[14], this paper proposes a mechanism that satisfies these criteria.

This mechanism is as follows. The IMF could: (*i*) impose *ex ante* the conditional-

[14] In the environment of lending with *ex post* conditionalities, a sovereign debtor in trouble faces a similar situation of a graduate of an university without a credible evaluation system. This graduate wants to enter job market. Without any credible evaluation, future employers are uncertain about his or her quality. To convince them, the graduate needs recommendation letters from his or her professors. However, without any evaluation, these professors can only write that this graduate will be good if he or she improves in certain respects. Will this corrects the beliefs of the future employers? The answer is negative because these letters do not contain any positive signal to correct their beliefs.

ities[15] to each sovereign debtor that is susceptible to financial collapse by specifying the policy adjustments specific to the country; (*ii*) evaluate credibly the fundamentals of the sovereign debtor and the degree of fulfillment of policy adjustments specified by the conditionalities; (*iii*) decide the loan of last resort based on the evaluation. Based on the model, the loan provided by the IMF or even its decision of lending increases both the precision and the estimation of private creditors on the fundamentals of the borrower, and thereby decrease all thresholds. Then, the loan of IMF serves as a positive signal to private creditors to roll over their loans in the sense that its loan is complementary to private loans.

A full analysis on conditionalities is beyond the scope of this chapter. See Jeanne, Ostry and Zettelmeyer (2008) for a model of moral hazard in which a similar conclusion is reached. Here, we sketch our proposal in a simple way. The system of ratings of IMF contains a sample $(y_1, y_2, ..., y_n; c_1, ..., c_m)$, where $(y_1, y_2, ..., y_n) = Y^n$ represents indicators of the fundamentals of the sovereign debtor with a likelihood mean of the sample $\bar{y}$, and $(c_1, ..., c_m) = C^m$ represents indicators measuring the degree of respecting the conditionalities with a likelihood mean of the sample $\bar{c}$. Higher $\bar{c}$ means higher degree of respect of conditionalities and the improvement of $\bar{y}$. Thus, $\bar{y}$ and $\bar{c}$ are bivariate normal with a covariance matrix

$$\begin{pmatrix} \frac{1}{n\sigma^y} & \frac{1}{\sigma^{cy}} \\ \frac{1}{\sigma^{cy}} & \frac{1}{m\sigma^c} \end{pmatrix}$$

where the covariance of $\bar{y}$ and $\bar{c}$ is $\frac{1}{\sigma^{cy}}$, $\bar{y}$ has precision $n\sigma^y$ and $\bar{c}$ has precision $m\sigma^c$. The final evaluation is normal with mean $\mu^y + \mu^c$ and variance $\frac{1}{n\sigma^y} + \frac{1}{m\sigma^c} + 2\frac{1}{\sigma^{cy}}$. When the sovereign debtor defaults, he can demand loans from the IMF.

The decision procedure of the IMF is similar to that of a national bankruptcy court. At a national level, a reorganization plan is voted on and the acceptance rule is

[15]Lending with *ex ante* conditionalities is not new. See Jeanne, Ostry and Zettelmeyer (2008) for a literature review.

that each class of creditors must approve the plan by a margin of at least two-thirds in amount and a simple majority in number of claims. If the plan is adopted by vote and satisfies other criteria, it can be confirmed by the bankruptcy court. If the plan is not adopted by vote, then the court may order the firm to be liquidated under Chapter 7 or adopt the plan using a procedure called cramdown. The requirements for cramdown are that, for a case of all creditors with equal priority, all creditors receive at least what they would get in liquidation. During the reorganization procedure, the decision of reorganize or liquidate the firm is made both by creditors and the bankruptcy court; still, the bankruptcy court has the last word.

The problem of the IMF is to test whether the fundamentals of the country, and the degree of respecting the conditionalities are high enough to lend. Let the critical switching points of the IMF denote by respectively by Y^* and C^*. If the fundamentals of the country and degree of respecting the conditionalities are higher respectively than these two switching points, then the IMF decide to lend. Otherwise, the IMF refuses to lend. Since, the critical switching points are set by the IMF, they give the margin to the IMF to make discretionary decisions. The system of evaluation, based both on the fundamentals of the country and on the degree of respecting the conditionalities, makes the decisions of IMF with commitment.

Without imposing a supranational bankruptcy court, this mechanism gives the same outcome as a domestic bankruptcy court's decision to confirm or not a reorganization plan with only one difference that the IMF can not convert the case into the liquidation. However, the IMF could refuse the loan by leaving the sovereign country on the state as before the demand of loans. Then the loan of IMF or even its lending decision serves as a positive signal to private creditors to roll over their loans in the sense that its loan is complementary to private loans. And most importantly, this mechanism encourages each sovereign debtor to improve its creditworthiness and makes the allocation of private capital more efficient.

2.6 Conclusion

Even abstracting from signaling, the presence of a better informed or a more optimistic large creditor increases the willingness of small creditors to roll over, and thereby reduces the probability of project failure. The equilibrium outcome can be explained by the behavior of relying on the information or the estimate of others. Relying on the information of others aggregates the information, but only partially because the anticipation of one type of creditors about the information of others coincides with the true value at a very weak probability. Thus, inefficient liquidation or rolling over may occur because of strategic uncertainty about others' actions.

The major message of this chapter is that in order to solve the coordination problem it is essential for a mechanism to be able to correct credibly the beliefs of creditors about the fundamentals the debtor. The current practice of international lending of last resort with *ex post* conditionalities rarely has catalytic finance effects because it confirms the negative beliefs of creditors about the fundamentals of their distressed sovereign debtor. To correct their beliefs about the fundamentals of a solvent but illiquid sovereign debtor, inspired by academic continual evaluation of students, we propose a mechanism of international lending of last resort with *ex ante* conditionalities combined with a credible system of evaluation, which generates the same outcome as a national bankruptcy court.

APPENDIX B

B.1. Proof of Proposition 1

Differentiating equation (2.10) with respect to G_X, we have

$$\begin{aligned}\frac{d\theta^*}{dG_X} &= z - z\Phi + \frac{\sigma}{\sqrt{\alpha}} z\left(1-G_X\right)\phi(\cdot)\frac{d\theta^*}{dG_X} \\ \frac{d\theta^*}{dG_X} &= \frac{\left(1-\Phi\right)z}{1-\frac{\sigma}{\sqrt{\alpha}}z\left(1-G_X\right)\phi(\cdot)}\end{aligned}$$

Since $\frac{\sigma}{\sqrt{\alpha}}z\left(1-G_X\right)\phi(\cdot) < 1$,

$$\frac{d\theta^*}{dG_X} > 0 \text{ and } \frac{dX^*}{dG_X} > 0$$

which completes the proof.

B.2. Proof of Proposition 3

We can finish the argument by showing that the unique switching equilibrium is the only equilibrium strategy to survive the iterative elimination of strictly dominated strategies. Consider the expected payoff to rolling over for a small creditor conditional on signal X when all other small creditors follow the switching strategy around $\widehat{X}$, and when the large creditor plays his best response against this switching strategy, which is to switch at $Y(\widehat{X})$, obtained from (2.15). Denote this expected payoff by $u(X,\widehat{X})$. It is given by

$$u(X,\widehat{X}) = X_X\left(1-\bar{G}_X\right)\left[1-\left(G\left(\underline{\theta}\left(\widehat{X}\right)-X\right)+\int_{\underline{\theta}(\widehat{X})}^{\bar{\theta}(\widehat{X})} g\left(\theta-X\right)H\left(Y\left(\widehat{X}\right)-\theta\right)d\theta\right)\right] \tag{2.19}$$

where $\underline{\theta}\left(\widehat{X}\right)$ and $\overline{\theta}\left(\widehat{X}\right)$ indicate the value of $\underline{\theta}$ and $\overline{\theta}$ when small creditors follow the switching strategy around $\widehat{X}$. We allow $\widehat{X} \in \mathbb{R} \cup \{-\infty, \infty\}$ take the values $-\infty$ and ∞, by which the small creditors respectively never and always foreclose. As shown above, $u(.,.)$ is increasing in its first argument and decreasing in its second.

For sufficiently high values of X, rolling over is a dominant action for a small creditor, regardless of the actions of others, small or large. Denote by $\overline{X}^1$ the threshold value of X above which it is a dominant action to roll over for a small creditor. Since all creditors realize this, any strategy to foreclose above $\overline{X}^1$ is dominated by rolling over. Then, it cannot be rational for a small creditor to foreclose whenever his signal is higher than $\overline{X}^2$, where $\overline{X}^2$ solves

$$u(\overline{X}^2, \overline{X}^1) = \overline{L}$$

It is so, since the switching strategy around $\overline{X}^2$ is the best reply to the switching strategy around $\overline{X}^1$ played by other small creditors and to that of the large creditor $Y(\overline{X}^1)$, and since even the small creditor that assumes the lowest possibility of the continuation of the project believes that the incidence of continuation is higher than that implied by the switching strategy around $\overline{X}^1$and $Y(\overline{X}^1)$. Since the payoff to rolling over is increasing in the incidence of continuation by the other creditors, any strategy that refrains from rolling over for signals higher than $\overline{X}^2$ is strictly dominated. Since

$$u\left(\overline{X}^1, \infty\right) = u\left(\overline{X}^2, \overline{X}^1\right) = \overline{L}$$

monotonicity of u implies $\overline{X}^1 > \overline{X}^2$. Thus, suppose $\overline{X}^{k-1} > \overline{X}^k$, monotonicity implies that $\overline{X}^k > \overline{X}^{k+1}$. We can generate a decreasing sequence

$$\overline{X}^1 > \overline{X}^2 > \overline{X}^3 ... > \overline{X}^k > ...$$

where any strategy that refrains from rolling over for signal $X > \overline{X}^k$ does not survive k rounds of deletion of dominated strategies. Since the sequence is bounded, assuming

$\overline{X}$ is the largest solution to $u(X, X) = \overline{L}$, then monotonicity of u implies that

$$\overline{X} = \lim_{k\to\infty} \overline{X}^k$$

Any strategy that refrains from rolling over for signal higher than $\overline{X}$ does not survive iterated dominance.

Conversely, if $\underline{X}$ is the smallest solution to $u(X, X) = \overline{L}$, any strategy that refrains from foreclosing for a signal below $\underline{X}$ does not survive iterative elimination. If there is a unique solution to $u(X, X) = \overline{L}$, then the smallest solution is the largest solution. Therefore, there is only one strategy that remains after eliminating all iteratively dominated strategies. This strategy is the only equilibrium strategy. This completes the argument.

B.3. Proof of Proposition 4

Differentiating equations (2.13) and (2.14) and rearranging, we get

$$\begin{aligned}
\frac{d\underline{\theta}}{dp} &= -zG_X - z\left(1-G_X\right)G\left(X^* - \underline{\theta}\right) + z(1-p)\left(1-G_X\right)g\left(X^* - \underline{\theta}\right)\left(\frac{dX^*}{dp} - \frac{d\underline{\theta}}{dp}\right) \\
\frac{dX^*}{dp} &= \left(1 + \frac{1}{z(1-p)\left(1-G_X\right)g\left(X^* - \underline{\theta}\right)}\right)\frac{d\underline{\theta}}{dp} + \\
&\quad \frac{G_X + \left(1-G_X\right)G\left(X^* - \underline{\theta}\right)}{(1-p)\left(1-G_X\right)g\left(X^* - \underline{\theta}\right)}
\end{aligned}$$

$$\begin{aligned}
\frac{d\overline{\theta}}{dp} &= z\left[1 - \left(G_X + \left(1-G_X\right)G\left(X^* - \overline{\theta}\right)\right)\right] + \\
&\quad z(1-p)\left(1-G_X\right)g\left(X^* - \overline{\theta}\right)\left(\frac{dX^*}{dp} - \frac{d\overline{\theta}}{dp}\right) \\
\frac{dX^*}{dp} &= \left(1 + \frac{1}{z(1-p)\left(1-G_X\right)g\left(X^* - \overline{\theta}\right)}\right)\frac{d\overline{\theta}}{dp} \\
&\quad - \frac{1 - \left(G_X + \left(1-G_X\right)G\left(X^* - \overline{\theta}\right)\right)}{(1-p)\left(1-G_X\right)g\left(X^* - \overline{\theta}\right)}
\end{aligned}$$

To ease notation, define c_1 and c_2 as follows

$$
\begin{aligned}
c_1 &= \left(1 + \frac{1}{z(1-p)(1-G_X)\,g(X^* - \underline{\theta})}\right)^{-1} < 1 \\
c_2 &= \left(1 + \frac{1}{z(1-p)(1-G_X)\,g\left(X^* - \overline{\theta}\right)}\right)^{-1} < 1 .
\end{aligned}
$$

Then

$$
\begin{aligned}
\frac{dX^*}{dp} &= \frac{1}{c_1}\frac{d\underline{\theta}}{dp} + \frac{G_X + (1-G_X)\,G(X^* - \underline{\theta})}{(1-p)(1-G_X)\,g(X^* - \underline{\theta})} \\
\frac{dX^*}{dp} &= \frac{1}{c_2}\frac{d\overline{\theta}}{dp} - \frac{1 - \left(G_X + (1-G_X)\,G\left(X^* - \overline{\theta}\right)\right)}{(1-p)(1-G_X)\,g\left(X^* - \overline{\theta}\right)}
\end{aligned}
$$

Using $\underline{\delta} \equiv \underline{\theta} - X^*$ and $\overline{\delta} \equiv \overline{\theta} - X^*$, we have

$$
\begin{aligned}
\frac{d\underline{\delta}}{dp} &= \frac{d\underline{\theta}}{dp} - \frac{dX^*}{dp} \\
&= c_1\left(\frac{dX^*}{dp} - \frac{G_X + (1-G_X)\,G(X^* - \underline{\theta})}{(1-p)(1-G_X)\,g(X^* - \underline{\theta})}\right) - \frac{dX^*}{dp} \\
&= (c_1 - 1)\frac{dX^*}{dp} - c_1\frac{G_X + (1-G_X)\,G(X^* - \underline{\theta})}{(1-p)(1-G_X)\,g(X^* - \underline{\theta})}
\end{aligned}
$$

$$
\begin{aligned}
\frac{d\overline{\delta}}{dp} &= \frac{d\overline{\theta}}{dp} - \frac{dX^*}{dp} \\
&= c_2\left(\frac{dX^*}{dp} + \frac{1 - \left(G_X + (1-G_X)\,G\left(X^* - \overline{\theta}\right)\right)}{(1-p)(1-G_X)\,g\left(X^* - \overline{\theta}\right)}\right) - \frac{dX^*}{dp} \\
&= (c_2 - 1)\frac{dX^*}{dp} + c_2\frac{1 - \left(G_X + (1-G_X)\,G\left(X^* - \overline{\theta}\right)\right)}{(1-p)(1-G_X)\,g\left(X^* - \overline{\theta}\right)}
\end{aligned}
$$

Now differentiate equation (2.18), and we have

$$\frac{d\underline{\delta}}{dp}g(\underline{\delta}) + \int_{\underline{\delta}}^{\overline{\delta}} g(k)h\left(\underline{\delta} - k - H^{-1}\left(1-\overline{\lambda}\right)\right)dk\frac{d\underline{\delta}}{dp} +$$
$$g(\overline{\delta})H\left(\underline{\delta} - \overline{\delta} - H^{-1}\left(1-\overline{\lambda}\right)\right)\frac{d\overline{\delta}}{dp} - g(\underline{\delta})H\left(\underline{\delta} - \underline{\delta} - H^{-1}\left(1-\overline{\lambda}\right)\right)\frac{d\underline{\delta}}{dp} = 0$$

$$\Leftrightarrow \frac{d\underline{\delta}}{dp}g(\underline{\delta})\left(1 - H\left(-H^{-1}\left(1-\overline{\lambda}\right)\right)\right) + \int_{\underline{\delta}}^{\overline{\delta}} g(k)h\left(\underline{\delta} - k - H^{-1}\left(1-\overline{\lambda}\right)\right)dk\frac{d\underline{\delta}}{dp} +$$
$$g(\overline{\delta})H\left(\underline{\delta} - \overline{\delta} - H^{-1}\left(1-\overline{\lambda}\right)\right)\frac{d\overline{\delta}}{dp} = 0$$

$$\Leftrightarrow \frac{d\underline{\delta}}{dp}\left(g(\underline{\delta})\overline{\lambda} + \int_{\underline{\delta}}^{\overline{\delta}} g(k)h\left(\underline{\delta} - k - H^{-1}\left(1-\overline{\lambda}\right)\right)dk\right)$$
$$+g(\overline{\delta})H\left(\underline{\delta} - \overline{\delta} - H^{-1}\left(1-\overline{\lambda}\right)\right)\frac{d\overline{\delta}}{dp} = 0$$

Let

$$\begin{aligned} c_3 &= g(\underline{\delta})\overline{\lambda} + \int_{\underline{\delta}}^{\overline{\delta}} g(k)h\left(\underline{\delta} - k - H^{-1}\left(1-\overline{\lambda}\right)\right)dk > 0 \\ c_4 &= g(\overline{\delta})H\left(\underline{\delta} - \overline{\delta} - H^{-1}\left(1-\overline{\lambda}\right)\right) > 0 \end{aligned}$$

Then

$$\frac{d\underline{\delta}}{dp}c_3 + \frac{d\overline{\delta}}{dp}c_4 = 0$$

$$c_3(c_1-1)\frac{dX^*}{dp} - c_1c_3\frac{G_\chi + (1-G_\chi)G\left(X^* - \underline{\theta}\right)}{(1-p)(1-G_\chi)g\left(X^* - \underline{\theta}\right)} +$$
$$c_4(c_2-1)\frac{dX^*}{dp} + c_2c_4\frac{1 - \left(G_\chi + (1-G_\chi)G\left(X^* - \overline{\theta}\right)\right)}{(1-p)(1-G_\chi)g\left(X^* - \overline{\theta}\right)} = 0$$

$$\begin{aligned}
\frac{dX^*}{dp} &= \frac{c_1 c_3 \frac{G_X+(1-G_X)G(X^*-\underline{\theta})}{(1-p)(1-G_X)g(X^*-\underline{\theta})} - c_2 c_4 \frac{1-\left(G_X+(1-G_X)G\left(X^*-\overline{\theta}\right)\right)}{(1-p)(1-G_X)g\left(X^*-\overline{\theta}\right)}}{c_3(c_1-1)+c_4(c_2-1)} \\
&= c_3 \frac{z(1-p)(1-G_X)\,g(X^*-\underline{\theta})}{1+z(1-p)(1-G_X)\,g(X^*-\underline{\theta})} \frac{G_X+(1-G_X)\,G(X^*-\underline{\theta})}{(1-p)(1-G_X)\,g(X^*-\underline{\theta})} - \\
&\quad c_4 \frac{z(1-p)(1-G_X)\,g\left(X^*-\overline{\theta}\right)}{1+z(1-p)(1-G_X)\,g\left(X^*-\overline{\theta}\right)} \frac{1-\left(G_X+(1-G_X)\,G\left(X^*-\overline{\theta}\right)\right)}{(1-p)(1-G_X)\,g\left(X^*-\overline{\theta}\right)} \\
&= \left[\begin{array}{c} c_3 \frac{z[G_X+(1-G_X)G(X^*-\underline{\theta})]}{1+z(1-p)(1-G_X)g(X^*-\underline{\theta})} - \\ c_4 \frac{z\left[1-\left(G_X+(1-G_X)G\left(X^*-\overline{\theta}\right)\right)\right]}{1+z(1-p)(1-G_X)g\left(X^*-\overline{\theta}\right)} \end{array}\right] \frac{1}{c_3(c_1-1)+c_4(c_2-1)}
\end{aligned}$$

$$\frac{dX^*}{dp} < 0 \text{ if and only if } c_3 \frac{z\left[G_X+(1-G_X)\,G(X^*-\underline{\theta})\right]}{1+z(1-p)(1-G_X)\,g(X^*-\underline{\theta})} > c_4 \frac{z\left[1-\left(G_X+(1-G_X)\,G\left(X^*-\overline{\theta}\right)\right)\right]}{1+z(1-p)(1-G_X)\,g\left(X^*-\overline{\theta}\right)}$$

Then

$$\frac{d\underline{\theta}}{dp} < 0,\ \frac{d\overline{\theta}}{dp} < 0, \text{ and } \frac{dY^*}{dp} < 0$$

which completes the proof.

