Development of an improved model estimating the nutrient content of the bole for four boreal tree species

P. Rochon, D. Paré, and C. Messier

Abstract: An improved model for estimating nutrient contents of the commercial portion of tree boles was developed for four boreal tree species (Populus tremuloides Michx., Betula papyrifera Marsh., Picea glauca (Moench) Voss, and Abies balsamea (L.) Mill.). This model considers the spatial pattern of variation of nutrient concentrations inside the bole and its relationships with tree size. For all species–nutrient combinations, no significant pattern was found for vertical variations in nutrient concentrations, while two types of nonlinear models, using distance from the tree periphery as the independent variable, fit the pattern of horizontal (or radial) variations. These patterns of variability were used to estimate the global nutrient concentration of the bole by using mathematical integration. The values obtained with this method were generally lower, especially for large stems, than values obtained with traditional methods that do not consider the variability of nutrient concentrations inside the bole. This improved model would permit better estimates of the amounts of nutrients lost in biomass upon forest harvesting, as well as internal cycling of nutrients within the bole.

Résumé: Une méthode permettant d’estimer le contenu en nutriments de la portion commerciale du tronc d’un arbre a été développée pour quatre essences de la forêt boréale (Populus tremuloides Michx., Betula papyrifera Marsh., Picea glauca (Moench) Voss et Abies balsamea (L.) Mill.). Cette méthode considère la variabilité spatiale des concentrations de nutriments dans le tronc et sa relation avec la taille de l’arbre. Pour toutes les combinaisons nutriment–essence, aucun patron de variation verticale des concentrations n’était significatif. Par contre deux modèles non linéaires, utilisant la distance de la périphérie du tronc comme variable indépendante, permettent de décrire la variabilité horizontale (ou radiale) des concentrations. Les patrons de variabilité des concentrations d’éléments dans le fût ont été utilisés pour estimer la concentration globale en nutriment du fût à l’aide du calcul intégral. Les valeurs obtenues avec cette méthode sont généralement plus faibles que celles obtenues par des méthodes qui ne tiennent pas compte de la variabilité des concentrations dans le fût, surtout pour les arbres de forte taille. La méthode qui a été développée pourrait permettre de mieux considérer l’importance du cycle interne des nutriments dans le fût et aussi d’estimer de façon plus adéquate les quantités d’éléments perdues avec la biomasse lors de l’exploitation des forêts.

Introduction

The sustainability of forest management practices relies, among other things, on a positive geochemical balance. It is therefore imperative to have adequate estimates of the amounts of nutrients that are lost to harvesting. Forest harvesting can increase hydrological losses of nutrients. However, nutrients being exported in biomass generally represents the largest nutrient output from exploited ecosystems (Mann et al. 1988), and the nutrient content of tree boles represents the minimum amount that will be exported, independently of the harvesting method used.

In the literature, several methods for estimating the nutrient content of boles are found. Some authors have used a single set of nutrient concentrations for stemwood and another for stembark, which are multiplied by the biomass of these components. This is done independently of tree size and with little indication of the sampling design that was used inside the tree to determine nutrient concentrations (Hendrickson et al. 1987; Mentherr et al. 1993). Other studies, such as those involving the process-based ecosystem-level model forcyte-11 (Kimmis et al. 1990), use one set of nutrient concentrations for heartwood and another for sapwood. While this method could yield more accurate results, it is limited by the estimation of the biomass of these two components for which few equations are available. Another approach involves sampling trees of different sizes and the determination of nutrient concentrations for several tree size classes (Chatarpaul et al. 1985). Again, the position inside the tree from which the samples were taken to determine nutrient concentration was not specified. The average concentration of random samples within the bole could yield biased results because the distribution of nutrients in the bole does not follow a normal distribution (Clément and Janin 1976; Myre and Camiré 1994). The distribution of nutrient concentrations inside the bole generally reflects retranslocation from older tissues toward the cambial zone (Bamber and
The probability of vertical variation in nutrient concentration was not taken into account further.

The results were also analyzed graphically to observe patterns of variation that would be nonlinear, but no clear trends could be found. Therefore, vertical variation in nutrient concentration was not taken into account further.

The same analysis was performed on the horizontal plane, and the results indicated that there were significant relationships for all tree-species concentrations. However, the residuals of these regressions were not normally distributed. Nonlinear patterns of horizontal variation in nutrient concentrations in tree boles were obtained in several studies (Myre and Camiré 1994; Colin-Belgrand et al. 1993; Clément and Janin 1976; Bamber and Fukasawa 1985). Myre and Camiré (1994) observed that the concentrations of mobile elements (N, P) fitted a negative exponential function followed by a nil linear regression from the bark to the center of the stem. They also found that non-mobile elements (Ca, Mg) fitted a negative exponential function that was followed by a positive linear regression. These two patterns of variation are illustrated in Figs. 1 and 2 and fit our data well. Concentration of K showed an intermediate pattern between the two types of curve. It was fitted to the second curve type despite a weak slope for the linear portion (Table 2).

The first part of both curves follows an exponential decay pattern (eq. 1):

\[ y = a e^{-(x^2)} \]

where \( y \) represents nutrient concentration at any distance from the 4 cm of bark to the pith using a 0.97-cm drill bit. Again each sample consisted of eight subsamples.

Fresh samples were oven-dried at 70°C in a forced-draught oven. Oven-dried samples were ground in a Wiley mill to pass a 40-mesh screen and subjected to nutrient analysis. Total nitrogen (N) was determined by Kjeldahl distillation using a Tecator Kjeltrec distillation system. For the analysis of phosphorus (P), potassium (K), calcium (Ca), and magnesium (Mg), samples of 1 g were burned for 5 h at 550°C in a muffle furnace and the elements in the ash were recuperated with 10 mL of 2 M HCl and diluted to 50 mL with distilled water (Allen 1989). The P concentrations were determined by colorimetry, while cation concentrations were determined by atomic absorption.