B.4. Proof of Proposition 5

Now, we consider the precision of the large creditor's information. Rewriting equation (2.15) in terms of standard normal, we have

$$\Phi\left(\sqrt{\beta+\sigma}(\underline{\theta}-Y^*)\right) = 1-\overline{\lambda}$$

Differentiating with respect to the precision of the large lender β, we get

$$\phi\left(\sqrt{\beta+\sigma}(\underline{\theta}-Y^*)\right) \cdot \left[\sqrt{\beta+\sigma}\left(\frac{d\underline{\theta}}{d\beta}-\frac{dY^*}{d\beta}\right)+\frac{\underline{\theta}-Y^*}{2\sqrt{\beta+\sigma}}\right] = 0.$$

Defining $\delta^* = Y^* - X^*$, using $\underline{\delta} \equiv \underline{\theta} - X^*$ and rearranging, we obtain

$$\begin{aligned}
\sqrt{\beta+\sigma}\left(\frac{d\underline{\delta}}{d\beta}-\frac{d\delta^*}{d\beta}\right)+\frac{\underline{\delta}-\delta^*}{2\sqrt{\beta+\sigma}} &= 0 \\
\sqrt{\beta+\sigma}\frac{d\delta^*}{d\beta} &= \sqrt{\beta+\sigma}\frac{d\underline{\delta}}{d\beta}+\frac{\underline{\delta}-\delta^*}{2\sqrt{\beta+\sigma}}
\end{aligned}$$

Moreover as for the size of the large creditor:

$$
\begin{aligned}
\frac{d\underline{\theta}}{d\beta} &= z(1-p)\left(1-G_X\right)g\left(X^*-\underline{\theta}\right)\left(\frac{dX^*}{d\beta}-\frac{d\underline{\theta}}{d\beta}\right) \\
\frac{dX^*}{d\beta} &= \left(1+\frac{1}{z(1-p)\left(1-G_X\right)g\left(X^*-\underline{\theta}\right)}\right)\frac{d\underline{\theta}}{d\beta}
\end{aligned}
$$

$$
\begin{aligned}
\frac{d\overline{\theta}}{d\beta_\theta} &= z(1-p)\left(1-G_X\right)g\left(X^*-\overline{\theta}\right)\left(\frac{dX^*}{d\beta}-\frac{d\overline{\theta}}{d\beta}\right) \\
\frac{dX^*}{d\beta} &= \left(1+\frac{1}{z(1-p)\left(1-G_X\right)g\left(X^*-\overline{\theta}\right)}\right)\frac{d\overline{\theta}}{dp}
\end{aligned}
$$

Then

$$
\begin{aligned}
\frac{dX^*}{d\beta} &= \frac{1}{c_1}\frac{d\underline{\theta}}{d\beta} \\
\frac{dX^*}{d\beta} &= \frac{1}{c_2}\frac{d\overline{\theta}}{d\beta}
\end{aligned}
$$

$$
\begin{aligned}
\frac{d\underline{\delta}}{d\beta} &= (c_1-1)\frac{dX^*}{d\beta} \\
\frac{d\overline{\delta}}{d\beta} &= (c_2-1)\frac{dX^*}{d\beta}
\end{aligned}
$$

Differentiate equation (2.18), and we have

$$
g(\underline{\delta})\frac{d\underline{\delta}}{d\beta}+\int_{\underline{\delta}}^{\overline{\delta}} g(k)\phi\left(\sqrt{\beta+\sigma}\left(\delta^*-k\right)\right)\left(\sqrt{\beta+\sigma}\frac{d\delta^*}{d\beta}+\frac{(\delta^*-k)}{2\sqrt{\beta+\sigma}}\right)dk+
$$
$$
g(\overline{\delta})\Phi\left(\sqrt{\beta+\sigma}\left(\underline{\delta}-\overline{\delta}-H^{-1}\left(1-\overline{\lambda}\right)\right)\right)\frac{d\overline{\delta}}{d\beta}-g(\underline{\delta})\Phi\left(\sqrt{\beta+\sigma}\left(\underline{\delta}-\underline{\delta}-\Phi^{-1}\left(1-\overline{\lambda}\right)\right)\right)\frac{d\underline{\delta}}{d\beta}=0
$$

$$\Leftrightarrow g(\underline{\delta})\frac{d\underline{\delta}}{d\beta} + \int_{\underline{\delta}}^{\overline{\delta}} g(k)\phi\left(\sqrt{\beta+\sigma}\,(\delta^* - k)\right) dk\sqrt{\beta+\sigma}\frac{d\underline{\delta}}{d\beta} +$$

$$\int_{\underline{\delta}}^{\overline{\delta}} g(k)\phi\left(\sqrt{\beta+\sigma}\,(\delta^* - k)\right)\frac{\underline{\delta} - \delta^*}{2\sqrt{\beta+\sigma}}dk +$$

$$\int_{\underline{\delta}}^{\overline{\delta}} g(k)\phi\left(\sqrt{\beta+\sigma}\,(\delta^* - k)\right)\frac{(\delta^* - k)}{2\sqrt{\beta+\sigma}}dk + g(\overline{\delta})\Phi\left(\sqrt{\beta+\sigma}\left(\underline{\delta} - \overline{\delta} - H^{-1}\left(1 - \overline{\lambda}\right)\right)\right)\frac{d\overline{\delta}}{d\beta}$$

$$-g(\underline{\delta})\Phi\left(\sqrt{\beta+\sigma}\left(\underline{\delta} - \underline{\delta} - \Phi^{-1}\left(1 - \overline{\lambda}\right)\right)\right)\frac{d\underline{\delta}}{d\beta} = 0$$

$$\Leftrightarrow \frac{d\underline{\delta}}{d\beta}\left(g(\underline{\delta})\overline{\lambda} + \int_{\underline{\delta}}^{\overline{\delta}} g(k)\phi\left(\sqrt{\beta+\sigma}\left(\underline{\delta} - \overline{\delta} - H^{-1}\left(1 - \overline{\lambda}\right)\right)\right) dk\sqrt{\beta+\sigma}\right) +$$

$$g(\overline{\delta})\Phi\left(\sqrt{\beta+\sigma}\left(\underline{\delta} - \overline{\delta} - H^{-1}\left(1 - \overline{\lambda}\right)\right)\right)\frac{d\overline{\delta}}{d\beta} + \int_{\underline{\delta}}^{\overline{\delta}} g(k)\phi\left(\sqrt{\beta+\sigma}\,(\delta^* - k)\right)\frac{\underline{\delta} - k}{2\sqrt{\beta+\sigma}}dk = 0$$

Let $c_5 = g(\underline{\delta})\overline{\lambda} + \int_{\underline{\delta}}^{\overline{\delta}} g(k)\phi\left(\sqrt{\beta+\sigma}\left(\underline{\delta} - \overline{\delta} - H^{-1}\left(1 - \overline{\lambda}\right)\right)\right) dk\sqrt{\beta+\sigma}$, Then

$$c_5\frac{d\underline{\delta}}{d\beta} + c_4\frac{d\overline{\delta}}{d\beta} + \int_{\underline{\delta}}^{\overline{\delta}} g(k)\phi\left(\sqrt{\beta+\sigma}\,(\delta^* - k)\right)\frac{\underline{\delta} - k}{2\sqrt{\beta+\sigma}}dk = 0$$

$$c_5(c_1 - 1)\frac{dX^*}{d\beta} + c_4(c_2 - 1)\frac{dX^*}{d\beta} +$$

$$\int_{\underline{\delta}}^{\overline{\delta}} g(k)\phi\left(\sqrt{\beta+\sigma}\,(\delta^* - k)\right)\frac{\underline{\delta} - k}{2\sqrt{\beta+\sigma}}dk = 0$$

$$\frac{dX^*}{d\beta} = \frac{-\int_{\underline{\delta}}^{\overline{\delta}} g(k)\phi\left(\sqrt{\beta+\sigma}\,(\delta^* - k)\right)(k - \underline{\delta})dk}{2\sqrt{\beta+\sigma}\left[c_5(1 - c_1) + c_4(1 - c_2)\right]} < 0$$

Then

$$\frac{d\underline{\theta}}{d\beta} < 0,\ \frac{d\overline{\theta}}{d\beta} < 0, \text{ and } \frac{dY^*}{d\beta} < 0$$

This completes the argument.

B.5. Proof of Proposition 6

Differentiating equation (2.15) with respect to H_χ, we get

$$-\left(1-H(\underline{\theta}-Y^*)\right)-(1-H_\chi)\,h\left(\underline{\theta}-Y^*\right)\cdot\left(\frac{d\underline{\theta}}{dH_\chi}-\frac{dY^*}{dH_\chi}\right)=0$$

Defining $\delta^*=Y^*-X^*$, using $\underline{\delta}\equiv\underline{\theta}-X^*$ and rearranging, we obtain

$$\begin{aligned}
\frac{d\delta^*}{dH_\chi}-\frac{d\underline{\delta}}{dH_\chi} &= \frac{1-H(\underline{\theta}-Y^*)}{(1-H_\chi)\,h\left(\underline{\theta}-Y^*\right)}\\
(1-H_\chi)\,h\left(\underline{\theta}-Y^*\right)\frac{d\delta^*}{dH_\chi} &= (1-H_\chi)\,h\left(\underline{\theta}-Y^*\right)\frac{d\underline{\delta}}{dH_\chi}+1-H(\underline{\theta}-Y^*)
\end{aligned}$$

Moreover as for the size of the large creditor:

$$\begin{aligned}
\frac{d\underline{\theta}}{dH_\chi} &= z(1-p)\left(1-G_\chi\right)g\left(X^*-\underline{\theta}\right)\left(\frac{dX^*}{dH_\chi}-\frac{d\underline{\theta}}{dH_\chi}\right)\\
\frac{dX^*}{dH_\chi} &= \left(1+\frac{1}{z(1-p)\left(1-G_\chi\right)g\left(X^*-\underline{\theta}\right)}\right)\frac{d\underline{\theta}}{dH_\chi}
\end{aligned}$$

$$\begin{aligned}
\frac{d\overline{\theta}}{dH_\chi} &= z(1-p)\left(1-G_\chi\right)g\left(X^*-\overline{\theta}\right)\left(\frac{dX^*}{dH_\chi}-\frac{d\overline{\theta}}{dH_\chi}\right)\\
\frac{dX^*}{dH_\chi} &= \left(1+\frac{1}{z(1-p)\left(1-G_\chi\right)g\left(X^*-\overline{\theta}\right)}\right)\frac{d\overline{\theta}}{dH_\chi}
\end{aligned}$$

Then

$$\begin{aligned}
\frac{dX^*}{dH_\chi} &= \frac{1}{c_1}\frac{d\underline{\theta}}{dH_\chi}\\
\frac{dX^*}{dH_\chi} &= \frac{1}{c_2}\frac{d\overline{\theta}}{dH_\chi}
\end{aligned}$$

$$\begin{aligned}
\frac{d\underline{\delta}}{dH_\chi} &= (c_1-1)\frac{dX^*}{dH_\chi}\\
\frac{d\overline{\delta}}{dH_\chi} &= (c_2-1)\frac{dX^*}{dH_\chi}
\end{aligned}$$

Differentiate equation (2.18), and we have

$$g(\underline{\delta})\frac{d\underline{\delta}}{dH\chi}+\int_{\underline{\delta}}^{\overline{\delta}} g(k)h\left(\delta^{*}-k\right)\frac{d\delta^{*}}{dH\chi}dk+$$
$$g(\overline{\delta})H\left(\underline{\delta}-\overline{\delta}-H^{-1}\left(1-\overline{\lambda}\right)\right)\frac{d\overline{\delta}}{dH\chi}-g(\underline{\delta})H\left(\underline{\delta}-\underline{\delta}-H^{-1}\left(1-\overline{\lambda}\right)\right)\frac{d\underline{\delta}}{dH\chi}=0$$

$$\Leftrightarrow g(\underline{\delta})\frac{d\underline{\delta}}{dH\chi}+\int_{\underline{\delta}}^{\overline{\delta}} g(k)h\left(\delta^{*}-k\right)\left[\frac{d\underline{\delta}}{dH\chi}+\frac{1-H(\underline{\theta}-Y^{*})}{\left(1-H_{\chi}\right)h\left(\underline{\theta}-Y^{*}\right)}\right]dk+$$
$$g(\overline{\delta})H\left(\underline{\delta}-\overline{\delta}-H^{-1}\left(1-\overline{\lambda}\right)\right)\frac{d\overline{\delta}}{dH\chi}-g(\underline{\delta})H\left(\underline{\delta}-\underline{\delta}-H^{-1}\left(1-\overline{\lambda}\right)\right)\frac{d\underline{\delta}}{dH\chi}=0$$

$$\Leftrightarrow \frac{d\underline{\delta}}{dH\chi}\left(g(\underline{\delta})\overline{\lambda}+\int_{\underline{\delta}}^{\overline{\delta}} g(k)h\left(\delta^{*}-k\right)dk\right)+g(\overline{\delta})H\left(\underline{\delta}-\overline{\delta}-H^{-1}\left(1-\overline{\lambda}\right)\right)\frac{d\overline{\delta}}{dH\chi}+$$
$$\int_{\underline{\delta}}^{\overline{\delta}} g(k)h\left(\delta^{*}-k\right)\frac{1-H(\underline{\theta}-Y^{*})}{\left(1-H_{\chi}\right)h\left(\underline{\theta}-Y^{*}\right)}dk=0$$

Let $c_5=g(\underline{\delta})\overline{\lambda}+\int_{\underline{\delta}}^{\overline{\delta}} g(k)h\left(\delta^{*}-k\right)dk$, Then

$$c_5\frac{d\underline{\delta}}{dH\chi}+c_4\frac{d\overline{\delta}}{dH\chi}+\int_{\underline{\delta}}^{\overline{\delta}} g(k)h\left(\delta^{*}-k\right)\frac{1-H(\underline{\theta}-Y^{*})}{\left(1-H_{\chi}\right)h\left(\underline{\theta}-Y^{*}\right)}dk=0$$

$$c_5(c_1-1)\frac{dX^{*}}{dH\chi}+c_4(c_2-1)\frac{dX^{*}}{dH\chi}+$$
$$\int_{\underline{\delta}}^{\overline{\delta}} g(k)h\left(\delta^{*}-k\right)\frac{1-H(\underline{\theta}-Y^{*})}{\left(1-H_{\chi}\right)h\left(\underline{\theta}-Y^{*}\right)}dk=0$$

$$\frac{dX^{*}}{dH\chi}=\frac{-\int_{\underline{\delta}}^{\overline{\delta}} g(k)h\left(\delta^{*}-k\right)\frac{1-H(\underline{\theta}-Y^{*})}{(1-H_{\chi})h(\underline{\theta}-Y^{*})}dk}{2\sqrt{\beta+\sigma}\left[c_5(1-c_1)+c_4(1-c_2)\right]}>0$$

Then,

$$\frac{d\underline{\theta}}{dH\chi}>0,\ \frac{d\overline{\theta}}{dH\chi}>0,\text{ and }\frac{dY^{*}}{dH\chi}>0$$

B.6. Proof of Proposition 7

Differentiating equations (2.13) and (2.14) and rearranging, we get

$$\begin{aligned}
\frac{d\underline{\theta}}{dG_X} &= (1-p)z\left[1-G\left(X^*-\underline{\theta}\right)\right] + z(1-p)\left(1-G_X\right)g\left(X^*-\underline{\theta}\right)\left(\frac{dX^*}{dG_X}-\frac{d\underline{\theta}}{dG_X}\right) \\
\frac{dX^*}{dG_X} &= \left(1+\frac{1}{z(1-p)\left(1-G_X\right)g\left(X^*-\underline{\theta}\right)}\right)\frac{d\underline{\theta}}{dp} + \frac{(1-p)z\left[1-G\left(X^*-\underline{\theta}\right)\right]}{(1-p)\left(1-G_X\right)g\left(X^*-\underline{\theta}\right)}
\end{aligned}$$

$$\begin{aligned}
\frac{d\overline{\theta}}{dG_X} &= (1-p)z\left[1-G\left(X^*-\overline{\theta}\right)\right] + z(1-p)\left(1-G_X\right)g\left(X^*-\overline{\theta}\right)\left(\frac{dX^*}{dG_X}-\frac{d\overline{\theta}}{dG_X}\right) \\
\frac{dX^*}{dG_X} &= \left(1+\frac{1}{z(1-p)\left(1-G_X\right)g\left(X^*-\overline{\theta}\right)}\right)\frac{d\overline{\theta}}{dG_X} + \frac{(1-p)z\left[1-G\left(X^*-\overline{\theta}\right)\right]}{(1-p)\left(1-G_X\right)g\left(X^*-\overline{\theta}\right)}
\end{aligned}$$

Then

$$\begin{aligned}
\frac{dX^*}{dG_X} &= \frac{1}{c_1}\frac{d\underline{\theta}}{dp} - \frac{(1-p)z\left[1-G\left(X^*-\underline{\theta}\right)\right]}{(1-p)\left(1-G_X\right)g\left(X^*-\underline{\theta}\right)} \\
\frac{dX^*}{dG_X} &= \frac{1}{c_2}\frac{d\overline{\theta}}{dp} - \frac{(1-p)z\left[1-G\left(X^*-\overline{\theta}\right)\right]}{(1-p)\left(1-G_X\right)g\left(X^*-\overline{\theta}\right)}
\end{aligned}$$

Using $\underline{\delta} \equiv \underline{\theta} - X^*$ and $\overline{\delta} \equiv \overline{\theta} - X^*$, we have

$$\begin{aligned}
\frac{d\underline{\delta}}{dG_X} &= \frac{d\underline{\theta}}{dG_X} - \frac{dX^*}{dG_X} \\
&= c_1\left(\frac{dX^*}{dG_X} + \frac{(1-p)z\left[1-G\left(X^*-\underline{\theta}\right)\right]}{(1-p)\left(1-G_X\right)g\left(X^*-\underline{\theta}\right)}\right) - \frac{dX^*}{dG_X} \\
&= (c_1-1)\frac{dX^*}{dG_X} + c_1\frac{(1-p)z\left[1-G\left(X^*-\underline{\theta}\right)\right]}{(1-p)\left(1-G_X\right)g\left(X^*-\underline{\theta}\right)}
\end{aligned}$$

$$\begin{aligned}
\frac{d\overline{\delta}}{dG_X} &= \frac{d\overline{\theta}}{dG_X} - \frac{dX^*}{dG_X} \\
&= c_2\left(\frac{dX^*}{dG_X} + \frac{(1-p)z\left[1-G\left(X^*-\overline{\theta}\right)\right]}{(1-p)\left(1-G_X\right)g\left(X^*-\overline{\theta}\right)}\right) - \frac{dX^*}{dG_X} \\
&= (c_2-1)\frac{dX^*}{dG_X} + c_2\frac{(1-p)z\left[1-G\left(X^*-\overline{\theta}\right)\right]}{(1-p)\left(1-G_X\right)g\left(X^*-\overline{\theta}\right)}
\end{aligned}$$

Now differentiate equation (2.18), and we have

$$\frac{d\underline{\delta}}{dG_\chi}g(\underline{\delta}) + \int_{\underline{\delta}}^{\overline{\delta}} g(k)h\left(\underline{\delta} - k - H^{-1}\left(1-\overline{\lambda}\right)\right)dk\frac{d\underline{\delta}}{dG_\chi} +$$
$$g(\overline{\delta})H\left(\underline{\delta} - \overline{\delta} - H^{-1}\left(1-\overline{\lambda}\right)\right)\frac{d\overline{\delta}}{dG_\chi} - g(\underline{\delta})H\left(\underline{\delta} - \underline{\delta} - H^{-1}\left(1-\overline{\lambda}\right)\right)\frac{d\underline{\delta}}{dG_\chi} = 0$$

$$\Leftrightarrow \frac{d\underline{\delta}}{dG_\chi}g(\underline{\delta})\left(1 - H\left(-H^{-1}\left(1-\overline{\lambda}\right)\right)\right) + \int_{\underline{\delta}}^{\overline{\delta}} g(k)h\left(\underline{\delta} - k - H^{-1}\left(1-\overline{\lambda}\right)\right)dk\frac{d\underline{\delta}}{dG_\chi} +$$
$$g(\overline{\delta})H\left(\underline{\delta} - \overline{\delta} - H^{-1}\left(1-\overline{\lambda}\right)\right)\frac{d\overline{\delta}}{dG_\chi} = 0$$

$$\Leftrightarrow \frac{d\underline{\delta}}{dG_\chi}\left(g(\underline{\delta})\overline{\lambda} + \int_{\underline{\delta}}^{\overline{\delta}} g(k)h\left(\underline{\delta} - k - H^{-1}\left(1-\overline{\lambda}\right)\right)dk\right) +$$
$$g(\overline{\delta})H\left(\underline{\delta} - \overline{\delta} - H^{-1}\left(1-\overline{\lambda}\right)\right)\frac{d\overline{\delta}}{dG_\chi} = 0$$

Then

$$\frac{d\underline{\delta}}{dG_\chi}c_3 + \frac{d\overline{\delta}}{dG_\chi}c_4 = 0$$

$$c_3(c_1-1)\frac{dX^*}{dG_\chi} + c_1c_3\frac{(1-p)z\left[1-G\left(X^*-\underline{\theta}\right)\right]}{(1-p)\left(1-G_\chi\right)g\left(X^*-\underline{\theta}\right)} +$$
$$c_4(c_2-1)\frac{dX^*}{dG_\chi} + c_2c_4\frac{(1-p)z\left[1-G\left(X^*-\overline{\theta}\right)\right]}{(1-p)\left(1-G_\chi\right)g\left(X^*-\overline{\theta}\right)} = 0$$

$$\frac{dX^*}{dG_\chi} = \frac{-c_1c_3\frac{(1-p)z\left[1-G\left(X^*-\underline{\theta}\right)\right]}{(1-p)\left(1-G_\chi\right)g\left(X^*-\underline{\theta}\right)} - c_2c_4\frac{(1-p)z\left[1-G\left(X^*-\overline{\theta}\right)\right]}{(1-p)\left(1-G_\chi\right)g\left(X^*-\overline{\theta}\right)}}{c_3(c_1-1)+c_4(c_2-1)} > 0$$

Then

$$\frac{d\underline{\theta}}{dG_\chi} > 0,\ \frac{d\overline{\theta}}{dG_\chi} > 0, \text{ and } \frac{dY^*}{dG_\chi} > 0$$

which completes the proof.

B.7. Proof of Proposition 8

Differentiating equation (2.15) with respect to Y_X, we get

$$(1 - H_x)(1 - H(\underline{\theta} - Y^*)) + Y_X h_X (1 - H(\underline{\theta} - Y^*)) - \\ Y_X(1 - H_x) h(\underline{\theta} - Y^*) \cdot \left(\frac{d\underline{\theta}}{dY_X} - \frac{dY^*}{dY_X}\right) = 0$$

Defining $\delta^* = Y^* - X^*$, using $\underline{\delta} \equiv \underline{\theta} - X^*$ and rearranging, we obtain

$$\begin{aligned} \frac{d\underline{\delta}}{dY_X} - \frac{d\delta^*}{dY_X} &= \frac{(1 - H_x)(1 - H(\underline{\theta} - Y^*)) + Y_X h_X (1 - H(\underline{\theta} - Y^*))}{Y_X(1 - H_x) h(\underline{\theta} - Y^*)} \\ \frac{d\delta^*}{dY_X} &= \frac{d\underline{\delta}}{dY_X} - \frac{(1 - H_x)(1 - H(\underline{\theta} - Y^*)) + Y_X h_X (1 - H(\underline{\theta} - Y^*))}{Y_X(1 - H_x) h(\underline{\theta} - Y^*)} \end{aligned}$$

Moreover as for the size of the large creditor:

$$\begin{aligned} \frac{d\underline{\theta}}{dY_X} &= z(1-p)(1-G_X)\, g(X^* - \underline{\theta}) \left(\frac{dX^*}{dY_X} - \frac{d\underline{\theta}}{dY_X}\right) \\ \frac{dX^*}{dY_X} &= \left(1 + \frac{1}{z(1-p)(1-G_X)\, g(X^* - \underline{\theta})}\right) \frac{d\underline{\theta}}{dY_X} \end{aligned}$$

$$\begin{aligned} \frac{d\overline{\theta}}{dY_X} &= z(1-p)(1-G_X)\, g\left(X^* - \overline{\theta}\right) \left(\frac{dX^*}{dY_X} - \frac{d\overline{\theta}}{dY_X}\right) \\ \frac{dX^*}{dY_X} &= \left(1 + \frac{1}{z(1-p)(1-G_X)\, g\left(X^* - \overline{\theta}\right)}\right) \frac{d\overline{\theta}}{dY_X} \end{aligned}$$

Then

$$\begin{aligned} \frac{dX^*}{dY_X} &= \frac{1}{c_1} \frac{d\underline{\theta}}{dY_X} \\ \frac{dX^*}{dY_X} &= \frac{1}{c_2} \frac{d\overline{\theta}}{dY_X} \end{aligned}$$

$$\begin{aligned}\frac{d\underline{\delta}}{dY_\chi} &= (c_1-1)\frac{dX^*}{dY_\chi}\\ \frac{d\overline{\delta}}{dY_\chi} &= (c_2-1)\frac{dX^*}{dY_\chi}\end{aligned}$$

Differentiate equation (2.18), and we have

$$g(\underline{\delta})\frac{d\underline{\delta}}{dY_\chi} + \int_{\underline{\delta}}^{\overline{\delta}} g(k)h\left(\delta^*-k\right)\frac{d\delta^*}{dY_\chi}dk +$$
$$g(\overline{\delta})H\left(\underline{\delta}-\overline{\delta}-H^{-1}\left(1-\overline{\lambda}\right)\right)\frac{d\overline{\delta}}{dY_\chi} - g(\underline{\delta})H\left(\underline{\delta}-\underline{\delta}-H^{-1}\left(1-\overline{\lambda}\right)\right)\frac{d\underline{\delta}}{dY_\chi} = 0$$

$$\Leftrightarrow g(\underline{\delta})\frac{d\underline{\delta}}{dY_\chi} + \int_{\underline{\delta}}^{\overline{\delta}} g(k)h\left(\delta^*-k\right)\left[\begin{array}{c}\frac{d\underline{\delta}}{dY_\chi}-\\ \frac{(1-H_\chi)(1-H(\underline{\theta}-Y^*))+Y_\chi h_\chi(1-H(\underline{\theta}-Y^*))}{Y_\chi(1-H_\chi)h(\underline{\theta}-Y^*)}\end{array}\right]dk +$$
$$g(\overline{\delta})H\left(\underline{\delta}-\overline{\delta}-H^{-1}\left(1-\overline{\lambda}\right)\right)\frac{d\overline{\delta}}{dY_\chi} - g(\underline{\delta})H\left(\underline{\delta}-\underline{\delta}-H^{-1}\left(1-\overline{\lambda}\right)\right)\frac{d\underline{\delta}}{dY_\chi} = 0$$

$$\Leftrightarrow \frac{d\underline{\delta}}{dY_\chi}\left(g(\underline{\delta})\overline{\lambda} + \int_{\underline{\delta}}^{\overline{\delta}} g(k)h\left(\delta^*-k\right)dk\right) + g(\overline{\delta})H\left(\underline{\delta}-\overline{\delta}-H^{-1}\left(1-\overline{\lambda}\right)\right)\frac{d\overline{\delta}}{dY_\chi}$$
$$-\int_{\underline{\delta}}^{\overline{\delta}} g(k)h\left(\delta^*-k\right)\frac{\left(1-H_\chi\right)\left(1-H(\underline{\theta}-Y^*)\right)+Y_\chi h_\chi\left(1-H(\underline{\theta}-Y^*)\right)}{Y_\chi(1-H_\chi)h\left(\underline{\theta}-Y^*\right)}dk = 0$$

Let $c_5 = g(\underline{\delta})\overline{\lambda} + \int_{\underline{\delta}}^{\overline{\delta}} g(k)h\left(\delta^*-k\right)dk$, Then

$$c_5\frac{d\underline{\delta}}{dY_\chi} + c_4\frac{d\overline{\delta}}{dY_\chi} -$$
$$\int_{\underline{\delta}}^{\overline{\delta}} g(k)h\left(\delta^*-k\right)\frac{\left(1-H_\chi\right)\left(1-H(\underline{\theta}-Y^*)\right)+Y_\chi h_\chi\left(1-H(\underline{\theta}-Y^*)\right)}{Y_\chi(1-H_\chi)h\left(\underline{\theta}-Y^*\right)}dk = 0$$

$$c_5(c_1-1)\frac{dX^*}{dY_\chi} + c_4(c_2-1)\frac{dX^*}{dY_\chi} -$$
$$\int_{\underline{\delta}}^{\overline{\delta}} g(k)h\left(\delta^*-k\right)\frac{\left(1-H_\chi\right)\left(1-H(\underline{\theta}-Y^*)\right)+Y_\chi h_\chi\left(1-H(\underline{\theta}-Y^*)\right)}{Y_\chi(1-H_\chi)h\left(\underline{\theta}-Y^*\right)}dk = 0$$

$$\frac{dX^*}{dY_\chi} = \frac{\int_{\underline{\delta}}^{\overline{\delta}} g(k)h\left(\delta^*-k\right)\frac{(1-H_\chi)(1-H(\underline{\theta}-Y^*))+Y_\chi h_\chi(1-H(\underline{\theta}-Y^*))}{Y_\chi(1-H_\chi)h(\underline{\theta}-Y^*)}dk}{c_5(1-c_1)+c_4(1-c_2)} < 0$$

Then,

$$\frac{d\underline{\theta}}{dY_\chi} < 0,\ \frac{d\overline{\theta}}{dY_\chi} < 0, \text{ and } \frac{dY^*}{dY_\chi} < 0$$

B.8. Proof of Proposition 9

Differentiating equation (2.15) with respect to β_χ, we get

$$\frac{1}{2\sqrt{\alpha_\chi+\beta_\chi}}\phi_\chi(\cdot)\left(Y_\chi-\overline{L}\right)\left(1-H(\underline{\theta}-Y^*)\right)-$$
$$(1-H_\chi)\,h\left(\underline{\theta}-Y^*\right)\cdot\left(\frac{d\underline{\theta}}{d\beta_\chi}-\frac{dY^*}{d\beta_\chi}\right)=0$$

Defining $\delta^* = Y^* - X^*$, using $\underline{\delta} \equiv \underline{\theta} - X^*$ and rearranging, we obtain

$$\frac{d\delta^*}{d\beta_\chi} = \frac{d\underline{\delta}}{d\beta_\chi} - \frac{\frac{1}{2\sqrt{\alpha_\chi+\beta_\chi}}\phi_\chi(\cdot)\left(Y_\chi-\overline{L}\right)\left(1-H(\underline{\theta}-Y^*)\right)}{(1-H_\chi)\,h\left(\underline{\theta}-Y^*\right)}$$

Moreover as for the size of the large creditor:

$$\begin{aligned}
\frac{d\underline{\theta}}{d\beta_\chi} &= z(1-p)\left(1-G_\chi\right)g\left(X^*-\underline{\theta}\right)\left(\frac{dX^*}{d\beta_\chi}-\frac{d\underline{\theta}}{d\beta_\chi}\right)\\
\frac{dX^*}{d\beta_\chi} &= \left(1+\frac{1}{z(1-p)\left(1-G_\chi\right)g\left(X^*-\underline{\theta}\right)}\right)\frac{d\underline{\theta}}{d\beta_\chi}
\end{aligned}$$

$$\begin{aligned}
\frac{d\overline{\theta}}{d\beta_\chi} &= z(1-p)\left(1-G_\chi\right)g\left(X^*-\overline{\theta}\right)\left(\frac{dX^*}{d\beta_\chi}-\frac{d\overline{\theta}}{d\beta_\chi}\right)\\
\frac{dX^*}{dH_\chi} &= \left(1+\frac{1}{z(1-p)\left(1-G_\chi\right)g\left(X^*-\overline{\theta}\right)}\right)\frac{d\overline{\theta}}{d\beta_\chi}
\end{aligned}$$

Then

$$\begin{aligned}\frac{dX^*}{d\beta_\chi} &= \frac{1}{c_1}\frac{d\underline{\theta}}{d\beta_\chi}\\ \frac{dX^*}{dH_\chi} &= \frac{1}{c_2}\frac{d\overline{\theta}}{dH_\chi}\end{aligned}$$

$$\begin{aligned}\frac{d\underline{\delta}}{d\beta_\chi} &= (c_1-1)\frac{dX^*}{d\beta_\chi}\\ \frac{d\overline{\delta}}{d\beta_\chi} &= (c_2-1)\frac{dX^*}{d\beta_\chi}\end{aligned}$$

Differentiate equation (2.18), and we have

$$g(\underline{\delta})\frac{d\underline{\delta}}{d\beta_\chi} + \int_{\underline{\delta}}^{\overline{\delta}} g(k)h\left(\delta^* - k\right)\frac{d\delta^*}{d\beta_\chi}dk +$$
$$g(\overline{\delta})H\left(\underline{\delta} - \overline{\delta} - H^{-1}\left(1-\overline{\lambda}\right)\right)\frac{d\overline{\delta}}{d\beta_\chi} - g(\underline{\delta})H\left(\underline{\delta} - \underline{\delta} - H^{-1}\left(1-\overline{\lambda}\right)\right)\frac{d\underline{\delta}}{d\beta_\chi} = 0$$

$$\Leftrightarrow g(\underline{\delta})\frac{d\underline{\delta}}{d\beta_\chi} + \int_{\underline{\delta}}^{\overline{\delta}} g(k)h\left(\delta^* - k\right)\left[\begin{array}{c}\frac{d\underline{\delta}}{d\beta_\chi} - \\ \frac{\frac{1}{2\sqrt{\alpha_\chi+\beta_\chi}}\phi_\chi(\cdot)\left(Y_\chi - \overline{L}\right)\left(1-H(\underline{\theta}-Y^*)\right)}{(1-H_\chi)h(\underline{\theta}-Y^*)}\end{array}\right]dk +$$
$$g(\overline{\delta})H\left(\underline{\delta} - \overline{\delta} - H^{-1}\left(1-\overline{\lambda}\right)\right)\frac{d\overline{\delta}}{d\beta_\chi} - g(\underline{\delta})H\left(\underline{\delta} - \underline{\delta} - H^{-1}\left(1-\overline{\lambda}\right)\right)\frac{d\underline{\delta}}{d\beta_\chi} = 0$$

$$\Leftrightarrow \frac{d\underline{\delta}}{d\beta_\chi}\left(g(\underline{\delta})\overline{\lambda} + \int_{\underline{\delta}}^{\overline{\delta}} g(k)h\left(\delta^* - k\right)dk\right) + g(\overline{\delta})H\left(\underline{\delta} - \overline{\delta} - H^{-1}\left(1-\overline{\lambda}\right)\right)\frac{d\overline{\delta}}{d\beta_\chi} -$$
$$\int_{\underline{\delta}}^{\overline{\delta}} g(k)h\left(\delta^* - k\right)\frac{\frac{1}{2\sqrt{\alpha_\chi+\beta_\chi}}\phi_\chi(\cdot)\left(Y_\chi - \overline{L}\right)\left(1-H(\underline{\theta}-Y^*)\right)}{\left(1-H_\chi\right)h\left(\underline{\theta}-Y^*\right)}dk = 0$$

Let $c_5 = g(\underline{\delta})\overline{\lambda} + \int_{\underline{\delta}}^{\overline{\delta}} g(k)h\left(\delta^* - k\right)dk$, Then

$$c_5\frac{d\underline{\delta}}{d\beta_\chi} + c_4\frac{d\overline{\delta}}{d\beta_\chi} - \int_{\underline{\delta}}^{\overline{\delta}} g(k)h\left(\delta^* - k\right)\frac{\frac{1}{2\sqrt{\alpha_\chi+\beta_\chi}}\phi_\chi(\cdot)\left(Y_\chi - \overline{L}\right)\left(1-H(\underline{\theta}-Y^*)\right)}{\left(1-H_\chi\right)h\left(\underline{\theta}-Y^*\right)}dk = 0$$

$$c_5(c_1-1)\frac{dX^*}{d\beta_X}+c_4(c_2-1)\frac{dX^*}{d\beta_X}-$$

$$\int_{\underline{\delta}}^{\overline{\delta}} g(k)h\left(\delta^*-k\right)\frac{\frac{1}{2\sqrt{\alpha_X+\beta_X}}\phi_X(\cdot)\left(Y_X-\overline{L}\right)\left(1-H(\underline{\theta}-Y^*)\right)}{\left(1-H_X\right)h\left(\underline{\theta}-Y^*\right)}dk=0$$

$$\frac{dX^*}{d\beta_X}=\frac{\int_{\underline{\delta}}^{\overline{\delta}} g(k)h\left(\delta^*-k\right)\frac{\frac{1}{2\sqrt{\alpha_X+\beta_X}}\phi_X(\cdot)\left(Y_X-\overline{L}\right)\left(1-H(\underline{\theta}-Y^*)\right)}{\left(1-H_X\right)h\left(\underline{\theta}-Y^*\right)}dk}{c_5(1-c_1)+c_4(1-c_2)}<0$$

Then,

$$\frac{d\underline{\theta}}{d\beta_X}<0,\ \frac{d\overline{\theta}}{d\beta_X}<0,\text{ and }\frac{dY^*}{d\beta_X}<0$$

B.9. Proof of Proposition 10

Differentiating equation (2.15) with respect to z, we get

$$\begin{aligned}
\frac{d\underline{\theta}}{dz} &= (1-p)\left[G_X+(1-G_X)G\left(X^*-\underline{\theta}\right)\right]+\\
&\quad z(1-p)\left(1-G_X\right)g\left(X^*-\underline{\theta}\right)\left(\frac{dX^*}{dz}-\frac{d\underline{\theta}}{dz}\right)\\
\frac{dX^*}{dz} &= \left(1+\frac{1}{z(1-p)\left(1-G_X\right)g\left(X^*-\underline{\theta}\right)}\right)\frac{d\underline{\theta}}{dz}-\\
&\quad \frac{G_X+(1-G_X)G\left(X^*-\underline{\theta}\right)}{z\left(1-G_X\right)g\left(X^*-\underline{\theta}\right)}
\end{aligned}$$

$$\begin{aligned}
\frac{d\overline{\theta}}{dz} &= \left[p+(1-p)\left(G_X+(1-G_X)G\left(X^*-\overline{\theta}\right)\right)\right]+\\
&\quad z(1-p)\left(1-G_X\right)g\left(X^*-\overline{\theta}\right)\left(\frac{dX^*}{dz}-\frac{d\overline{\theta}}{dz}\right)\\
\frac{dX^*}{dz} &= \left(1+\frac{1}{z(1-p)\left(1-G_X\right)g\left(X^*-\overline{\theta}\right)}\right)\frac{d\overline{\theta}}{dz}-\\
&\quad \frac{p+(1-p)\left(G_X+(1-G_X)G\left(X^*-\overline{\theta}\right)\right)}{z(1-p)\left(1-G_X\right)g\left(X^*-\overline{\theta}\right)}
\end{aligned}$$

Then

$$\begin{aligned}\frac{dX^*}{dz} &= \frac{1}{c_1}\frac{d\underline{\theta}}{dz} - \frac{G_X + (1-G_X)\,G\left(X^* - \underline{\theta}\right)}{z\left(1-G_X\right) g\left(X^* - \underline{\theta}\right)} \\ \frac{dX^*}{dz} &= \frac{1}{c_2}\frac{d\overline{\theta}}{dz} - \frac{p + (1-p)\left(G_X + (1-G_X)\,G\left(X^* - \overline{\theta}\right)\right)}{z(1-p)\left(1-G_X\right) g\left(X^* - \overline{\theta}\right)}\end{aligned}$$

Using $\underline{\delta} \equiv \underline{\theta} - X^*$ and $\overline{\delta} \equiv \overline{\theta} - X^*$, we have

$$\begin{aligned}\frac{d\underline{\delta}}{dz} &= \frac{d\underline{\theta}}{dz} - \frac{dX^*}{dz} \\ &= c_1\left(\frac{dX^*}{dz} + \frac{G_X + (1-G_X)\,G\left(X^* - \underline{\theta}\right)}{z\left(1-G_X\right) g\left(X^* - \underline{\theta}\right)}\right) - \frac{dX^*}{dz} \\ &= (c_1 - 1)\frac{dX^*}{dz} + c_1\frac{G_X + (1-G_X)\,G\left(X^* - \underline{\theta}\right)}{z\left(1-G_X\right) g\left(X^* - \underline{\theta}\right)}\end{aligned}$$

$$\begin{aligned}\frac{d\overline{\delta}}{dz} &= \frac{d\overline{\theta}}{dz} - \frac{dX^*}{dz} \\ &= c_2\left(\frac{dX^*}{dz} + \frac{p + (1-p)\left(G_X + (1-G_X)\,G\left(X^* - \overline{\theta}\right)\right)}{z(1-p)\left(1-G_X\right) g\left(X^* - \overline{\theta}\right)}\right) - \frac{dX^*}{dz} \\ &= (c_2 - 1)\frac{dX^*}{dz} + c_2\frac{p + (1-p)\left(G_X + (1-G_X)\,G\left(X^* - \overline{\theta}\right)\right)}{z(1-p)\left(1-G_X\right) g\left(X^* - \overline{\theta}\right)}\end{aligned}$$

Now differentiate equation (2.18), and we have

$$\frac{d\underline{\delta}}{dz}g(\underline{\delta}) + \int_{\underline{\delta}}^{\overline{\delta}} g(k)h\left(\underline{\delta} - k - H^{-1}\left(1-\overline{\lambda}\right)\right) dk\frac{d\underline{\delta}}{dz} + g(\overline{\delta})H\left(\underline{\delta} - \overline{\delta} - H^{-1}\left(1-\overline{\lambda}\right)\right)\frac{d\overline{\delta}}{dz} - g(\underline{\delta})H\left(\underline{\delta} - \underline{\delta} - H^{-1}\left(1-\overline{\lambda}\right)\right)\frac{d\underline{\delta}}{dz} = 0$$

$$\Leftrightarrow \frac{d\underline{\delta}}{dz}g(\underline{\delta})\left(1 - H\left(-H^{-1}\left(1-\overline{\lambda}\right)\right)\right) + \int_{\underline{\delta}}^{\overline{\delta}} g(k)h\left(\underline{\delta} - k - H^{-1}\left(1-\overline{\lambda}\right)\right) dk\frac{d\underline{\delta}}{dz} + g(\overline{\delta})H\left(\underline{\delta} - \overline{\delta} - H^{-1}\left(1-\overline{\lambda}\right)\right)\frac{d\overline{\delta}}{dz} = 0$$

$$\frac{d\underline{\delta}}{dz}\left(g(\underline{\delta})\overline{\lambda} + \int_{\underline{\delta}}^{\overline{\delta}} g(k)h\left(\underline{\delta} - k - H^{-1}\left(1-\overline{\lambda}\right)\right) dk\right) + g(\overline{\delta})H\left(\underline{\delta} - \overline{\delta} - H^{-1}\left(1-\overline{\lambda}\right)\right)\frac{d\overline{\delta}}{dz} = 0$$

Then

$$\frac{d\underline{\delta}}{dz}c_3 + \frac{d\overline{\delta}}{dz}c_4 = 0$$

$$c_3(c_1 - 1)\frac{dX^*}{dz} + c_1c_3\frac{G_X + (1 - G_X)\,G\,(X^* - \underline{\theta})}{z\,(1 - G_X)\,g\,(X^* - \underline{\theta})} + c_4(c_2 - 1)\frac{dX^*_{\theta}}{dz} +$$
$$c_2c_4\frac{p + (1 - p)\left(G_X + (1 - G_X)\,G\left(X^* - \overline{\theta}\right)\right)}{z(1 - p)\,(1 - G_X)\,g\left(X^* - \overline{\theta}\right)} = 0$$

$$\frac{dX^*}{dz} = \frac{-c_1c_3\frac{G_X+(1-G_X)G(X^*-\underline{\theta})}{z(1-G_X)g(X^*-\underline{\theta})} - c_2c_4\frac{p+(1-p)\left(G_X+(1-G_X)G\left(X^*-\overline{\theta}\right)\right)}{z(1-p)(1-G_X)g\left(X^*-\overline{\theta}\right)}}{c_3(c_1 - 1) + c_4(c_2 - 1)} > 0$$

Then

$$\frac{d\underline{\theta}}{dz} > 0,\ \frac{d\overline{\theta}}{dz} > 0, \text{ and } \frac{dY^*}{dz} > 0$$

which completes the proof.

CHAPTER III

FULL OR PARTIAL WAGE INDEXATION: BAYESIAN ESTIMATION OF A SMALL OPEN ECONOMY MODEL

Abstract

Empirical work with dynamic stochastic general equilibrium (DSGE) models such as Christiano, Eichenbaum and Evans (2005) imposes full price and wage indexation to lagged inflation. This specification has the advantage of matching the positive trend inflation observed in the data without considering the resource costs induced by inefficient price and wage dispersions. However, this chapter finds this specification is not empirically robust by developing a small open economy model in which partial indexation is allowed and estimating it on Canadian data using Bayesian estimation techniques. The model incorporates a number of nominal and real frictions proved to be of importance for explaining the dynamics of an open economy. The results show: (*i*) wages are indexed only partially to lagged inflation, which is close to the average proportion of wage settlements with cost-of-living clauses in Canada during the sample period; (*ii*) the empirical performance drops with a version of the model in which wages are fully indexed to lagged inflation; (*iii*) the model with partial indexation accounts well for the dynamics of the Canadian economy.

Key words: Small open economy; Bayesian inference.

3.1 Introduction

Empirical work with DSGE models in which prices and wages are adjusted in the fashion of Calvo (1983) often imposes full indexation of unadjusted prices and wages to lagged inflation[1] (See, for example, Christiano, Eichenbaum and Evans, 2005; Altig et al., 2005). This specification has the advantage of generating a simple solution with a positive steady-state inflation rate[2], an empirical fact, without considering the resource costs induced by inefficient price and wage dispersions. However, there is good reason to ask whether this specification is empirically robust, and if it is not, whether there is a specification with a better empirical performance.

This chapter addresses these concerns. Theoretically we know from Gray (1976) that full indexation is not optimal, and indexation is rather partial than full. More importantly, there is no empirical evidence of full indexation either for prices or for wages. Instead, this paper develops a small open economy model in which partial indexation is allowed and estimates it on Canadian data using Bayesian estimation techniques. The specification adopted in this paper is indeed based on empirical evidence at both the macro and micro level. First, since there is no evidence of price indexation using aggregate data from Canada (Justiniano and Preston, 2004), in the model, prices are not indexed. Next, from Canadian microdata of wage contracts[3], the average proportion of wage settlements with cost-of-living clauses for the last twenty-five years is around 15 percent[4]. Therefore, wages are indexed to lagged inflation in the model, and the wage indexation parameter is estimated rather than calibrated to unity in the estimation procedure.

In an economy with trend inflation, some important issues arise when moving from a specification with full indexation to a specification with partial indexation. First

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[1]Some other variants of full indexation can be found in the literature such as full indexation to both lagged and leading inflation as in Adolfson et al. (2007) or to trend inflation as in Yun (1996).

[2]Ascari (2004) shows the assumption that log-linearize a model with price adjustment in the fashion of Calvo (1983) around a positive inflation steady state or around a zero steady state inflation should have negligible effect both on the steady state and on the dynamic properties of the model is invalid. Other works with a positive steady-state inflation are King and Wolman (1996), Chari et al. (2002), and Schmitt-Grohe and Uribe (2007).

[3]This database is constructed by Human Resource and Social Development Department Canada.

[4]Other contracts include often annual wage adjustments that may take into account other factors for example expected inflation, general expected increase of productivity, incentive mechanism, etc.

of all, to match trend inflation, around 3 percent in Canada for the last twenty-five years, the estimated model should feature a positive steady-state inflation. By definition the steady state is characterized by relative price movements. Second, the indexation pattern decides whether this steady state features price dispersions. On the one hand, when unadjusted prices are indexed fully on lagged inflation, this automatic updating rule coincides with the steady state optimal pricing rule; therefore, there are no price dispersions in the steady sate. On the other hand, when prices are indexed partially to lagged inflation, the nominal prices charged by firms are dispersed around the average price prevailing in the economy. Though their nominal prices are fixed, trend inflation leads to different relative prices according to which firms produce different levels of output. Subsequently, due to the non-linearities in the utility and production function, price dispersions cause an aggregate output loss, which is similar to the deadweight loss induced by a distortionary tax on final goods.

The same argument applies to wage adjustment. When unadjusted wages are fully indexed to lagged inflation, only one optimal salary is charged in the steady state; consequently, there are no wage dispersions. In contrast, with partial indexation households offer different levels of labor to differentiated labor markets according to their relative wages. This generates an aggregate labor loss, which is similar to the deadweight loss induced by a distortionary labor income tax.

These inefficient resource costs induced by price and wage dispersions matter for both the steady state and the dynamics of the economy[5]. The model therefore explicitly considers these costs. In order to compare with previous studies, together, the model features four nominal frictions: *(i)* sticky prices, *(ii)* sticky wages, *(iii)* cash-in-advance constraints for firms, and *(iv)* cash-in-advance on the wage bill of firms; and four real frictions: *(i)* variable capital utilization, *(ii)* capital adjustment costs, *(iii)* habit persistence, and *(iv)* imperfect competition in goods and factor markets. The empirical role of these frictions is assessed in the estimation procedure.

[5]See some interesting studies on welfare costs of price and wage dispersions such as Amano et al. (2007), and Amano, Ambler and Rebei (2007).

The model contains nine shocks of which two are permanent and the others are temporary. The two permanent shocks are a neutral technology shock and an investment-specific technology shock. Fisher (2006) argues that the investment-specific technology shock is important in explaining the observed decline in the relative price of investment goods in terms of consumption goods in the postwar U.S. economy, which is a feature of the Canadian data as well. These two permanent shocks induce two common stochastic trends in the real variables of the model, allowing for the raw data to estimate the model by taking into account the cointegration between the real variables. Additional temporary shocks, such as government purchases, monetary policy and a set of structural shocks from the open economy aspects are included in the model.

The model is estimated using Bayesian estimation techniques[6]. Smets and Wouters (2003, 2007) show that the forecasting performance of models estimated with Bayesian methods on U.S. data and Euro area data is quite good compared to standard vector autoregressive models. Indexation parameters are estimated in their models; nevertheless, resource costs are not considered. Adolfson et al. (2007) estimate a rich DSGE open economy model on Euro area data using Bayesian estimation techniques. Their estimated model succeeds in accounting for most of the dynamics of an open economy; still, prices and wages are indexed fully to lagged and leading inflation. For the Canadian economy, Dib (2003) shows that a model estimated on Canadian data using maximum likelihood techniques has forecasting properties that are comparable to those of a vector autoregression. Ambler, Dib and Rebei (2004) use the same estimation techniques to estimate the structural parameters and use them to analyze optimal monetary policy. These two models are linearized around zero steady-state inflation, and neither prices nor wages are indexed.

The model is estimated on quarterly data from Canada for the small open economy and U.S. data to approximate the rest of the world. Together, eleven time series

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[6]Some important contributions to the literature on Bayesian estimation of DSGE models are Dejong, Ingram, and Whiteman (2000), Schorfheide (2000), Fernandez-Villaverde and Rubio-Ramirez (2004), Landon-Lane (1998), Otrok (2001), and An and Schorfheide (2007).

from 1981Q3 to 2006Q4 are used to estimate the model. Since nine structural shocks are included in the model, two error terms are added to the observation equations to avoid the problem of stochastic singularity[7].

The principal results of the estimation performed here are as follows. The estimate of the wage indexation parameter is quite low at 10 percent, which is close to the average proportion of wage settlements with cost-of-living clauses (about 15 percent in Canada during the sample period)[8]. This result is quite robust to changes in model structure. Between a version of the model with partial wage indexation and that with full wage indexation, the posterior odds ratio is largely in favor of the version of the model with partial indexation, which does a good job of accounting for the dynamics of the economy.

The results support the idea that the investment-specific technology shock is important in explaining business cycles; it accounts for about 30 percent of aggregate fluctuations at business-cycle frequencies in the model. Fisher (2006) shows that the investment-specific technology shock accounts for about 50 percent of the business cycles in the postwar U.S. economy; however, Altig et al. (2005) find smaller numbers. The estimates show that shocks to neutral technology exhibit a high degree of serial correlation, while shocks to investment specific technology do not. The unconditional standard deviation of the growth rate of neutral technology is roughly 0.88 percent, and that of investment specific technology is about 1.88. Altig et al. (2005) find the values of 0.16 and 0.31 percent for the U.S. economy. Thus these two shocks are substantially more volatile than their estimates.

Finally, the household re-optimizes wages on average once every 3.3 quarters.

[7]The issue of stochastic singularity arises when a DSGE model generates predictions about a larger number of observable endogenous variables than exogenous shocks used in the model. There are two strategies to deal with this problem: (*i*) use at most as many observable variables as structural shocks (Adding two more shocks into the model raises two problems: first these two shocks are not well identified; second, the empirical performance is worse than the strategy adding two error terms.); (*ii*) add error terms to the observation equations.

[8]The minor difference between the estimated result and the result from microdata is due to the fact that wage contracts in microdata cover around 11 percent of workers in Canada, while the model is estimated on aggregated data.

Firms, including domestic firms and importing firms, on average re-optimize prices every 3 quarters[9]. The model yields the values of the elasticity of substitution between domestic and imported goods in the range of 1.4 to 1.5, which is in line with the results reported in Collard and Dellas (2002).

The chapter is organized as follows. Section 3.2 describes the model economy. Section 3.3 discusses data, measurement equations, calibrated parameters and prior distributions for the parameters chosen to estimate. Section 3.4 first reports the estimation results and validates the model fit. Then it shows the impulse responses from different shocks. Section 3.5 provides some conclusions.

3.2 The small open economy model

This section describes the model economy and displays the problems of firms and households. The model economy is open and populated by households, firms, commercial banks, a government and a central bank. The representative household consumes a basket of domestically produced goods and imported goods, rents capital to the domestic firms, and decides how much to invest. Labor decisions are decided by a central authority within the households which supplies labor monopolistically to a continuum of labor markets[10]. There are two categories of firms operating in this open economy: domestic and importing firms. The domestic goods are produced in two stages. The intermediate domestic firms use capital and labour inputs to produce differentiated intermediate goods. The final producer uses a continuum of these intermediate goods in its production to produce a homogeneous final good, and sell its products in both the domestic and foreign markets. The importing firms differentiate homogeneous goods bought in the world market and sell them in the domestic market. A fraction of domestic

[9]This estimate is consistent with the results reported by Ambler et al. (2004) on Canadian data.

[10]This modeling strategy can be applied to an environment with highly centralized labor unions. Since in Canada, the degree of unionization remains around 35 percent in the sample period, this assumption is only an approximation rather than an accurate description of the reality. However, this assumption has the advantage that it avoids the need to assume the existence of financial markets allowing agents fully insure against employment risk if each household is assumed to supply a differentiated type of labor input as in Erceg, et al. (2000).

output is exported.

3.2.1 Firms

Domestic firms

The final good, Y_t, is produced using a constant elasticity of substitution technology

$$Y_t = \left[\int_0^1 Y_t(i)^{\frac{\eta-1}{\eta}} di\right]^{\frac{\eta}{\eta-1}}$$

where $\eta > 1$ represents the elasticity of demand for each intermediate good, $Y_t(i)$. Under perfect competition, the firm takes input prices $P_t(i)$ and output prices P_t as given. The profit maximization leads to the demand functions of intermediate goods

$$Y_t(i) = \left(\frac{P_t(i)}{P_t}\right)^{-\eta} Y_t$$

By substituting the demand function into the production function, the price index for the domestic composite goods is given by

$$P_t = \left[\int_0^1 P_t(i)^{1-\eta} di\right]^{\frac{1}{1-\eta}}$$

There is a continuum of intermediate goods producers under monopolistic competition. They have identical production function given by

$$Y_t(i) = K_t(i)^{\theta} \left(z_t h_t(i)\right)^{1-\theta} - \psi z_t^* \tag{3.1}$$

where $\theta \in (0, 1)$, $h_t(i)$ and $K_t(i)$ denote time t labour and capital inputs employed by firm i. The variable, z_t, is a permanent neutral technology shock. The fixed costs[11], ψ,

[11] It implies that the production function exhibits increasing returns to scale and ensures a realistic profit-to-output ratio in steady state.

are subject to permanent shocks, z_t^*, with

$$\frac{z_t^*}{z_t} = \Upsilon_t^{\frac{\theta}{1-\theta}}$$

where Υ_t represents a permanent investment-specific technology shock . Let $\mu_{z,t} = z_t/z_{t-1}$ denote the gross growth rate of the neutral technology shock. Similarly $\mu_{z^*,t}$ and $\mu_{\Upsilon,t}$ are respectively the gross growth rate of z_t^* and Υ_t. We have the following relationship

$$\mu_{z^*,t} = (\mu_{\Upsilon,t})^{\frac{\theta}{1-\theta}} \mu_{z,t}$$

The percentage deviation of z_t, denoted by $\ln(\frac{\mu_{z,t}}{\mu_z})$, follows a law of motion

$$\ln(\frac{\mu_{z,t}}{\mu_z}) = \rho_{\mu_z} \ln(\frac{\mu_{z,t-1}}{\mu_z}) + \epsilon_{\mu_z,t}$$

where $\rho_{\mu_z} \in (0,1)$, and $\epsilon_{\mu_z,t}$ is an $i.i.d$ innovation with mean zero, standard deviation σ_{μ_z} and bounded support. Similarly, the variable $\ln(\frac{\mu_{\Upsilon,t}}{\mu_\Upsilon})$ involves according to

$$\ln(\frac{\mu_{\Upsilon,t}}{\mu_\Upsilon}) = \rho_{\mu_\Upsilon} \ln(\frac{\mu_{\Upsilon,t-1}}{\mu_\Upsilon}) + \epsilon_{\mu_\Upsilon,t}$$

where $\epsilon_{\mu_\Upsilon,t}$ has the same properties as $\epsilon_{\mu_z,t}$ with standard deviation σ_{μ_Υ}.

Wage payments are subject to a cash-in-advance constraint

$$M_t^f(i) \geq vW_t h_t(i)$$

where $M_t^f(i)$ represents the monetary assets hold by firm i, v is a parameter indicating the fraction of the wage bill that must be backed with monetary assets and W_t is the nominal wage rate.

The problem of the firm i at time t is to choose contingent plans for $P_t(i)$, $h_t(i)$, and $K_t(i)$ so as to maximize the present discounted value of dividend payments.

$$\max_{\{P_t(i),h_t(i),K_t(i)\}} E_t \sum_{s=0}^{\infty} r_{t,t+s} \left\{ \begin{array}{c} P_t(i)^{1-\eta} P_{t+s}^{\eta} Y_{t+s} - R_{t+s}^{k} K_{t+s}(i) - \\ W_{t+s} h_{t+s}(i) - (1 - R_{t+s}^{-1}) v W_{t+s} h_{t+s}(i) \end{array} \right\}$$

subject to the technological constraint (3.1). Here, $r_{t,t+s} \equiv \Pi_{k=1}^{s} r_{t+k-1,t+k}$ for $s \geq 1$ denotes the stochastic nominal discount factor between t and $t+s$, and $r_{t,t} \equiv 1$. The variable, R_t^k, is the gross nominal rental rate per unit of capital services and $(1 - R_t^{-1}) v W_t h_t(i)$ represents the financial costs of holding money to satisfy the working-capital constraint. Let $P_t mc_t(i)$ be the Lagrange multiplier associated with constraint (3.1). The first-order conditions with respect to labor services and capital are

$$W_t \left[1 + \nu \frac{R_t - 1}{R_t} \right] = P_t mc_t(i)(1-\theta) K_t(i)^{\theta} h_t(i)^{-\theta} \tag{3.2}$$

$$R_t^k = P_t mc_t(i) \theta K_t(i)^{\theta-1} h_t(i)^{1-\theta} \tag{3.3}$$

As this economy features two types of permanent shocks and a unit-root in the price level, a number of variables are not stationary. Let small letters indicate stationarized variables[12]. For convenience, let $\mu_{I,t} = \mu_{z,t} \mu_{\Upsilon,t}^{\frac{1}{1-\theta}}$. Since all firms have access to the same factor prices and the same production technology, the marginal costs are identical across firms. Therefore, in equilibrium the above first-order conditions are stationarized as

$$mc_t(1-\theta) \left(\frac{u_t k_t}{\mu_{I,t}} \right)^{\theta} (h_t^d)^{-\theta} = w_t \left[1 + v \frac{R_t - 1}{R_t} \right]$$

$$mc_t \theta \left(\frac{u_t k_t}{\mu_{I,t}} \right)^{\theta - 1} (h_t^d)^{1-\theta} = r_t^k$$

The price setting problem of the intermediate firms is à la Calvo (1983) and Yun

[12]The variables are stationarized in the following way: $r_t^k \equiv \frac{\Upsilon_t R_t^k}{P_t}$, $w_t \equiv \frac{W_t}{z_t^* P_t}$, $k_t \equiv \frac{K_t}{z_{t-1} \Upsilon_{t-1}^{\frac{1}{1-\theta}}}$

(1996). Each period, a fraction $\alpha \in [0,1)$ of firms is not allowed to optimally set the nominal prices of the goods they produce. The remaining $1-\alpha$ firms choose prices optimally. Let $\tilde{P}_t$ denote the price to reoptimize, the first-order condition with respect to $\tilde{P}_t$ is expressed recursively and stationarized[13] as

$$x_t^1 = y_t mc_t \tilde{p}_t^{-\eta-1} + \alpha\beta E_t \frac{\mu_{\lambda,t+1}\lambda_{t+1}}{\lambda_t}\left(\frac{\tilde{p}_t}{\tilde{p}_{t+1}}\right)^{-\eta-1} \pi_{t+1}^{\eta}\mu_{z^*,t+1}x_{t+1}^1$$

$$x_t^2 = y_t \tilde{p}_t^{-\eta} + \alpha\beta E_t \frac{\mu_{\lambda,t+1}\lambda_{t+1}}{\lambda_t}\left(\frac{\tilde{p}_t}{\tilde{p}_{t+1}}\right)^{-\eta} \pi_{t+1}^{\eta-1}\mu_{z^*,t+1}x_{t+1}^2$$

and

$$\eta x_t^1 = (\eta-1)x_t^2$$

where x_t^1 and x_t^2 are two auxiliary variables. Optimizing firms set nominal prices so as to satisfy that average future expected marginal revenues are equal to average future expected marginal costs.

A fraction of domestic output is exported. Foreign demands for domestic consumption good, $C_{x,t}$ and for domestic investment good, $I_{x,t}$ are given by

$$C_{x,t} = \omega_f \left(\frac{P_t}{e_t P_t^*}\right)^{-\eta_f} C_t^*, \quad I_{x,t} = \omega_f \left(\frac{P_t}{e_t P_t^*}\right)^{-\eta_f} I_t^*$$

where C_t^* and I_t^* are respectively the foreign consumption and investment, P_t^* is the foreign price level in foreign currency, and e_t is the nominal exchange rate, defined as the price of foreign currency in terms of domestic currency. The parameter, ω_f, represents the fraction of domestic exports in foreign spending, and η_f is the elasticity of demand for domestic consumption and investment goods[14]. The foreign output is assumed equal

[13]The variables are stationarized as $y_t \equiv Y_t/z_t^*$, $\lambda_t \equiv z_t^*\Lambda_t$, $x_t^1 \equiv X_t^1/z_t^*$, $x_t^2 \equiv X_t^2/z_t^*$, $\tilde{p}_t \equiv \tilde{P}_t/P_t$, $\pi_t = P_t/P_{t-1}$, and $\mu_{\lambda,t} = 1/\mu_{z^*,t}$.

[14]The fraction, ω_f, and the elasticity, η_f, are assumed to the same for consumption and investment goods. This allows to use foreign output as the only demand variable without tracing how much of the exporting goods are used for consumption and investment purposes respectively.

to the sum of the foreign consumption and investment, $Y_t^* = C_t^* + I_t^*$. We scale Y_t^* with z_t^f, which follows a similar process as z_t^*. Then the stationarized exports are

$$c_{x,t} + i_{x,t} = \omega_f s_t^{\eta_f} y_t^*$$

where $s_t \equiv e_t P_t^*/P_t$ is the real exchange rate.

Importing firms

The importing firms transform the imported products into differentiated consumption and investment goods, $C_{m,t}(i)$ and $I_{m,t}(i)$. In each sector there is a continuum of importing firms under monopolistic competition. The final imported consumption good is a composite of differentiated imported consumption goods such as

$$C_{m,t} = \left[\int_0^1 C_{m,t}(i)^{\frac{\eta_{cm}-1}{\eta_{cm}}} di\right]^{\frac{\eta_{cm}}{\eta_{cm}-1}}$$

where η_{cm} represents the elasticity of demand for each consumption good. The firm i then faces the demand function given by

$$C_{m,t}(i) = \left(\frac{P_{cm,t}(i)}{P_{cm,t}}\right)^{-\eta_{cm}} C_{m,t}$$

Equivalently, for the final import investment

$$I_{m,t} = \left[\int_0^1 I_{m,t}(i)^{\frac{\eta_{im}-1}{\eta_{im}}} di\right]^{\frac{\eta_{im}}{\eta_{im}-1}}, \; I_{m,t}(i) = \left(\frac{P_{im,t}(i)}{P_{im,t}}\right)^{-\eta_{im}} I_{m,t}$$

where η_{im} represents the elasticity of demand for each investment good. In order to allow for incomplete exchange rate pass-through to the imported consumption and investment prices, the local currency price is sticky à la Calvo. Each period, a fraction $\alpha_{cm} \in [0,1)$ of firms is not allowed to optimally set the nominal prices of the consumption goods. The remaining $1-\alpha_{cm}$ firms choose prices optimally. Equivalently, a fraction $\alpha_{im} \in [0,1)$ of firms is not allowed to optimally set the nominal prices of the investment goods.

The remaining $1-\alpha_{im}$ firms choose prices optimally. Let $\tilde{P}_{cm.t}$ and $\tilde{P}_{im.t}$ denote the prices to reoptimize for respectively imported consumption and investment goods. The first-order condition relative to $\tilde{P}_{cm.t}$ is stationarized as follows:

$$\begin{aligned}
x^1_{cm,t} &= c_{m,t} e_t p^*_t \left(\frac{\tilde{p}_{cm,t}}{p_{cm,t}}\right)^{-\eta_{cm}-1} + \alpha_{cm}\beta E_t \frac{\lambda_{t+1}}{\lambda_t}\left(\frac{\tilde{p}_{cm,t}}{\tilde{p}_{cm,t+1}}\right)^{-\eta_{cm}-1} \pi_{t+1}^{1+\eta_{cm}} x^1_{cm,t+1} \\
x^2_{cm,t} &= c_{m,t}\left(\frac{\tilde{p}_{cm,t}}{p_{cm,t}}\right)^{-\eta_m} p_{cm,t} + \beta\alpha_{cm} E_t \frac{\lambda_{t+1}}{\lambda_t}\left(\frac{\tilde{p}_{cm,t}}{\tilde{p}_{cm,t+1}}\right)^{-\eta_m} \pi_{t+1}^{\eta_{cm}} x^2_{cm,t+1}
\end{aligned}$$

and

$$\eta_{cm} x^1_{cm,t} = (\eta_{cm}-1) x^2_{cm,t}$$

The first-order condition relative to $\tilde{P}_{im.t}$ is stationarized as follow

$$\begin{aligned}
x^1_{im,t} &= i_{m,t} e_t p^*_t \left(\frac{\tilde{p}_{im,t}}{p_{im,t}}\right)^{-\eta_{im}-1} + \alpha_{im}\beta E_t \frac{\lambda_{t+1}}{\lambda_t}\left(\frac{\tilde{p}_{im,t}}{\tilde{p}_{im,t+1}}\right)^{-\eta_{im}-1} \pi_{t+1}^{1+\eta_{im}} x^1_{im,t+1} \\
x^2_{im,t} &= i_{m,t}\left(\frac{\tilde{p}_{im,t}}{p_{im,t}}\right)^{-\eta_m} p_{im,t} + \beta\alpha_{im} E_t \frac{\lambda_{t+1}}{\lambda_t}\left(\frac{\tilde{p}_{im,t}}{\tilde{p}_{im,t+1}}\right)^{-\eta_{im}} \pi_{t+1}^{\eta_{im}} x^2_{im,t+1}
\end{aligned}$$

and

$$\eta_{im} x^1_{im,t} = (\eta_{im}-1) x^2_{im,t}$$

Small letters of prices in the above conditions are scaled by domestic price P_t; $x^1_{cm,t}$, $x^2_{cm,t}$, $x^1_{im,t}$, and $x^2_{im,t}$ are auxiliary variables. Under flexible prices ($\alpha_{cm} = \alpha_{im} = 0$), the above optimality conditions reduce to static relations that equate marginal costs to marginal revenues period by period.

3.2.2 Household

The economy is populated by a large representative family with a continuum of members. The household's preferences are described by the utility function

$$E_0 \sum_{t=0}^{\infty} \beta^t U(C_t, \frac{V_t}{P_t}, h_t)$$

where E_t denotes the mathematical expectations operator conditional on information available at time t, $\beta \in (0,1)$ a subjective discount factor, C_t per capita consumption, V_t/P_t per capita real assets the household chooses to hold in non-interest bearing form, and h_t per capita hours worked. The functional form of period utility is given by

$$U(C_t, \frac{V_t}{P_t}, H_t) = \zeta_{c,t} \ln(C_t - bC_{t-1}) + A_v \frac{\left(\frac{V_t}{z_t^* P_t}\right)^{1-\phi_v}}{1-\phi_v} - A_L \frac{h_t^{1+\phi_L}}{1+\phi_L}$$

where $\zeta_{c,t}$ is a consumption preference shock, the constant $b > 0$ captures habit persistence, and per capita real assets are scaled with z_t^* in order to render them stationary. Here, ϕ_v represents the real assets demand elasticity and ϕ_L the labour supply elasticity; A_v and A_L are scale factors that determine real money balances and hours worked in the steady state. The percent deviation of preference shock follows an AR(1) process

$$\ln(\zeta_{c,t}/\zeta_c) = \rho_{\zeta_c} \ln(\zeta_{c,t-1}/\zeta_c) + \varepsilon_{\zeta_{c,t}}$$

where ζ_c denotes the steady state of $\zeta_{c,t}$.

The consumption bundle, C_t, is a constant elasticity of substitution (CES) index composed of domestically produced goods, $C_{d,t}$, and imported goods, $C_{m,t}$, according to

$$C_t = \left((1-\omega_c)^{\frac{1}{\eta_c}} C_{d,t}^{\frac{\eta_c - 1}{\eta_c}} + \omega_c^{\frac{1}{\eta_c}} C_{m,t}^{\frac{\eta_c - 1}{\eta_c}}\right)^{\frac{\eta_c}{\eta_c - 1}} \tag{3.4}$$

where ω_c is the share of imported consumption goods in the economy and $\eta_c > 0$ denotes the intratemporal elasticity of substitution between foreign and domestic consumption goods. By maximizing (3.4) subject to the budget $C_{d,t} + p_{cm,t}C_{m,t} = p_{c,t}C_t$, where $p_{cm,t} = P_{cm,t}/P_t$ and $p_{c,t} = P_{c,t}/P_t$, we obtain the following consumption demand functions

$$C_{d,t} = (1-\omega_c)p_{c,t}^{\eta_c}C_t, \; C_{m,t} = \omega_c\left(\frac{p_{cm,t}}{p_{c,t}}\right)^{-\eta_c}C_t$$

where the consumption price index (CPI) is given by

$$p_{c,t} = \left((1-\omega_c) + \omega_c p_{cm,t}^{1-\eta_c}\right)^{\frac{1}{1-\eta_c}}$$

The household owns physical capital, K_t, which accumulates according to the following law of motion

$$K_{t+1} = (1-\delta)K_t + I_t\left[1 - S\left(\frac{I_t}{I_{t-1}}\right)\right]$$

where i_t denotes gross investment and δ is a parameter denoting the rate of depreciation of physical capital. S is an investment adjustment cost function[15]. The owner of physical capital can control the intensity at which this factor is utilized. Investment goods are produced from final goods by means of a linear technology whereby $1/\Upsilon_t$ units of final goods yield one unit of investment goods. Let u_t measure capacity utilization in period t. Using the stock of capital with intensity u_t entails a cost of $\Upsilon_t^{-1}a(u_t)K_t$ units of the composite final goods[16].

The household rents the capital stock to firms at the nominal rental rate R_t^k per unit of capital. Thus, total income is given by $R_t^k u_t K_t$. The investment good is a composite good.

$$I_t = \left((1-\omega_i)^{\frac{1}{\eta_i}} I_{d,t}^{\frac{\eta_i-1}{\eta_i}} + \omega_i^{\frac{1}{\eta_i}} I_{m,t}^{\frac{\eta_i-1}{\eta_i}}\right)^{\frac{\eta_i}{\eta_i-1}}$$

[15]The functional form for investment adjustment costs is $S\left(\frac{I_t}{I_{t-1}}\right) = \frac{\kappa}{2}\left(\frac{I_t}{I_{t-1}} - \mu_I\right)^2$, where μ_I is the steady-state growth rate of investment. This function is assumed to satisfy in the steady state $S(\mu_I) = S'(\mu_I) = 0$ and $S''(\mu_I) > 0$. Adolfson et al. (2007) use another specification but still satisfies these conditions.

[16]The functional form for capital utilization is $a(u_t) = \gamma_1(u_t - 1) + \frac{\gamma_2}{2}(u_t - 1)^2$.

where ω_i is the share of imported investment goods in the economy and $\eta_i > 0$ denotes the intratemporal elasticity of substitution between domestic and foreign investment goods in the domestic country. As with consumption, the investment demand functions are

$$I_{d,t} = (1-\omega_i)\Upsilon_t^{-1} p_{i,t}^{\eta_i} I_t, \quad I_{m,t} = \omega_i \Upsilon_t^{-1} \left(\frac{p_{im,t}}{p_{i,t}}\right)^{-\eta_i} I_t$$

where the investment goods price index is given by

$$p_{i,t} = \left((1-\omega_i) + \omega_i p_{im,t}^{1-\eta_i}\right)^{\frac{1}{1-\eta_i}}$$

Labor decisions are decided by a central authority within the household, a union, which supplies labor monopolistically to a continuum of labor markets of measure 1 indexed by $j \in [0,1]$. In each labor market j, the union faces a demand for labor given by $(W_t(j)/W_t)^{-\tilde{\eta}} h_t^d$, where $\tilde{\eta} > 1$ represents the elasticity of demand for each type of labor. Here $W_t(j)$ denotes the nominal wage charged by the union in labor market j at time t, W_t is an index of nominal wages prevailing in the economy, and h_t^d is a measure of aggregate labor demand by firms. In each particular labor market, the union takes W_t and h_t^d as exogenous. The supply of labor is

$$h_t(j) = \left(\frac{W_t(j)}{W_t}\right)^{-\tilde{\eta}} h_t^d \tag{3.5}$$

The total number of hours allocated to the different labor markets must satisfy the resource constraint $h_t = \int_0^1 h_t(j)dj$. Combining this restriction with equation (3.5), we obtain

$$h_t = h_t^d \int_0^1 \left(\frac{W_t(j)}{W_t}\right)^{-\tilde{\eta}} dj$$

The household holds their financial assets in the form of cash, V_t, bank deposits, $M_t - V_t$, a domestic complete set of nominal state-contingent assets and foreign bonds, B_{t+1}^*. She pays lump-sum taxes, τ_t, and receives nominal lump-sum transfers from the government in the amount N_t per period. The household's period-by-period budget constraint is

$$\begin{aligned}
&e_t B^*_{t+1} + P_{c,t}C_t + \Upsilon_t^{-1} P_{i,t} I_t + M_{t+1} + \tau_t \\
&= \Omega(A_{t-1}, \tilde{\phi}_{t-1}) R^*_{t-1} e_t B^*_t + R_{t-1}(M_t - V_t) + V_t + N_t + D_t \\
&+ \left[R^k_t u_t - \Upsilon_t^{-1} P_{i,t} a(u_t)\right] K_t + \Phi_t + h^d_t \int_0^1 W_t(j) \left(\frac{W_t(j)}{W_t}\right)^{-\tilde{\eta}} dj
\end{aligned}$$

where Φ_t denotes dividends received, and D_t the household's net cash income from participating in state contingent securities at time t. R_{t-1} and R^*_{t-1} are respectively gross nominal domestic and foreign interest rates. $\ln(\Omega(A_t, \phi_t)) = \exp(-\phi_a(A_t - \bar{A}) + \phi_t)$ is a risk premium[17] on foreign bond holdings, $A_t \equiv \frac{e_t B^*_{t+1}}{P_t}$ represents the real aggregate net foreign asset position of the domestic economy, and ϕ_t a time varying shock to the risk premium. The risk premium function satisfies $\Omega_1 < 0$ and $\Omega(0,0) = 1$.

The household chooses processes for C_t, h_t, B^*_{t+1}, V_t, $W_t(j)$, K_{t+1}, I_t, u_t, and M_{t+1} as to maximize the utility function subject to constraints. The Lagrangian associated with the household's optimization problem is

$$h = E_t \sum_{t=0}^{\infty} \beta^t \left\{ \begin{array}{c}
\zeta_{c,t} \ln(C_t - bC_{t-1}) + A_v \frac{\left(\frac{V_t}{z_t^* P_t}\right)^{1-\phi_v}}{1-\phi_v} - A_L \frac{h_t^{1+\phi_L}}{1+\phi_L} \\
+\Lambda_t \left[\begin{array}{c}
\Omega(A_{t-1}, \tilde{\phi}_{t-1}) R^*_{t-1} e_t B^*_t + R_{t-1}(M^h_t - V_t) + V_t \\
+N_t + D_t + h^d_t \int_0^1 W_t(j) \left(\frac{W_t(j)}{W_t}\right)^{-\tilde{\eta}} dj \\
+ \left[R^k_t u_t - \Upsilon_t^{-1} P_{i,t} a(u_t)\right] K_t + \Phi_t \\
-C_t P_{c,t} - \tau_t - \Upsilon_t^{-1} P_{i,t} I_t - M^h_{t+1} - e_t B^*_{t+1}
\end{array} \right] \\
+ \frac{\Lambda_t W_t}{\tilde{\mu}_t} \left[h_t - h^d_t \int_0^1 \left(\frac{W_t(j)}{W_t}\right)^{-\tilde{\eta}} dj \right] \\
+\Lambda_t Q_t \left[(1-\delta) K_t + I_t \left[1 - S\left(\frac{I_t}{I_{t-1}}\right)\right] - K_{t+1} \right]
\end{array} \right\}$$

[17]The introduction of the risk-premium is needed in order to ensure a unique steady state in the model. Without it, the time paths of domestic consumption and wealth follow random walks, see Schimitt-Grohé and Uribe (2003) for further details. The functional form of the risk premium is similar to the one used by Ambler et al. (2004).

where Λ_t is the Lagrangian multiplier relative to the budget constraint, $\frac{\Lambda_t W_t}{\tilde{\mu}_t}$ the Lagrangian multiplier relative to the labour resource constraint and $\Lambda_t Q_t$ to the law of motion of capital.

The first-order conditions with respect to C_t, M_{t+1}, V_t, B^*_{t+1}, h_t, K_{t+1}, I_t, and u_t are given by[18]

$$\frac{\zeta_{c,t}}{c_t - \frac{bc_{t-1}}{\mu_{z^*,t}}} - b\beta E_t \frac{\zeta_{c,t+1}}{\mu_{z^*,t+1}c_{t+1} - bc_t} = \lambda_t p_{c,t} \tag{3.6}$$

$$\lambda_t = \beta E_t \left[\frac{\lambda_{t+1}\mu_{\lambda,t+1}}{\pi_{t+1}} R_t \right] \tag{3.7}$$

$$A_v v_t^{-\phi_v} = \lambda_t (R_{t-1} - 1) \tag{3.8}$$

$$e_t \lambda_t = \beta E_t \frac{\mu_{\lambda,t+1}\lambda_{t+1}}{\pi_{t+1}} e_{t+1} R^*_t \Omega_t \tag{3.9}$$

$$A_L h_t^{\phi_L} = \frac{\lambda_t w_t}{\tilde{\mu}_t} \tag{3.10}$$

$$\lambda_t q_t = \beta E_t \frac{\mu_{\lambda,t+1}}{\mu_{\Upsilon,t+1}} \lambda_{t+1} \left\{ \left[r^k_{t+1} u_{t+1} - p_{i,t+1} a(u_{t+1}) \right] + q_{t+1}(1-\delta) \right\} \tag{3.11}$$

[18]The following variables are stationarized as

$$c_t \equiv \frac{C_t}{z^*_t}, m_{t+1} \equiv \frac{M_{t+1}}{P_t z^*_t}, v_t \equiv \frac{V_t}{P_t z^*_t}, i_t \equiv \frac{I_t}{z_{t-1}\Upsilon_{t-1}^{\frac{1}{1-\theta}}}, q_t \equiv \Upsilon_t Q_t, b^*_{t+1} \equiv \frac{B^*_{t+1}}{P^*_t z^*_t}$$

$$p_{i,t}\lambda_t = \lambda_t q_t \left[1 - \frac{\kappa}{2}\left(\frac{i_t\mu_{I,t}}{i_{t-1}} - \mu_I\right)^2 - \frac{i_t\mu_{I,t}}{i_{t-1}}\kappa\left(\frac{i_t\mu_{I,t}}{i_{t-1}} - \mu_I\right)\right] + \beta E_t \frac{\mu_{\lambda,t+1}}{\mu_{\Upsilon,t+1}}\lambda_{t+1}q_{t+1}\left(\frac{i_{t+1}\mu_{I,t+1}}{i_t}\right)^2 \kappa\left(\frac{i_{t+1}\mu_{I,t+1}}{i_t} - \mu_I\right) \tag{3.12}$$

$$r_t^k = p_{i,t}\left[\gamma_1 + \gamma_2(u_t - 1)\right] \tag{3.13}$$

By combining the household's first-order condition for M_{t+1} (3.7) and for B^*_{t+1} (3.9), we obtain the following uncovered interest rate parity (UIP) condition :

$$\frac{R_t}{\Omega_t R_t^*} = E_t \frac{s_{t+1}}{s_t}\frac{\pi_{t+1}}{\pi^*_{t+1}} \tag{3.14}$$

3.2.3 Wage decisions

Wage stickiness is introduced in the model by assuming that each period the household cannot set the nominal wage optimally in a fraction $\tilde{\alpha} \in [0, 1]$ of randomly chosen labor markets. In these markets, the wage rate is indexed to the previous period's consumer-price inflation $\pi_{c,t} \equiv P_{c,t}/P_{c,t-1}$, which can be expressed in terms of final goods price as $\pi_{c,t} \equiv \frac{p_{c,t}}{p_{c,t-1}}\pi_t$ according to the rule $W_t(j) = W_{t-1}(j)(\mu_{z^*}\pi_{c,t-1})^{\tilde{\chi}}$, where $\tilde{\chi} \in [0, 1]$ is a parameter measuring the degree of wage indexation, and μ_{z^*} the long-term growth rate of wage. If $\tilde{\chi} = 1$, which means a complete wage indexation, the presence of μ_{z^*} implies that there are no distortions from wage dispersion along the steady state growth path; for $\tilde{\chi} \in [0, 1)$, wage dispersion entails resource costs in equilibrium.

In any labor market j where the wage is set optimally in period t, the real wage in that period is $\tilde{w}_t$; if in period $t+1$ wages are not reoptimized in that market, the real wage is $\tilde{w}_t(\mu_{z^*}\tilde{\pi}_{c,t})^{\tilde{\chi}}/\pi_{t+1}$. In general, s period after the last reoptimization, the real wage is $\tilde{w}_t\Pi_{k=1}^{s}\frac{(\mu_{z^*}\tilde{\pi}_{c,t+k-1})^{\tilde{\chi}}}{\pi_{t+k}}$. The first-order condition with respect to $\tilde{w}_t$ is expressed recursively as:

$$f_t^1 = \frac{\tilde{\eta}-1}{\tilde{\eta}}\tilde{w}_t\lambda_t\left(\frac{w_t}{\tilde{w}_t}\right)^{\tilde{\eta}} h_t^d + \tilde{\alpha}\beta E_t\left(\frac{\pi_{t+1}}{(\mu_{z^*}\pi_{c,t})^{\tilde{\chi}}}\right)^{\tilde{\eta}-1}\left(\frac{\mu_{z^*,t+1}\tilde{w}_{t+1}}{\tilde{w}_t}\right)^{\tilde{\eta}-1} f_{t+1}^1$$

$$f_t^2 = A_L h_t^{\phi_L}\left(\frac{w_t}{\tilde{w}_t}\right)^{\tilde{\eta}} h_t^d + \tilde{\alpha}\beta E_t\left(\frac{\pi_{t+1}}{(\mu_{z^*}\pi_{c,t})^{\tilde{\chi}}}\right)^{\tilde{\eta}}\left(\frac{\mu_{z^*,t+1}\tilde{w}_{t+1}}{\tilde{w}_t}\right)^{\tilde{\eta}} f_{t+1}^2$$

and

$$f_t^1 = f_t^2$$

where f_t^1 and f_t^2 are auxiliary variables and $A_L h_t^{\phi_L}$ is the marginal disutility of labor. As for intermediate goods producers, the union sets wages so as to satisfy that average future expected marginal revenues are equal to average future expected marginal costs.

3.2.4 Fiscal and monetary policy

Each period, the government consumes G_t units of the composite good. G_t is stationarized as $g_t \equiv G_t/z_t^*$, which follows an exogenous stochastic process

$$\ln(g_t/g) = \rho_g \ln(g_{t-1}/g) + \epsilon_{g,t}$$

The period-by-period budget constraint of the government is given by

$$g_t + n_t = R_{t-1}(\mu_{z^*,t+1}\pi_{t+1}m_{t+1} - m_t) + \tau_t$$

Monetary policy is approximated with the following rule

$$\log(R_t/R) = \rho_r \log(R_{t-1}/R) + (1-\rho_r)\left[b_\pi \log(\pi_{c,t}/\pi_c) + b_y \log(y_t/y)\right] + \varepsilon_{R,t}$$

where ρ_r is a nominal interest rate smoothing parameter. The central bank adjusts the nominal interest in response to deviations of inflation and output from the respective targets.

3.2.5 Market clearing conditions

There are four markets in this open economy: (*i*) final goods market; (*ii*) labour market; (*iii*) loan market; (*iv*) foreign bond market. In equilibrium all these markets must clear. Because of the price dispersion in the equilibrium, the final goods market clears when the demand from the households, the government and the foreign market scaled with the degree of price dispersion equals to the production of the final good firm. Similarly, because of the wage dispersion at the equilibrium, the labour market clears when the aggregate demand of labour scaled with the degree of wage dispersion meets the supply of labour. The loan market is in equilibrium when the demand for liquidity from firms equals the supply of liquidity both from the households and the central bank. The foreign bond market clears when the trade balance is met.

Final goods market

At the firm level, the supply must equal the demand. Let Y_t denote the demand, then

$$Y_t = C_{d,t} + C_{x,t} + G_t + \Upsilon_t^{-1}\left(I_{d,t} + I_{x,t} + a(u_t)K_t\right)$$

$$K_t(i)^{\theta}\left(z_t h_t(i)\right)^{1-\theta} - \psi z_t^* = Y_t\left(\frac{P_t(i)}{P_t}\right)^{-\eta}$$

Integrating over all firms and taking into account the facts that the capital-labor ratio is common across firms, the aggregate demand for the composite labor input and the aggregate effective level of capital. Let $d_t \equiv \int_0^1 \left(\frac{P_t(i)}{P_t}\right)^{-\eta} di$. Then the resource constraint in the present model is given by the following stationary expressions

$$y_t = (1-\omega_c)p_{c,t}^{\eta_c}c_t + (1-\omega_i)p_{i,t}^{\eta_i}i_t + \frac{1}{\mu_{I,t}}a(u_t)k_t + g_t + \omega_f s_t^{\eta_f}y_t^*$$

$$\left(\frac{u_t k_t}{\mu_{I,t}}\right)^{\theta}(h_t^d)^{1-\theta} - \psi = y_t d_t \text{ and } d_t = (1-\alpha)\tilde{p}_t^{-\eta} + \alpha\pi_t^{\eta}d_{t-1}$$

where d_t measures the degree of price dispersion and $d_t \geq 1$. When prices are fully flexible, $\alpha = 0$, we have $\tilde{p}_t = 1$ and $d_t = 1$.

From the price index, we have

$$1 = \alpha \pi_t^{\eta - 1} + (1-\alpha)\tilde{p}_t^{1-\eta}$$

which implies that in the steady state: *i)* if the price is flexible, then $\tilde{p} = 1$ and $\pi = 1$; *ii)* if there is relative price movements, then π can be greater than unity.

Labor market

Let $\tilde{d}_t \equiv \int_0^1 \left(\frac{\tilde{W}_t}{W_t}\right)^{-\tilde{\eta}} dj$, which measures the degree of wage dispersion across different type of labor. Thus

$$h_t = \tilde{d}_t h_t^d \text{ and } \tilde{d}_t = (1-\tilde{\alpha})\left(\frac{\tilde{w}_t}{w_t}\right)^{-\tilde{\eta}} + \tilde{\alpha}\left(\frac{w_{t-1}}{w_t}\right)^{-\tilde{\eta}}\left(\frac{\pi_t}{(\mu_{z^*}\pi_{c,t-1})^{\tilde{\chi}}}\right)^{\tilde{\eta}} \tilde{d}_{t-1}$$

From the wage index

$$w_t = \left[(1-\tilde{\alpha})\tilde{w}_t^{1-\tilde{\eta}} + \tilde{\alpha}\left(\frac{w_{t-1}}{\mu_{z^*,t}}\right)^{1-\tilde{\eta}}\left(\frac{(\mu_{z^*}\pi_{c,t-1})^{\tilde{\chi}}}{\pi_t}\right)^{1-\tilde{\eta}}\right]^{\frac{1}{1-\tilde{\eta}}}$$

which implies that in the steady state: *i)* if wage is fully indexed to lagged inflation (*i.e.* $\tilde{\chi} = 1$), then $w = \tilde{w}$ and there is no relative wage distortion; *ii)* if wage is partially indexed to lagged inflation (*i.e.* $\tilde{\chi} \in (0,1)$), then $w \neq \tilde{w}$ and there is relative wage distortion.

Loan market, trade balance and foreign economy

In equilibrium, the money market clears by the following condition

$$vW_t h_t^d = \mu_t M_t - V_t$$

where μ_t is the growth rate of money. The stationarized form is

$$\nu w_t h_t^d = \frac{\mu_t m_t}{\pi_t \mu_{z^*,t}} - v_t$$

In equilibrium, the evolution of net foreign assets level satisfies

$$e_t B_{t+1}^* = P_t(C_{x,t} + \Upsilon_t^{-1} I_{x,t}) - e_t P_t^*(C_{m,t} + \Upsilon_t^{-1} I_{m,t}) + \Omega_{t-1} R_{t-1}^* e_t B_t^*$$

or equivalently, in its stationarized form,

$$b_{t+1}^* = \omega_f s_t^{\eta_f - 1} y_t^* - \omega_c \left(\frac{p_{cm,t}}{p_{c,t}}\right)^{-\eta_c} c_t - \omega_i \left(\frac{p_{im,t}}{p_{i,t}}\right)^{-\eta_i} i_t + \Omega_{t-1} R_{t-1}^* \frac{b_t^*}{\pi_t^* \mu_{z^*,t}}$$

The foreign output, inflation and interest rate are exogenously given as

$$\ln(y_t^*/y^*) = \rho_{y^*} \ln(y_{t-1}^*/y^*) + \varepsilon_{y^*,t}$$

$$\ln(\pi_t^*/\pi^*) = \rho_{\pi^*} \ln(\pi_{t-1}^*/\pi^*) + \varepsilon_{\pi^*,t}$$

$$\ln(R_t^*/R^*) = \rho_{R^*} \ln(R_{t-1}^*/R^*) + \varepsilon_{R^*,t}$$

where $\varepsilon_{i,t}$, $i \in \{y^*, \pi^*, R^*\}$, is $i.i.d$ innovation with mean zero, standard deviation σ_i.

3.3 Estimation

The model is estimated using Bayesian estimation techniques. Bayesian estimation starts from forming a prior distribution over the parameter to estimate. This prior distribution can be interpreted as a set of information prior to observing the data used in the estimation. The precision of this set of information is defined by the inverse of the variance of the prior distribution. The prior is then updated by using the observed data via Bayes theorem to the posterior distribution, from which the inference is based on the actual occurring data. This section describes the data, measurement equations, calibrated parameters and prior distributions.

3.3.1 Data and measurement equations

The model is estimated on quarterly data from Canada for the small open economy and U.S. data to approximate the rest of the world. All data range from 1981Q3 to 2006Q4. The set of observed variables contains eleven variables: the real GDP, consumption, investment, wage, the GDP deflator, the consumption deflator, the investment deflator, the nominal short-run interest rate, foreign output, foreign inflation and the foreign interest rate[19]. The real variables are stationarized by taking the first difference. The vector of observed variables is given by

$$Y_{obs} = \begin{bmatrix} \Delta\ln(Y_t) \;\; \Delta\ln(C_t) \;\; \Delta\ln(I_t) \;\; \Delta\ln(W_t/P_t) \;\; \pi_t^{obs}\ldots \\ \pi_{c,t}^{obs} \;\; \pi_{i,t}^{obs} \;\; R_t^{obs} \;\; \Delta\ln(Y_t^*) \;\; \pi_t^{*,obs} \;\; R_t^{*,obs} \end{bmatrix}$$

To estimate a DSGE model, the posterior distribution of the estimated parameters is obtained in three steps. First, the likelihood function is computed by using the Kalman filter to the model solution and the observation equations[20]. The likelihood function combined with the prior distribution gives the posterior kernel, which is in logarithm term the sum of the likelihood and the prior. Second, the posterior mode is obtained by maximizing the posterior kernel with respect to the estimated parameters. This is computed by a modified version of the numerical optimization routine--Christopher Sims' optimizer csminwel. Finally, the posterior distribution is simulated by the Metropolis-Hastings algorithm.

The measurement equations, relating the model variables to the observed vari-

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[19]See Appendix C for more details about the data and data source.

[20]See details in Appendix C

ables, are given by

$$
\begin{aligned}
\Delta \ln(Y_t) &= \mu_{z^*,t}\frac{y_t}{y_{t-1}},\ \Delta \ln(C_t) = \mu_{z^*,t}\frac{c_t}{c_{t-1}} \\
\Delta \ln(I_t) &= \mu_{I,t}\frac{i_t}{i_{t-1}},\ \Delta \ln(W_t/P_t) = \mu_{z^*,t}\frac{w_t}{w_{t-1}} \\
\pi_t^{obs} &= \pi_t,\ \pi_{c,t}^{obs} = \pi_{c,t} \\
\pi_{i,t}^{obs} &= \pi_{i,t},\ R_t^{obs} = R_t \\
\Delta \ln(Y_t^*) &= \mu_{z^*,t}\frac{y_t^*}{y_{t-1}^*},\ \pi_t^{*,obs} = \pi_t^*,\ R_t^{*,obs} = R_t^*
\end{aligned}
$$

For real variables, both observed variables and model variables are expressed in terms of their growth rates. Since the model variables are stationarized, their growth rates are scaled with the growth rates of technology shocks. Nominal model variables are taken directly into their corresponding observed variables.

3.3.2 Calibrated parameters

A number of parameters are calibrated throughout the estimation procedure. Most of them are fixed to meet the steady state values of the observed variables, which are the sample mean of these. The ratio of V/M is equal to the average ratio of $M1/M2$ of our sample 0.18. The constant in cash utility function A_v and the curvature parameter related to money demand ϕ_v are set to ensure the cash to money ratio. The fraction of the firms' wage financed in advance, ν, is calculated in the steady state as 0.51. The ratio of G/Y is equal to the ratio of government expenditure over the GPD of our sample, 0.24. The growth rate of money, μ, is equal to the average growth rate of $M2$ of the sample, 1.015.

To match the sample mean of the investment-output 0.24 and labour income-output rate 0.68, the depreciation rate δ is set to 0.025 and the share of capital in production θ to 0.32. The constant in the labour disutility function A_L is set to 12.96 to ensure that the agents devote around 30 percent of their time to work in steady state. In line with standard calibrations used in literature, the labour supply elasticity

ϕ_L is set to 1, and the markup power in the wage setting is set to 1.05, which implies the elasticity of differentiated labour, $\tilde{\eta} = 21$; the markup power in the domestic price setting is set to 1.2, implying that the intratemporal elasticity of substitution between domestic goods, $\eta = 6$; the markup power in the importing price setting is also set to 1.2.

The parameter μ_Υ is set to be 1.0022, which corresponds to the inverse of the average growth rate of the price of investment relative to the GDP deflator. The steady state growth of real per capita GDP, μ_{z^*}, is equal to the average growth rate of per capita GDP in our sample 1.0029. From its definition,

$$\mu_{z^*} = \mu_z \left(\mu_\Upsilon\right)^{\frac{\theta}{1-\theta}}$$

we can calculate the steady state growth rate of neutral technological shock $\mu_z = 1.0009$.

The discount factor β is set to $1.03^{-1/4}$, which implies a steady state annualized real interest rate of 4 percent. This value corresponds to the sample mean of annualized real interest rate. The shares of imports[21] ω_c and ω_i are calibrated to match the sample average of the import-output ratio 0.32 and $\omega_i = 0.56$. This implies that $\omega_c = 0.40$. The imported investment is approximated by the sum of imported machinery and equipment and imported automotive products. Since the capacity utilization is not included as an observable variable, following Altig et al. (2005), the ratio of the capacity utilization parameters are set so that $\frac{\gamma_2}{\gamma_1} = 1.46$ which implies in the steady state $\gamma_1 = 0.04$ and $\gamma_2 = 0.06$.

[21] In the Canadian data, there is a clear upward trend in the import and export shares. In terms of the model these strong increases in import and export shares imply that import and export grow more rapidly than output. Since import and export are not included as observable variables, this is not an issue for the estimation. However, if these two variables are included as observable variables, one should remove the excessive trend of import and export in the data or render the model to take this fact into account.

3.3.3 Prior distributions of the estimated parameters

Prior distributions of the non-calibrated parameters describe the available information before using the data to estimate these parameters. This information can be obtained from presample analysis and earlier studies at both the macro and micro level. The marginal prior distributions for the 34 parameters are summarized in Table 3.3. For all the parameters bounded between 0 and 1 the beta distribution is employed. This applies to the nominal stickiness parameters α, the wage indexation parameter $\tilde{\chi}$, the habit persistence b, and the persistence parameters of the shock processes, the interest rate smoothing parameter and of the foreign variables ρ. The domestic price stickiness parameter, α, is set so that the average length between price adjustments is 2.86 quarters. The import and export price stickiness parameters are set so that the average length between price adjustments is 2 quarters. The wage stickiness parameter is set higher so that the average length between wage adjustments is one year. The wage indexation parameter is set to 0.2. The prior means of most of autoregressive coefficients are set to 0.8 The rest of them are set to 0.85.

For parameters assumed to be positive, such as the standard deviations of the shocks, σ, the inverse gamma distribution is used. The data is let quite freely to determine the sizes of the shocks and the prior mean on the risk premium parameter by setting the prior standard deviations to 0.1, which gives the degrees of freedom[22] larger than 2. For the risk premium parameter, ϕ_b, and the substitution elasticity parameters, η, the gamma distribution[23] is employed.

For response parameters in the monetary policy we use normal distribution. The prior mean on the inflation coefficient, b_π, is set to 1.6, with a standard deviation of 0.1, and the output reaction to 0.1, with a standard deviation of 0.05. The interest rate

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[22]For the density of the inverse gamma distribution $f(x) \propto x^{-\alpha-1}e^{-\frac{1}{\beta x}}$, the shape and scale parameters are defined as: $\alpha = 2 + \frac{\mu^2}{\sigma^2}$, $\beta = \frac{1}{\mu\left(1+\frac{\mu^2}{\sigma^2}\right)}$, where μ and σ are mean and standard deviation.

[23]For a gamma distribution, $f(x) \propto x^{\alpha-1}e^{-\frac{x}{\beta}}$, and the shape and scale parameters are defined as: $\alpha = \frac{\mu^2}{\sigma^2}$, $\beta = \frac{\sigma^2}{\mu}$.

smoothing parameter is set to 0.45, with a standard deviation of 0.1.

3.4 Properties of the Estimated Model

3.4.1 Posterior distributions of the estimated parameters

The results of the posterior distribution are reported in Table 3.4. It shows the posterior mode with the approximate posterior standard deviation obtained by maximizing the posterior kernel. The mean along with the 5th and 95th percentiles of the posterior distribution are reported. For comparison, the prior mean is also reported. The same results are showed in Figure 3.1-3.4 that report the prior and posterior distributions.

From the results in Table 3.4, the estimate of $\tilde{\alpha}$ implies that the wage contracts are re-optimized, on average, once every 3.3 quarters. The estimated sticky domestic price parameter implies that intermediate firms re-optimize prices every 6.1 quarters. For the importing sectors, the sticky price parameter is much lower with respectively 1.2 quarters stickiness for consumption goods and 1.7 quarters for investment goods. On average, firms re-optimize their prices every 3 quarters. Ambler et al. (2004) estimate a small open economy model with less observed variables using maximum likelihood techniques and find that the wage contracts are fixed on average for 5.7 quarters, domestic prices for 1.79 quarters, and importing goods prices for 2.23 quarters. As far as there is no empirical evidence about the average duration of wage contracts, we are not able to tell which result is closer to the data.

The elasticity of substitution between foreign and domestic consumption goods, η_c, is close to η_i and η_f, around 1.48, which is favorable to the value of 1.5 used in the international real business cycle literature. The estimate of the wage indexation parameter, $\tilde{\chi}$, is quite low, of 10 percent, which is close to the average proportion of wage settlements with cost-of-living clauses of about 15 percent in Canada during the sample period. The estimated habit persistence parameter, b, is 0.78, given a prior of

0.5. This value is higher than the estimate of 0.65, reported in Altig et al. (2005), and the value of 0.7 reported in Boldrin, Christiano, and Fisher (2001). The estimated parameters of the monetary rule[24] imply a low interest rate smoothing of 0.24, a weak reaction to output fluctuations with 0.13, and a strong response to inflation deviation with 1.32. Clarida, Gali and Gertler (1999) find these values using U.S. post-1979 data are the following: 0.80 of interest rate smoothing, 0.30 of response to inflation deviation and 0.02 to output deviation. This implies that the interest rate in Canada is less smoothed than in the United States, which is consistent with cross country evidence reported in Drew and Plantier (2000).

The persistence parameters in the unit-root neutral and the investment specific technology process are estimated respectively to be 0.94 and 0.21. These values are close to the estimates of 0.9 and 0.24 reported by Altig et al. (2005) despite the fact that the U.S. data are used in their estimation. These estimates imply that shocks to neutral technology exhibit a high degree of serial correlation, while shocks to investment specific technology do not. The standard deviations are estimated to be, in percentage, 0.31 and 1.84, which imply that a one-standard deviation neutral technology shock drives z_t up by 0.31 percent in the period of the shock and by 5.17 percent in the long run. A one-standard deviation investment specific technology shock drives Υ_t up by 1.84 percent in the period of the shock and by 2.32 percent in the long run. The unconditional standard deviation of the growth rate of neutral technology is roughly 0.88 percent, and that of investment specific technology is about 1.88. Altig et al. (2005) find the growth rate of neutral and investment specific technology have respectively the standard deviation of 0.16 and 0.31 percent. Thus these two shocks are substantially more volatile than their estimates.

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[24] I also estimate a version of the model with a monetary rule from which the central bank reacts to the real exchange rate with the prior mean of zero and standard deviation of 0.1. The estimated coefficient measuring the response to the real exchange rate is quite weak with a value of 0.02, associated with a log marginal likelihood of 3881. Comparing to the model without the reaction to the real exchange rate, the model fit is dropped.

3.4.2 Model fit

The conformity of the model and the data is assessed in three dimensions. First, the Kalman filtered one-sided estimates of the observed variables are compared with the actual variables in Figure 3.5. For the real variables, we report their growth rates. As the figure shows, the in-sample fit of the model is satisfactory. In Figure 3.6, the estimates of the unobserved shocks are reported. All the shock series are centered around zero.

Second, the relative model comparison when the wage indexation parameter is calibrated to unity and some of the nominal and real frictions are turned off favors the bench mark model. With full indexation, $\tilde{\chi} = 1$, this implies that in equilibrium the optimal wage $\tilde{w}_t = w_t$. However, if one wants to estimate the wage indexation parameter, $\tilde{\chi}$, with a prior different from unity, there are distortions induced by wage dispersion in equilibrium. Table 3.5 shows the posterior mode and marginal likelihood when there is: *i)* full wage indexation; *ii)* no domestic price stickiness; *iii)* no wage stickiness; *iv)* no variable capital utilization; *v)* no investment adjustment cost; *vi)* no working capital channel; *vii)* the law of one price in imported consumption goods; *viii)* the law of one price in imported investment goods; *ix)* no habit formation.

From the estimates of a version of the model in which the wage indexation parameter is calibrated to unity, the log marginal likelihood[25] drops to 3215 from 3388 of the bench mark model, and the posterior odds ratio is largely in favor of the bench model with estimated wage indexation parameter. The results show that all the nominal and real frictions increase the fit of the model, in particular wage stickiness, price stickiness in the investment goods import sector, habit formation and investment adjustment costs

[25] The marginal likelihood of a model i is defined as $p(\tilde{Y}_T|\theta_i) = \int L_i(\theta_i; \tilde{Y}_T)p(\theta_i)d\theta_i$, where $L(\theta_i; \tilde{Y}_T)$ is the likelihood function of the model's parameter vector θ_i conditional on the set of observed data $\tilde{Y}_T$ and $p(\theta_i)$ is the prior distribution of the model's parameters. $p(\tilde{Y}_T|\theta_i)$ is relative measure of model fit and should be compared across competing models. This can be done through the ratio of the posterior model distributions, which is also called as the posterior odds ratio, defined as for two models i and j $\frac{p(\theta_i|\tilde{Y}_T)}{p(\theta_j|\tilde{Y}_T)} = \frac{p(\theta_i)p(\tilde{Y}_T|\theta_i)}{p(\theta_j)p(\tilde{Y}_T|\theta_j)}$ or by the Bayes factor defined as $\frac{p(\tilde{Y}_T|\theta_i)}{p(\tilde{Y}_T|\theta_j)}$, which is also the ratio of the posterior odds ratio to the prior odds ratio.

are important. It should be noticed that the bench mark model is preferable to a version without variable capital utilization; the posterior odd ratio is 2181 in favor of the model with variable capital utilization. Between the bench mark model and the model without working capital channel, the posterior odd ratio of 6.7 in favor of the model with working capital channel. Although the price stickiness parameter related to the import consumption goods is not particularly high, it still appears to be of importance of the model's performance with a posterior odd ratio of 2567 in favor of the model with price stickiness in imported consumption goods.

Finally, the unconditional moments are reported in Table 3.6 and Table 3.7 between the data and the different versions of the model. From the results of the benchmark model, the standard deviations for the series are fairly well captured by the model, but it is hard for the model to reproduce the first-order autocorrelation coefficient for the growth rate of consumption. In the data, the first-order autocorrelation coefficient for the growth rate of consumption is negative, while the model predicts a positive coefficient. However, this coefficient is sensitive to the time convention as illustrated by Campbell (1996). For other variables, in general, the first-order autocorrelation coefficients predicted by the model are higher than those in the data with the exception of the nominal interest rate whose coefficient matches perfectly to the data. In particular, the persistence for the domestic inflation remains with other versions of the model. The reason is that the nominal and real frictions are shut down only one by one. Turning to the correlations with output, the model does well for real variables, less successfully for nominal interest rate and inflation that are negatively correlated to output in the data, but positively correlated in the model. As Kiley (2002) points out that price setting in the fashion of Calvo needs a supply shock with a large variance to produce the negative correlation between output growth and inflation. To sum up, although there seems to be room for improvements in some aspects in the model, it is still fair to say that the model does a fairly good job in replicating conventional statistics.

3.4.3 Variance decompositions and impulse response functions

Table 3.8 reports the variance decomposition of the mean, 5th and 95th percentiles of the posterior distribution at business frequencies. The real variables such as output, consumption, investment, imports, exports and real wage are expressed in growth rate. Since the contribution of risk premium shock tends to be zero, its contribution is not reported.

The technology shocks account for about 76 percent of the output growth. Other shocks such as government purchases, monetary policy and consumption preference account for about 15 percent. This result supports the idea of predominant part of business cycles being due to technology shocks. For consumption growth, the neutral technology shock accounts for 62 percent, while the consumption preference for 21 percent. The technology shocks account for about 65 percent of the investment growth, while consumption preference shock accounts for 17 percent. The technology shocks account for 66 percent for exports growth. Foreign output accounts for 30 percent of the export growth, which depends on the foreign demand. The import growth is largely due to the neutral technology and consumption shocks. The real wage growth depends only on the technology shocks, which account for 100 percent. The technology shocks are also central sources of hours fluctuations, accounting for 64 percent of its volatility, while the monetary policy and consumption preference shocks account for 15 percent. For inflation, in the long run, the technology shocks and consumption preference shock are central sources of inflation volatility, accounting for about 90 percent. The same conclusion for consumption and investment deflator inflation. The technology shocks account for 69 percent for the volatility of interest rate, while monetary policy accounts for 11 percent and the foreign interest rate for 11 percent.

Figures 3.7-3.12 report the impulse functions of mean, 5th, and 95th percentiles for each estimated shock. The inflation and nominal interest rates are reported as annualized quarterly rates in levels. Output, real wages, investment, consumption, exports and imports are expressed in cumulative growth rates in percent. Hours and real exchange rate are expressed in level deviation from their respective steady-state values.

In Figure 3.7, after a positive neutral technology shock, output, consumption, investment, real wage, exports, imports and hours rise, supporting the results reported in Adlfson et al. (2007) for the European economy. Inflation, interest rate and real exchange rate follow the same pattern: rise after the shock, then decline. The rise of the real exchange rate means that a favorable neutral technology shock leads to a real exchange rate depreciation, which is consistent with the finding of Basu, Fernald and Kimball (2006) and Backus, Kehoe and Kydland (1994).

Turning to the effects of an investment specific technology shock in Figure 3.8, this shock has strong cyclical effects on output, consumption, investment, real wage, exports, imports and hours. In addition, inflation and interest rate rise after the shock, followed by an ongoing decline, supporting the results reported in Altig et al (2005) for the U.S. economy. As for the neutral technology shock, a favorable investment specific technology shock leads to a real exchange rate depreciation.

The dynamic effects of a monetary policy shock are depicted in Figure 3.9. Following an unanticipated temporary increase in the nominal interest rate, the model generates a very persistent negative response in output, consumption, investment and hours, while inflation is somewhat less inertial than the typical estimates in the VARs literature. This shock induces a sharp increase in the interest rate which then returns to its pre-shock level within 2 quarters. The real wage remains unaffected by the monetary policy shock. It should be noticed from the figure that the responses are not hump-shaped for real variables, which is not consistent with the VAR literature. The reason is that these variables are expressed in cumulative growth rates. Expressed in level, the model generates the hump-shaped responses for variables such as output, consumption, investment, imports and hours with the exception of exports. Finally the real exchange rate appreciate immediately on impact and then returns to zero from below, which is not consistent with a hump-shaped response of estimated VARs. The behavior of the real exchange rate in this model is implied by the uncovered interest parity condition: an initial appreciation is followed by a strong depreciation.

Figure 3.10 presents the dynamic responses to a government purchases shock. Following an increase of government purchases, output and hours worked rise, while consumption, investment, exports and imports decline. Given that in the model the share of government expenditures in GDP is 24 percent, the government multiplier implied by the model is higher than unity. Consistent with the finding of Edelberg, Eichenbaum and Fisher (1999) for the U.S. economy, the interest rate decreases on impact and then increases. Finally, this shock leads to a real exchange rate appreciation on impact, and the real wage is unaffected.

From Figure 3.11, the consumption preference shock accounts for a substantial amount of the cyclical behavior of consumption. Investment drops and households increase slowly their labour effort in order to finance the increase in consumption. The high value of habit persistence parameter b helps to generate the hump-shaped consumption response. The decline of imports implies that the household favors domestic produced consumption goods.

Finally, turning to the responses to a positive risk premium shock in Figure 3.12, this shock depreciates the real exchange rate, which makes imported consumption and investment goods more expensive. This forces in turn the total imports to decrease. The exchange rate depreciation makes the relative prices between the imported goods and the domestic goods increase, which affects the consumption and investment deflator inflation. But this effect is limited by the value of substitution elasticity around 1.46. Therefore, the consumption and investment deflator inflation increase after the shock.

3.5 Conclusion

This chapter explores the empirical performance of a small open economy model in which the wage indexation parameter is estimated rather than calibrated to unity. With this specification, resource costs induced by price and wage dispersions are considered. The estimate of this wage indexation parameter is quite low of around 10 percent, which is close to the average proportion of wage settlements with cost-of-living clauses in Canada during the sample period. Abstaining from using full indexation to generate inflation inertia, between a version of the model in which the wage indexation is calibrated to unity and a version of the model where this parameter is estimated, the posterior odds ratio is largely in favor of the latter. An extensive test is conducted for the role of various frictions and finds strong support for the nominal and real frictions in the model, in particular wage stickiness, price stickiness in the investment goods importing sector, habit formation and investment adjustment costs are important for the empirical success of the model. The model with partial wage indexation accounts well for the dynamics of the Canadian economy.

For future work, some issues remain to be addressed. First, following an unanticipated temporary increase in the nominal interest rate, the model generates a very persistent negative response in output, consumption, investment hours and inflation, while inflation drops on the impact. The reason for this result is that in the model there is no indexation to lagged inflation for firms and the estimate of wage indexation is quite low. New endogenous channel should be explored to generate inflation inertia. Next, the real exchange rate appreciate immediately on the impact and then returns to zero from below, which is not consistent with a hump-shaped response of estimated VARs. This fast response to a monetary policy shock in this model is driven by the uncovered interest parity condition. Moreover, more empirical evidence is needed for sticky wages in Canada concerning: (*i*) the average duration of wage contracts; (*ii*) how wage contracts are set; (*iii*) whether wage contracts are set in the fashion of Calvo. Finally, the optimal monetary policy is not analyzed in the context of the model.

APPENDIX C

C.1. Data and data sources

The Canadian data are retrieved from Cansim databank of Statistics of Canada. Series titles and series numbers used to construct the vector of observed variables and to calibrate certain parameters are presented in the following table. The series in terms of per capita, Y_t, C_t and I_t, are obtained by dividing each series by the labour force of 15 years and over.

The U.S. data are from the Federal Reserve Bank of St. Louis.

The series in terms of per capita, Y_t^*, is obtained by dividing the series by the labour force.

C.2. Model solution and likelihood function

To solve the model, I first compute the non-stochastic steady state. Second, Dynare is used to linearize the relevant equilibrium equations around this steady state and to solve the linearized model. equation to be solved can be written as

$$E_t \{\alpha_0 \Gamma_{t-1} + \alpha_1 \Gamma_t + \alpha_2 \Gamma_{t+1} + \beta_2 \Psi_{t+1} + \beta_1 \Psi_t\} = 0 \qquad (E.1)$$

where Γ_t is a vector of endogenous variables, Ψ_t is a vector of exogenous variables, and α_0, α_1, α_2, β_1 and β_2 are matrices of coefficients. Ψ_t follows

$$\Psi_t = \rho \Psi_{t-1} + \epsilon_t, \qquad \epsilon_t \sim N(0, \Sigma) \qquad (E.2)$$

Table 3.1 Canadian data

Variables	Series title	Series number
Y_t	Gross domestic product at market price	v1992067
C_t	Personal expenditure on non-durable goods +	v1992047
	Personal expenditure on services	v1992119
I_t	Personal expenditure on durable goods +	v1992045
	Machinery and equipment +	v1992056
	Residential structures +	v1992053
	Non-residential structures +	v1992055
	Business investment in inventories	v1992057
W_t	Wages, salaries, and supplementary labor income	v498076
P_t	GDP implicit price indexes	v1997756
$P_{c,t}$	Consumer price index	v735319
$P_{i,t}$	Business gross fixed capital formation	v1997745
	implicit price indexes	
R_t	Treasure bills: 3 month	v122531
Per capita	Labour force, 15 years and over ×	v2062810
	Labour force participation	v2062816
V_t	Gross M1	v37252
M_t	Gross M2	v41552786
$1-\theta$	Net domestic income at market price -	v1997472
	Taxes less subsidies, on factors of production	v1992216

Table 3.2 U.S. data

Variables	Series title	Series number
Y_t^*	Real domestic product	GDPC96
P_t^*	GDP implicit price deflator	GDPDEF
R_t^*	3-month treasury bill: secondary market rate	TB3MS
Per capita	Civilian noninstitutional population	CNP16OV

The solution of the model can be written as

$$\Gamma_t = A\Gamma_{t-1} + B\Psi_t \tag{E.3}$$

With the exogenous processes $(E.2)$, the solution of the model can be transformed to the following state-space representation for the unobserved state variables ξ_t

$$\xi_{t+1} = F\xi_t + \nu_{t+1}, \quad E(\nu_{t+1}\nu'_{t+1}) = Q$$

and the observation equation

$$\widetilde{Y}_t = H'\xi_t + A'_X X_t + \zeta_t, \quad \zeta_t \sim N(0, R)$$

where $\widetilde{Y}_t$ is a vector of observed variables, X_t a vector of predetermined variables and ζ_t a vector of measurement errors. Following Hamilton (1994), the Kalman updating equations are given by

$$\hat{\xi}_{t+1|t} = F\hat{\xi}_{t|t-1} + K_t\left(\widetilde{Y}_t - H'\xi_{t|t-1} - A'_X X_t\right)$$

$$P_{t+1|t} = FP_{t|t-1}F' - K_t H' P_{t|t-1}F' + Q$$

where $K_t \equiv FP_{t|t-1}H(H'P_{t|t-1}H + R)^{-1}$, $P_{t|t-1} \equiv E\left[(\xi_t - \hat{\xi}_{t|t-1})(\xi_t - \hat{\xi}_{t|t-1})'\right]$ and $\hat{\xi}_{t|t-1}$ is the linear projection of ξ_t on the data observed up to period $t-1$. For $t = 1, ..., T$ and with initial values ξ_1 and P_1, from the Kalman filter recursion, we can derive the log-likelihood function given by

$$\begin{aligned}\ln L(\theta|\tilde{\mathbf{Y}}_T) &= -\frac{Tn}{2}\ln(2\pi) - \frac{1}{2}\sum_{t=1}^{T}\ln|\Sigma_t| - \\ &\quad \frac{1}{2}\sum_{t=1}^{T}\left(\widetilde{Y}_t - H'\xi_{t|t-1} - A'_X X_t\right)'\Sigma_t^{-1}\left(\widetilde{Y}_t - H'\xi_{t|t-1} - A'_X X_t\right)\end{aligned}$$

where $\Sigma_t = H'P_{t|t-1}H + R$, θ is a vector of the parameters to estimate, $\tilde{\mathbf{Y}}_T$ a set of observed variables $\tilde{Y}_t$ and n the number of variables.

C.3. Tables and figures

Table 3.3 Prior distribution

Parameter		Domain	Density	Mean	Std. dev.
Wage stickiness	$\tilde{\alpha}$	$[0,1)$	*beta*	0.750	0.05
Domestic price stic.	α	$[0,1)$	*beta*	0.650	0.05
Import c price stic.	α_{cm}	$[0,1)$	*beta*	0.500	0.10
Import i price stic.	α_{im}	$[0,1)$	*beta*	0.500	0.10
Subst. elasticity con.	η_c	$\mathbb{R}^+$	*gamma*	1.700	0.10
Subst. elasticity invest.	η_i	$\mathbb{R}^+$	*gamma*	1.500	0.10
Subst. elasticity foreign	η_f	$\mathbb{R}^+$	*gamma*	1.500	0.10
Wage indexation	$\tilde{\chi}$	$[0,1]$	*beta*	0.200	0.10
Investment adj. costs	κ	$\mathbb{R}^+$	*normal*	2.490	0.10
Degree of habit persistence	b	$[0,1)$	*beta*	0.500	0.10
Risk premium	ϕ_b	$\mathbb{R}^+$	*gamma*	0.020	0.01
Government expenditures	ρ_g	$[0,1)$	*beta*	0.800	0.10
Neutral tech. shock	ρ_{μ_z}	$[0,1)$	*beta*	0.800	0.10
Invest. spec. tech. shock	ρ_{μ_Υ}	$[0,1)$	*beta*	0.500	0.10
Foreign output	ρ_{y^f}	$[0,1)$	*beta*	0.800	0.10
Foreign inflation	ρ_{π^*}	$[0,1)$	*beta*	0.800	0.10
Foreign interest rate	ρ_{r^*}	$[0,1)$	*beta*	0.800	0.10
Risk premium shock	ρ_ϕ	$[0,1)$	*beta*	0.850	0.10
Consumption pref. shock	ρ_{ζ_c}	$[0,1)$	*beta*	0.800	0.10
Government expenditures	σ_g	$\mathbb{R}^+$	*inv.gamma*	0.006	0.10
Monetary policy shock	σ_r	$\mathbb{R}^+$	*inv.gamma*	0.010	0.10
Foreign interest rate	σ_{r^*}	$\mathbb{R}^+$	*inv.gamma*	0.006	0.10
Foreign output	σ_{y^f}	$\mathbb{R}^+$	*inv.gamma*	0.006	0.10
Foreign inflation	σ_{π^*}	$\mathbb{R}^+$	*inv.gamma*	0.006	0.10
Risk premium shock	σ_ϕ	$\mathbb{R}^+$	*inv.gamma*	0.001	0.10
Neutral tech. shock	σ_{μ_z}	$\mathbb{R}^+$	*inv.gamma*	0.010	0.10
Invest. spec. tech. shock	σ_{μ_Υ}	$\mathbb{R}^+$	*inv.gamma*	0.020	0.10
Consumption pref. shock	σ_{ζ_c}	$\mathbb{R}^+$	*inv.gamma*	0.010	0.10
Measure. error interest	σ_{r_obs}	$\mathbb{R}^+$	*inv.gamma*	0.010	0.10
Measure. error investment	σ_{inv_obs}	$\mathbb{R}^+$	*inv.gamma*	0.010	0.10
Output response	by	$\mathbb{R}$	*normal*	0.100	0.05
Inflation response	$b\pi$	$\mathbb{R}$	*normal*	1.600	0.10
Interest rate smoothing	ρ_r	$\mathbb{R}$	*beta*	0.450	0.10

Table 3.4 Posterior distribution

Parameters		Prior	Posterior distribution				
		Mean	Mode	Std.dev. (Hessian)	Mean	5%	95%
Wage stickiness	$\tilde{\alpha}$	0.750	0.700	0.033	0.693	0.656	0.714
Domestic price stic.	α	0.650	0.843	0.031	0.836	0.807	0.866
Import c price stic.	α_{cm}	0.500	0.117	0.052	0.128	0.094	0.177
Import i price stic.	α_{im}	0.500	0.430	0.189	0.401	0.144	0.517
Subst. elasticity con.	η_c	1.700	1.484	0.109	1.488	1.336	1.649
Subst. elasticity invest.	η_i	1.500	1.476	0.010	1.460	1.290	1.600
Subst. elasticity foreign	η_f	1.500	1.456	0.098	1.454	1.287	1.619
Wage indexation	χ	0.200	0.080	0.096	0.099	0.020	0.166
Investment adj. costs	κ	2.490	2.592	0.097	2.595	2.434	2.771
Degree of habit persistence	b	0.500	0.772	0.046	0.775	0.743	0.832
Risk premium	ϕ_b	0.020	0.0002	0.002	0.0003	0.0001	0.0006
Government expenditures	ρ_g	0.800	0.998	0.001	0.996	0.995	0.999
Neutral tech. shock	ρ_{μ_z}	0.800	0.939	0.012	0.936	0.914	0.956
Invest. spec. tech. shock	ρ_{μ_Υ}	0.500	0.196	0.061	0.208	0.136	0.281
Foreign output	ρ_{y^f}	0.800	0.994	0.004	0.990	0.981	0.998
Foreign inflation	ρ_{π^*}	0.800	0.817	0.031	0.813	0.767	0.859
Foreign interest rate	ρ_{r^*}	0.800	0.927	0.011	0.929	0.913	0.954
Risk premium shock	ρ_ϕ	0.850	0.886	0.072	0.849	0.742	0.966
Consumption pref. shock	ρ_{ζ_c}	0.800	0.996	0.000	0.994	0.992	0.994
Government expenditures	σ_g	0.006	0.020	0.003	0.0208	0.0178	0.0232
Monetary policy shock	σ_r	0.010	0.008	0.002	0.0085	0.0071	0.0097
Foreign interest rate	σ_{r^*}	0.006	0.002	0.001	0.0017	0.0015	0.0019
Foreign output	σ_{y^f}	0.006	0.010	0.001	0.0098	0.0087	0.0110
Foreign inflation	σ_{π^*}	0.006	0.002	0.001	0.0025	0.0022	0.0028
Risk premium shock	σ_ϕ	0.001	0.005	0.002	0.0118	0.0023	0.0252
Neutral tech. shock	σ_{μ_z}	0.010	0.003	0.002	0.0031	0.0022	0.0043
Invest. spec. tech. shock	σ_{μ_Υ}	0.020	0.018	0.002	0.0184	0.0160	0.0206
Consumption pref. shock	σ_{ζ_c}	0.010	0.062	0.014	0.0611	0.0487	0.0760
Measure. error interest	σ_{r_obs}	0.010	0.007	0.001	0.0070	0.0061	0.0079
Measure. error investment	σ_{inv_obs}	0.010	0.037	0.003	0.0377	0.0324	0.0416
Output response	by	0.100	0.162	0.051	0.172	0.136	0.219
Inflation response	$b\pi$	1.600	1.743	0.095	1.734	1.583	1.852
Interest rate smoothing	ρ_r	0.450	0.226	0.056	0.236	0.148	0.339
LML		3386					

Note: the log marginal likelihood (LML) is computed numerically from the posterior draws using the modified harmonic estimator in Geweke (1999).

Table 3.5 Sensitivity analysis with respect to frictions

	Bench-mark	Full wage index.	No price sticki-ness	No wage sticki-ness	No var. cap. utilis.	No inv.. adj. . cost	No work. cap.	LOP con. imp.	LOP inv. imp.	No habit persist.
$\tilde{\alpha}$	0.700	0.64	0.722		0.764	0.688	0.701	0.697	0.721	0.745
α	0.843	0.82		0.575	0.862	0.434	0.846	0.376	0.871	0.876
α_{cm}	0.117	0.13	0.432	0.423	0.077	0.252	0.115		0.089	0.050
α_{im}	0.430	0.48	0.457	0.699	0.487	0.616	0.440	0.503		0.431
η_c	1.484	1.31	1.500	1.548	1.501	1.358	1.484	1.443	1.565	1.636
η_i	1.476	1.39	1.427	1.381	1.442	1.310	1.479	1.410	1.469	1.434
η_f	1.456	1.39	1.379	1.425	1.473	1.435	1.457	1.492	1.469	1.487
$\tilde{\chi}$	0.080		0.047	0.174	0.171	0.140	0.077	0.141	0.121	0.185
κ	2.592	2.47	2.551	2.544	2.549		2.594	2.560	2.561	2.574
b	0.772	0.93	0.529	0.950	0.634	0.681	0.775	0.781	0.655	
ϕ_b	0.0002	0.11	0.022	0.029	0.0009	0.169	0.0002	0.015	0.002	0.003
ρ_g	0.998	0.97	0.920	0.952	0.999	0.974	0.998	0.950	0.999	0.997
ρ_{μ_z}	0.939	0.99	0.897	0.995	0.963	0.972	0.941	0.956	0.963	0.977
ρ_{μ_Υ}	0.196	0.27	0.153	0.326	0.238	0.158	0.197	0.148	0.187	0.304
ρ_{y^f}	0.994	0.99	0.984	0.987	0.988	0.993	0.994	0.988	0.983	0.956
ρ_{π^*}	0.817	0.86	0.874	0.824	0.829	0.901	0.818	0.859	0.840	0.830
ρ_{r^*}	0.927	0.99	0.967	0.971	0.945	0.969	0.927	0.949	0.952	0.956
ρ_ϕ	0.886	0.99	0.985	0.995	0.886	0.988	0.849	0.994	0.885	0.886
ρ_{ζ_c}	0.996	0.98	0.998	0.864	0.811	0.961	0.996	0.963	0.983	0.999
σ_g	0.020	0.028	0.026	0.026	0.022	0.022	0.020	0.020	0.019	0.023
σ_r	0.008	0.006	0.003	0.003	0.0077	0.004	0.008	0.007	0.008	0.007
σ_{r^*}	0.002	0.002	0.002	0.002	0.002	0.002	0.002	0.002	0.002	0.002
σ_{y^f}	0.010	0.010	0.010	0.010	0.010	0.010	0.010	0.010	0.010	0.010
σ_{π^*}	0.002	0.002	0.002	0.002	0.002	0.002	0.002	0.002	0.002	0.002
σ_ϕ	0.005	0.269	2.259	1.162	0.005	0.312	0.003	1.356	0.005	0.005
σ_{μ_z}	0.003	0.003	0.002	0.002	0.002	0.003	0.003	0.002	0.002	0.002
σ_{μ_Υ}	0.018	0.022	0.018	0.022	0.020	0.021	0.018	0.020	0.020	0.020
σ_{ζ_c}	0.062	0.267	0.030	0.200	0.023	0.049	0.064	0.067	0.071	0.008
σ_{r_obs}	0.007	0.004	0.002	0.002	0.007	0.004	0.007	0.004	0.007	0.007
σ_{i_obs}	0.037	0.039	0.038	0.042	0.039	0.033	0.037	0.027	0.035	0.040
by	0.162	0.000	0.051	0.014	0.025	0.002	0.161	0.003	0.021	0.013
$b\pi$	1.743	1.621	1.505	1.519	1.729	1.650	1.744	1.820	1.700	1.450
ρ_r	0.226	0.268	0.737	0.727	0.196	0.584	0.223	0.201	0.210	0.144
LML	3388	3215	3375	3358	3378	3354	3384	3367	3347	3331

Note: the log marginal likelihood is computed numerically by Laplace approximation.

Table 3.6 Unconditional second moments in the data and in the estimated models (a)

	Data	Bench -mark model	No price sticki- ness	No wage sticki- ness	No var. cap. utilis.	No inv. adj. cost	No work. cap.
				Consumption growth rate			
$\sigma_{\Delta\ln(C_t)}/\sigma_{\Delta\ln(Y_t)}$	0.96	1.01	0.86	0.92	0.95	0.82	1
Corr. with $\Delta\ln(Y_t)$	0.63	0.60	0.658	0.665	0.38	0.46	0.58
		$(0.53, 0.67)$	$(0.59, 0.73)$	$(0.62, 0.71)$	$(0.34, 0.43)$	$(0.35, 0.56)$	$(0.52, 0.65)$
Autocorrelation	−0.06	0.82	0.608	0.91	0.76	0.73	0.82
		$(0.78, 0.85)$	$(0.52, 0.71)$	$(0.89, 0.92)$	$(0.71, 0.79)$	$(0.66, 0.82)$	$(0.79, 0.86)$
				Investment growth rate			
$\sigma_{\Delta\ln(I_t)}/\sigma_{\Delta\ln(Y_t)}$	4.78	3.73	3.89	2.67	5.65	5.08	3.77
Corr. with $\Delta\ln(Y_t)$	0.46	0.71	0.73	0.380	0.58	0.64	0.70
		$(0.66, 0.76)$	$(0.68, 0.77)$	$(0.33, 0.42)$	$(0.50, 0.65)$	$(0.61, 0.69)$	$(0.65, 0.75)$
Autocorrelation	0.35	0.88	0.82	0.85	0.90	0.42	0.89
		$(0.86, 0.91)$	$(0.80 - 0.86)$	$(0.83, 0.86)$	$(0.90, 0.91)$	$(0.37, 0.46)$	$(0.86, 0.91)$
				Real wage growth rate			
$\sigma_{\Delta\ln(w_t)}/\sigma_{\Delta\ln(Y_t)}$	1.11	0.61	0.57	0.84	0.78	0.76	0.62
Corr. with $\Delta\ln(Y_t)$	0.63	0.72	0.374	0.83	0.54	0.53	0.71
		$(0.67, 0.76)$	$(0.28, 0.46)$	$(0.81, 0.86)$	$(0.51, 0.56)$	$(0.41, 0.63)$	$(0.66, 0.74)$
Autocorrelation	0.1	0.55	0.34	0.79	0.55	0.62	0.55
		$(0.46, 0.62)$	$(0.29, 0.41)$	$(0.75, 0.82)$	$(0.47, 0.60)$	$(0.51, 0.73)$	$(0.48, 0.64)$
				Nominal interest rate			
$\sigma_{R_t}/\sigma_{\Delta\ln(Y_t)}$	1.18	0.97	0.98	1.43	1.63	1.26	0.96
Corr. with $\Delta\ln(Y_t)$	−0.27	0.08	0.15	0.60	0.34	0.39	0.11
		$(-0.01, 0.18)$	$(-0.01, 0.423)$	$(0.52, 0.69)$	$(0.25, 0.39)$	$(0.25, 0.58)$	$(-0.01, 0.21)$
Autocorrelation	0.90	0.90	0.94	0.99	0.94	0.97	0.91
		$(0.89, 0.92)$	$(0.93, 0.97)$	$(0.987, 0.994)$	$(0.93, 94)$	$(0.96, 0.99)$	$(0.90, 0.92)$
				Domestic inflation			
$\sigma_{\pi_t}/\sigma_{\Delta\ln(Y_t)}$	0.87	0.97	1.13	0.93	1.67	0.99	0.95
Corr. with $\Delta\ln(Y_t)$	−0.25	0.11	−0.13	0.43	0.11	0.15	0.13
		$(0.01, 0.21)$	$(-0.22, -0.03)$	$(0.31, 0.51)$	$(0.01, 0.17)$	$(-0.01, 0.32)$	$(0.03, 0.23)$
Autocorrelation	0.41	0.91	0.91	0.91	0.94	0.89	0.92
		$(0.90, 0.92)$	$(0.90, 0.93)$	$(0.87, 0.92)$	$(0.938, 0.944)$	$(0.86, 0.93)$	$(0.907, 0.921)$

Note: For the model, the median from the simulated distribution of moments is reported. The numbers in parenthesis are the 5th and 95th percentiles of the simulated distribution moments. $\Delta\ln$ means that a variable is expressed in growth rates.

Table 3.7 Unconditional second moments in the data and in the estimated models (b)

	Data	Bench -mark model	LOP con. imp.	LOP inv. imp.	No habit persist.	Full wage index.
Consumption growth rate						
$\sigma_{\Delta \ln(C_t)}/\sigma_{\Delta \ln(Y_t)}$	0.96	1.01	1.06	0.86	0.58	0.92
Corr. with $\Delta \ln(Y_t)$	0.63	0.60	0.60	0.69	0.81	0.68
		(0.53, 0.67)	(0.53, 0.69)	(0.58, 0.80)	(0.80, 0.83)	(0.63, 0.72)
Autocorrelation	−0.06	0.82	0.76	0.90	0.84	0.77
		(0.78, 0.85)	(0.69, 0.83)	(0.83, 0.92)	(0.79, 0.88)	(0.74, 0.80)
Investment growth rate						
$\sigma_{\Delta \ln(I_t)}/\sigma_{\Delta \ln(Y_t)}$	4.78	3.73	3.95	2.47	2.20	3.44
Corr. with $\Delta \ln(Y_t)$	0.46	0.71	0.66	0.67	0.80	0.74
		(0.66, 0.76)	(0.59, 0.75)	(0.66, 0.72)	(0.77, 0.83)	(0.70, 0.77)
Autocorrelation	0.35	0.88	0.78	0.93	0.95	0.85
		(0.86, 0.91)	(0.76, 0.80)	(0.91, 0.93)	(0.94, 0.96)	(0.84, 0.86)
Real wage growth rate						
$\sigma_{\Delta \ln(w_t)}/\sigma_{\Delta \ln(Y_t)}$	1.11	0.61	0.78	0.39	0.25	0.63
Corr. with $\Delta \ln(Y_t)$	0.63	0.72	0.53	0.37	0.36	0.68
		(0.67, 0.76)	(0.43, 0.60)	(0.25, 0.48)	(0.31, 0.41)	(0.65, 0.71)
Autocorrelation	0.1	0.55	0.50	0.50	0.53	0.51
		(0.46, 0.62)	(0.42, 0.61)	(0.49, 0.52)	(0.53, 0.54)	(0.45, 0.58)
Nominal interest rate						
$\sigma_{R_t}/\sigma_{\Delta \ln(Y_t)}$	1.18	0.97	1.39	1.81	1.52	0.84
Corr. with $\Delta \ln(Y_t)$	−0.27	0.08	0.039	0.72	0.85	−0.01
		(−0.01, 0.18)	(−0.09, 0.16)	(0.62, 0.83)	(0.82, 0.88)	(−0.07, 0.03)
Autocorrelation	0.90	0.90	0.91	0.99	0.99	0.91
		(0.89, 0.92)	(0.89, 0.93)	(0.98, 0.997)	(0.99, 0.995)	(0.90, 0.92)
Domestic inflation						
$\sigma_{\pi_t}/\sigma_{\Delta \ln(Y_t)}$	0.87	0.97	1.11	1.17	0.79	0.80
Corr. with $\Delta \ln(Y_t)$	−0.25	0.11	−0.01	0.11	0.04	0.04
		(0.01, 0.21)	(−0.15, 0.08)	(0.03, 0.19)	(−0.002, 0.08)	(−0.02, 0.11)
Autocorrelation	0.41	0.91	0.86	0.98	0.97	0.88
		(0.90, 0.92)	(0.83, 0.88)	(0.96, 0.99)	(0.965, 0.98)	(0.87, 0.89)

Note: For the model, the median from the simulated distribution of moments is reported. The numbers in parenthesis are the 5th and 95th percentiles of the simulated distribution moments. $\Delta \ln$ means that a variable is expressed in growth rates.

Table 3.8 Decomposition of the Variance of the Forecast Error

SH	Output gth			Con. gth			Inv. gth			Export gth			Import gth			Dom. inflation		
	Mean	5%	95%	Mean	5%	95%	Mean	5%	95%	Mean	5%	95%	Mean	5%	95%	Mean	5%	95%
GP	0.058	0.037	0.077	0.024	0.014	0.035	0.010	0.002	0.018	0.000	0.000	0.000	0.020	0.008	0.037	0.055	0.022	0.091
FR	0.054	0.033	0.080	0.090	0.059	0.127	0.141	0.096	0.198	0.000	0.000	0.001	0.094	0.064	0.131	0.017	0.007	0.027
FO	0.025	0.016	0.032	0.002	0.001	0.003	0.001	0.000	0.001	0.367	0.297	0.446	0.001	0.001	0.002	0.005	0.001	0.008
FI	0.007	0.002	0.012	0.018	0.007	0.030	0.029	0.009	0.046	0.001	0.000	0.001	0.026	0.014	0.042	0.003	0.000	0.004
NT	0.478	0.376	0.557	0.617	0.534	0.692	0.410	0.342	0.498	0.257	0.184	0.343	0.487	0.389	0.567	0.604	0.542	0.678
IS	0.289	0.221	0.361	0.037	0.022	0.051	0.241	0.173	0.315	0.307	0.248	0.369	0.089	0.061	0.123	0.206	0.202	0.209
MP	0.040	0.024	0.054	0.002	0.001	0.003	0.001	0.000	0.001	0.066	0.022	0.107	0.017	0.011	0.024	0.001	0.000	0.001
CP	0.048	0.028	0.065	0.210	0.150	0.279	0.168	0.082	0.239	0.002	0.000	0.004	0.265	0.184	0.352	0.110	0.050	0.191

	CPI inflation			Investment inflation			Interest rate			Real exchange rate			Hours			Real wages gth		
	Mean	5%	95%	Mean	5%	95%	Mean	5%	95%	Mean	5%	95%	Mean	5%	95%	Mean	5%	95%
GP	0.057	0.030	0.083	0.059	0.023	0.092	0.047	0.019	0.078	0.004	0.001	0.008	0.068	0.017	0.123	0.000	0.000	0.000
FR	0.108	0.073	0.144	0.040	0.011	0.069	0.110	0.069	0.145	0.011	0.006	0.014	0.021	0.008	0.031	0.000	0.000	0.000
FO	0.013	0.012	0.005	0.007	0.001	0.013	0.004	0.001	0.008	0.000	0.000	0.000	0.004	0.001	0.008	0.000	0.000	0.000
FI	0.031	0.016	0.045	0.008	0.001	0.016	0.049	0.031	0.069	0.018	0.011	0.025	0.004	0.001	0.006	0.000	0.000	0.000
NT	0.507	0.571	0.644	0.484	0.430	0.541	0.584	0.537	0.640	0.025	0.021	0.048	0.605	0.639	0.778	0.460	0.351	0.560
IS	0.126	0.114	0.135	0.309	0.302	0.315	0.106	0.103	0.109	0.007	0.004	0.010	0.241	0.219	0.259	0.540	0.440	0.649
MP	0.085	0.061	0.115	0.012	0.001	0.031	0.110	0.113	0.127	0.890	0.844	0.936	0.002	0.001	0.002	0.000	0.000	0.000
CP	0.073	0.050	0.153	0.081	0.023	0.167	0.040	0.020	0.108	0.035	0.012	0.060	0.055	0.026	0.172	0.000	0.000	0.000

Note: The table reports the posterior mean mean, 5th and 95th percentiles of the variance of the forecast errors at business cycle frequencies.
SH: shock; GP: government purchases; FR: foreign interest rate; FO: foreign output; FI: foreign inflation rate
NT: neutral technology; IS: investment-specific technology; MP: monetary policy; CP: consumption preference.

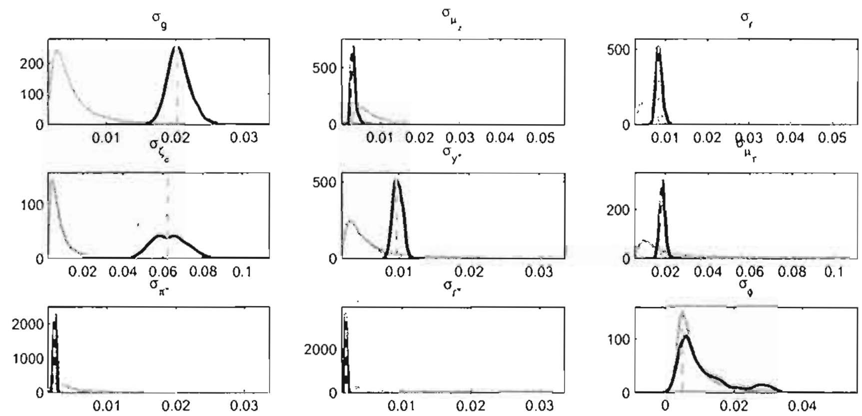

Figure 3.1 Priors (gray) and Posteriors (black)

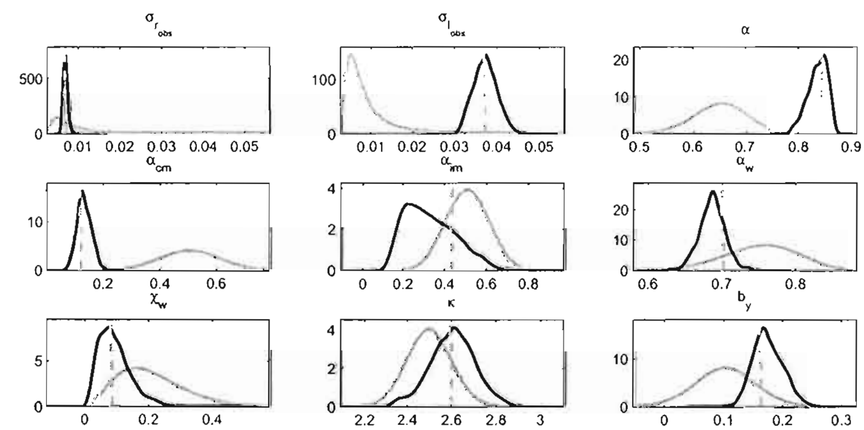

Figure 3.2 Priors (gray) and Posteriors (black)

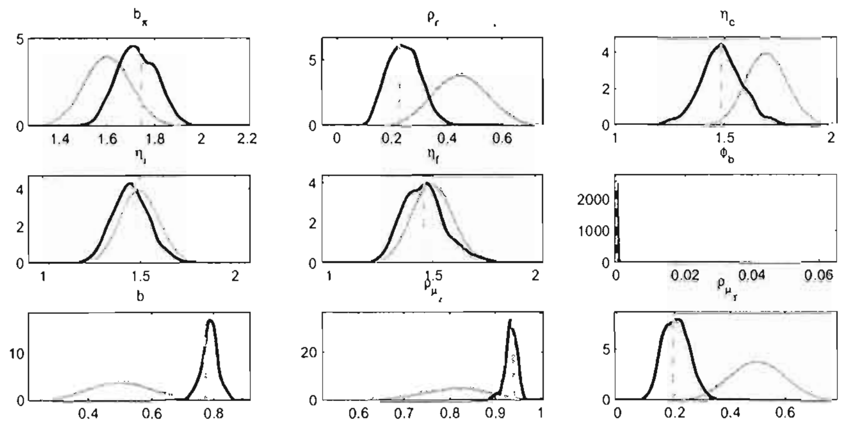

Figure 3.3 Priors (gray) and Posteriors (black)

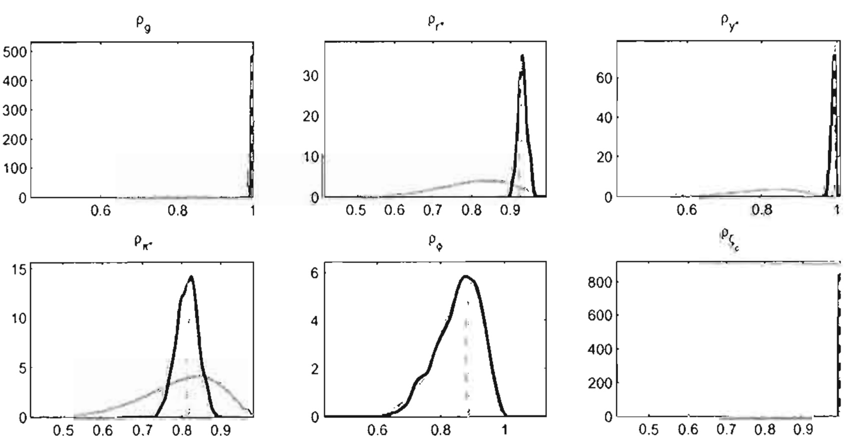

Figure 3.4 Priors (gray) and Posteriors (black)

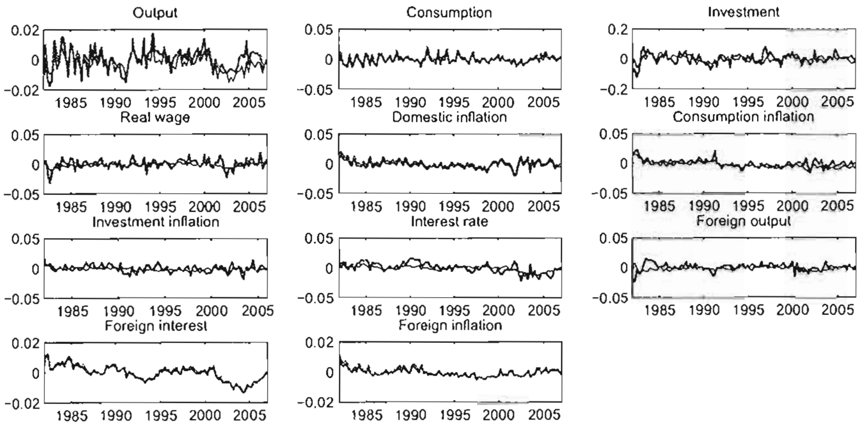

Figure 3.5 Data (bold) and predicted values (thin)

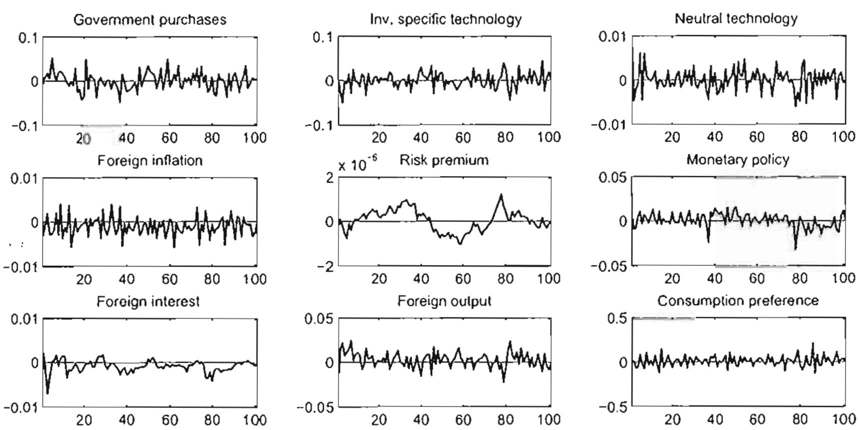

Figure 3.6 Smoothed estimates of the unobserved shocks

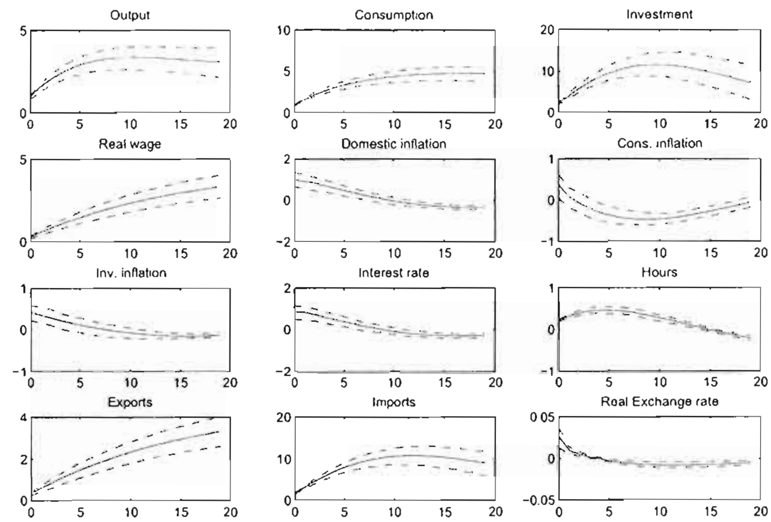

Figure 3.7 Impulse responses to a neutral technology shock (solid-mean, dashed-95% confidence interval)

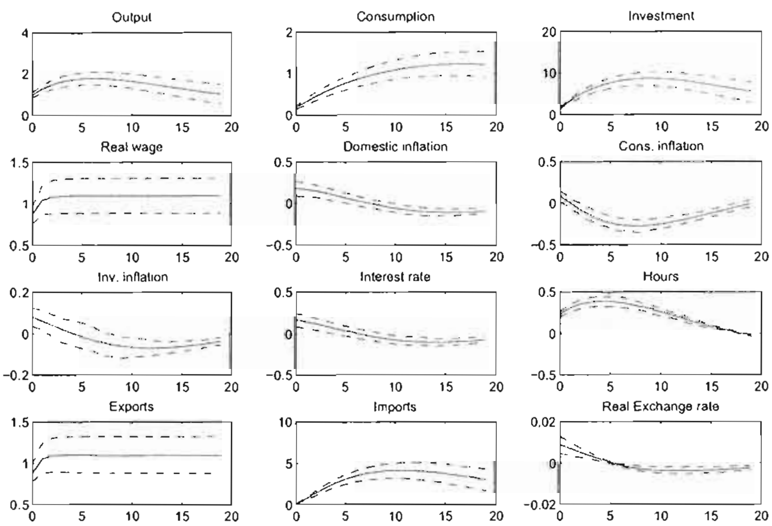

Figure 3.8 Impulse responses to an investment specific technology shock (solid-mean, dashed-95% confidence interval)

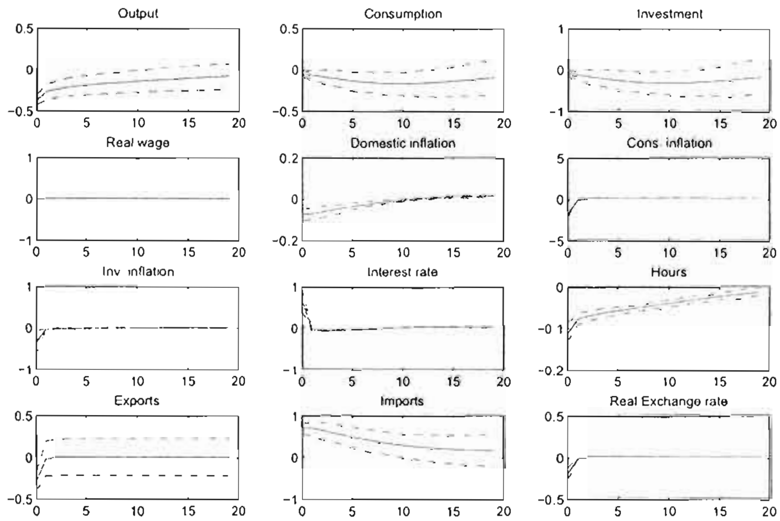

Figure 3.9 Impulse responses to a monetary policy shock (solid-mean, dashed-95% confidence interval)

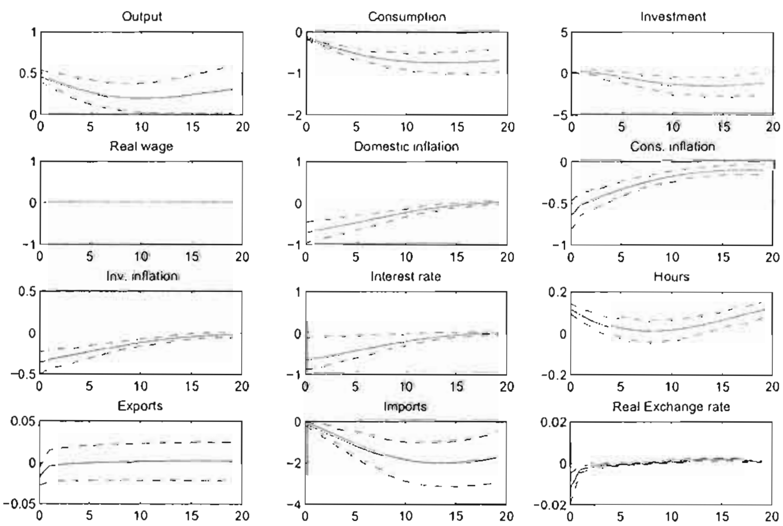

Figure 3.10 Impulse responses to a government purchases shock (solid-mean, dashed-95% confidence interval)

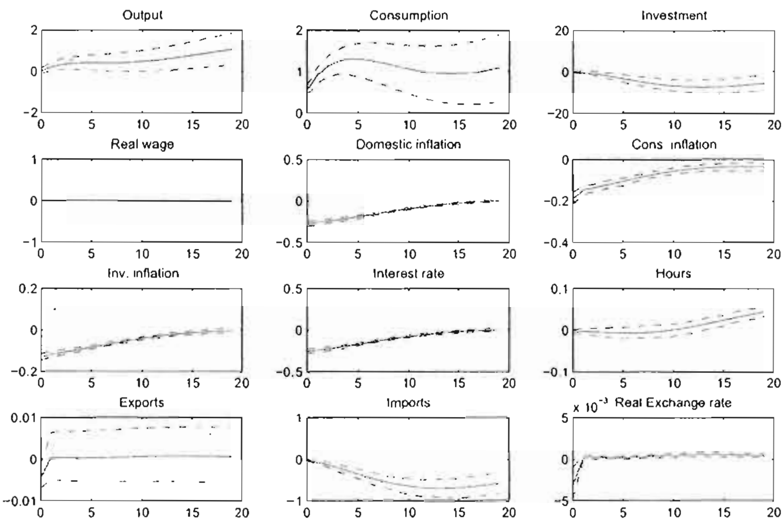

Figure 3.11 Impulse responses to a consumption preference shock (solid-mean, dashed-95% confidence interval)

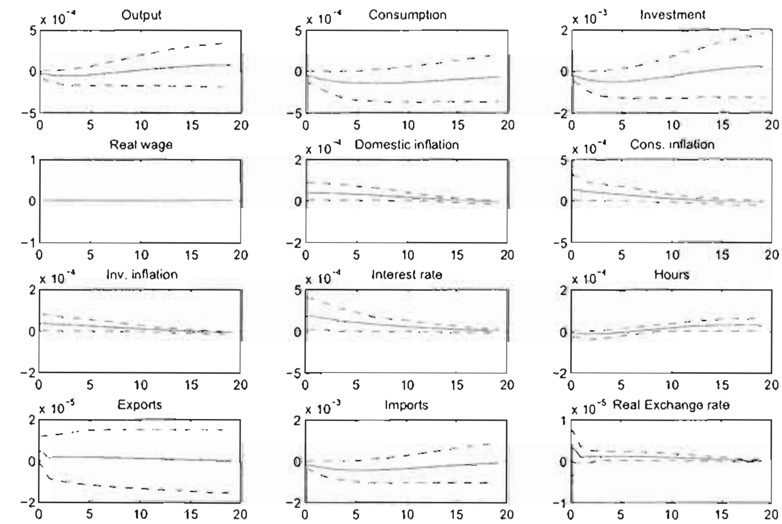

Figure 3.12 Impulse responses to a risk premium shock (solid-mean, dashed-95% confidence interval)

CONCLUSION

This dissertation studies three issues in international finance: (i) the effects of the limited contractual enforcement of sovereign debt with a renegotiation option on international risk sharing, (ii) the collective-action problem regarding to rolling over or foreclosing with multiple creditors and its policy implications for preventing and managing sovereign default, and (iii) the empirical performance of a small open economy model with partial price and wage indexation using Canadian data.

The first essay generalizes the permanent exclusion sanction used in previous studies to solve the cross-country consumption correlation puzzle. In a two-country model, a defaulting sovereign borrower is excluded from external finance temporarily and can renegotiate its debt. A country defaults if only if the expected value of serving its debt is strictly smaller than the expected value of partial default. A co-state variable, introduced to generalize the standard Bellman equation, captures a country's defaulting history. This modeling strategy transforms the problem into a solution of Nash bargaining problem. The results show: (i) international risk sharing is decreasing in the number of exclusion periods; (ii) the model with partial default generates the cross-country correlation of consumption that is lower than that of output.

The second essay characterizes the equilibrium of a global game with two types of creditors. Even abstracting from signaling, the presence of a better informed or a more optimistic large creditor increases the willingness of small creditors to roll over, and thereby reduces the probability of project failure. The equilibrium outcome can be explained by the behavior of relying on the information or the estimate of others. Relying on the information of others aggregates the information, but only partially because the anticipation of one type of creditors about the information of others coincides with the true value at a very weak probability. Thus, inefficient liquidation or rolling over may occur because of strategic uncertainty about others' actions.

To solve the coordination problem with multiple creditors it is essential for a mechanism to be able to correct credibly the beliefs of creditors. The model suggests that lending from a third party with *ex post* conditionalities confirms the negative beliefs of private creditors that the fundamentals of the debtor are not sound enough and the future improvement is uncertain. To correct credibly the beliefs of private creditors, we propose a mechanism of lending with *ex ante* conditionalities combining with a system of evaluation on both the fundamentals of the debtor and its degree of respecting the conditionalities. This mechanism is shown to be able to generate the same outcome as a national bankruptcy law.

The third essay estimates a small open economy model in which partial wage indexation is allowed using Canadian data. Previous empirical work imposes that unadjusted wages are completely indexed to lagged inflation. However, the empirical evidence for this specification is absent. Canadian collective wage bargaining contracts show only a small proportion of contacts contains cost-of-living clauses. It is important ask whether this specification is empirically robust, and if it is not, whether there is a specification with better empirical performance. To answer these questions, the wage indexation parameter is estimated along with other structural parameters, and the empirical performances of different versions of the model are assessed using Bayesian estimation techniques. The results show that aggregated data contain information about the value of wage indexation parameter, and wages are indexed only partially to lagged inflation, which is close to the average proportion of wage settlements with cost-of-living clauses in Canada during the sample period. In addition, the empirical performance drops with a version of the model in which wages are fully indexed to lagged inflation. Finally, the model with partial indexation accounts well for the dynamics of the Canadian economy, in particular it can replicate most of conventional statistics observed in data

Several new questions and possible extensions have been developed through this work. From the first essay, limited contractual enforcement of debt can be extended to labor contracts. The idea is to explore the monetary transmission mechanism through interactions between loan contracts and labor contracts whose levels of contractual enforcement are different and creditors are asymmetrically informed about the fundamentals of their debtor. This is motivated by the fact that firms in financial difficulties, often accentuated by coordination failure between creditors, lay off their workers in the first place. This observation reflects two things. First, between physical assets and workers, firms cannot sell physical assets that are often served as collaterals to get loans, but can lay off workers if layoff costs are less important than those of bankruptcy. This means the degree of respecting loan contracts is higher than that of labor contracts. Second, unemployment can be induced by imperfect collective action of creditors when they have to reconsider their investment decision such as rolling over or foreclosing their loans. The coordination problem in this case accentuates the monetary policy transmission through the credit channel. Previous work in the credit channel of monetary policy transmission as in Bernanke and Gertler (1995) does not explore neither the interactions between loan contracts and labor contracts, nor the coordination problem.

The second essay on coordination problem with multiple creditors gives rise to one interesting question : what happens when one or some of creditors could purchase the obligations owned by other creditors. This question in turn leads to characterize the equilibrium of the game in which owners of a common asset, imperfectly informed about both the value of the asset and what other owners know, compete to have the sole ownership using a common-value auction.

The setup is motivate by the following observed facts. First, when a project (for example, pipeline, mine, real estate, the purchasing of banks' troubled assets), financed through nonrecourse debt, runs into problems and thereby cannot deliver the expected cash flow, asymmetrically informed creditors have to reconsider their investment decisions. Second, their communication is limited to a kind of cheap talk in the sense that each creditor has a tendency to tell others to roll over their loans while he himself forecloses to have the first-move advantage. Last, when an informed creditor knows that the investment project will be profitable if it remains, he could propose to purchase the other part owned by a less informed one. The equilibrium of such game has not been characterized in previous studies. Engelbrechat-Wiggans, Milgrom and Weber (1983) analyze private value auction with two asymmetrically informed bidders. Burkart (1995) extends private value auction with partial ownership in takeover contests, while common value auction with partial ownership is analyzed by Bulow and Klemperer (1999). Frutos and Pechlivanos (2006) characterize the equilibrium of a second-price common-value auction with double uncertainty about the adversary.

Finally, through the third essay, at least two questions remain: (i) what the average duration of wage contracts is; (ii) whether wage contracts are fixed in the fashion of Calvo or Taylor (1983). The database of wage bargaining contracts in Canada provides some empirical evidence to address these concerns. Previous work uses this database exclusively to study downward nominal wage rigidity such as in Christofides and Stengos (2003).

After investigating empirically the sticky nominal wage, it is of central importance to ask what the optimal monetary policy is when: (i) the central bank and the private sector are asymmetrically informed about the true state of the economy; (ii) the central bank is uncertain how well the private sector is informed, and (iii) *vice versa*. The equilibrium here is in a context of incomplete information. The central bank maximizes social welfare by fixing short term nominal interest rate without knowing for certainty the information in the possession of the private sector. The same argument applies to the private sector. In previous studies, Shmitt-Grohe and Uribe (2007) analyze optimal monetary policy in a complete-information context; Svensson and Woodford (2004) characterize the optimal monetary policy when the central bank and the private sector are asymmetrically informed. However, the double uncertainty, a realistic feature, has not been addressed.

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