The coefficients for eq. 2 were estimated by the iterative least squares method for all species-nutrient combinations using the Gauss–Newton method in SAS software (SAS Institute Inc. 1985).
For mobile elements the variation becomes nil after the exponential portion. These low concentrations express the lower limit of nutrients left in the heartwood following retranslocation. In the model, this value can be expressed by the addition of a constant $k$.

For immobile nutrients, the variation becomes linear with a positive slope, and another term $(bx + k)$ will be added to eq. 1 to express this trend (eq. 2):

$$ y = a e^{-\left(\frac{x}{B}\right)^2} + bx + k $$

In eq. 2, $a$, $b$, and $k$ are parameters specific to each species-nutrient combination. This equation is simplified for mobile nutrients since parameter $b$ equals zero (eq. 3):

$$ y = a e^{-\left(\frac{x}{B}\right)^2} + k $$

All parameters in eq. 2 were significant under the threshold of $\alpha = 0.05$. The residuals were analyzed graphically in order to detect possible model inadequacies and to examine the validity of regression assumptions. The Shapiro–Wilk test was used to evaluate the normality of the residuals. Only the equation for the P concentration of aspen did not respect this regression assumption. For this reason, this equation was not further considered (Table 2). The values of $R^2$ and $R_{adj}^2$ varied between 0.70 and 0.83 for N, P, and K. They ranged from 0.50 to 0.84 for Ca and Mg (Table 2).
Mathematical integration over three dimensions was performed to evaluate global nutrient concentrations in the tree bole. The merchantable portion of the bole was considered as a truncated cone with an upper diameter of 9 cm. The radial dimension (from the center to the circumference) was the first one considered for integration. Equation 3 expresses nutrient concentrations from the circumference to the center and is inverted to allow integration. Therefore, the variable $x$, expressing the distance from the periphery to the center, is replaced by eq. 4, where $r$ represents the radius at the base of the bole, $H$ represents total height, $h$ represents the height of a sample, and $x$ represents the distance from the periphery:

$$\left(r - \frac{r}{H}h\right)^{-x}$$

The integration of all concentrations over a radius is given in eq. 5:

$$\int_{0}^{r - \frac{r}{H}h - x} A e^{\left(r - \frac{r}{H}h\right)^{2}} + B\left(r - \frac{r}{H}h - x\right) + Kx \, dx$$

Integration is then performed over the second dimension, on a circle that begins at 0 and finishes at $2\pi$ (eq. 6):

$$\int_{0}^{2\pi} \int_{0}^{r - \frac{r}{H}h - x} A e^{\left(r - \frac{r}{H}h\right)^{2}} + B\left(r - \frac{r}{H}h - x\right) + Kx \, dx \, d\theta$$

Integration is performed over the third dimension, which is the height (eq. 7):

$$\int_{0}^{H} \int_{0}^{2\pi} \int_{0}^{r - \frac{r}{H}h - x} A e^{\left(r - \frac{r}{H}h\right)^{2}} + B\left(r - \frac{r}{H}h - x\right) + Kx \, dx \, d\theta \, dh$$

This triple integral is then resolved

$$C = -H\pi A + \frac{1}{12}H\pi r^{3}B + \frac{1}{2}H\pi r^{2}K + \frac{1}{2}H\pi A e^{\frac{A}{2B}} + \frac{1}{2}H(\pi^{1/2})^{3}rA \text{ erf}(r) + \frac{1}{4}(\pi^{1/2})^{3}rA \text{ erf}(r)$$

where $H$ represents the total height and $r$ the radius at the base of the bole. $A$, $B$, and $K$ represent the three parameters of eq. 2.

$\text{erf}(r)$ represents the error function between 0 and $r$ (eq. 9):

$$\text{erf}(r) = \frac{2}{\pi^{1/2}} \int_{0}^{r} e^{-t^{2}} \, dt$$
Fig. 5. Estimated nutrient mass for aspen and birch of different sizes estimated by different authors: ---−−, Chatarpaul et al. (1985); -−−−, Hendrickson et al. (1987), Young (1985) for P in birch; −−−−, this study.

Since this equation integrates all the concentrations in a cone, it is necessary to subtract the top portion of the cone to obtain estimates of nutrient concentrations for the merchantable portion of the bole. Concentration is evaluated for a small cone with a maximum radius of 4.5 cm and a height estimated as proportional to the change in radius along the total tree length. To calculate the height of the small cone, total tree height \((H)\) in eq. 8 is replaced by \(H'\) defined in eq. 10:

\[
H' = 4.5 \frac{H}{r}
\]

where \(r\) represents the radius at the base of the tree. The replacement of \(H\) by \(H'\) gives eq. 11:

\[
C' = -H' \pi A + \frac{1}{12} H' \pi A 4.5^3 B + \frac{1}{3} H' \pi A 4.5^3 K + \frac{1}{2} H' \pi A e 4.5^3
\]

\[+ \frac{1}{2} H' (x^{1/2})^3 4.5 A erf(4.5) + \frac{1}{4} H' (x^{1/2})^3 4.5 A erf(4.5) \]

Therefore, the total of all concentrations in the merchantable portion of the tree bole (conc. \(x\) vol.) can be written (eq. 12).

\[
C_{eq,8} - C_{eq,11}
\]

To calculate the balanced mean concentration for the merchantable tree bole we need to divide the results of eq. 12 by the volume of the truncated bole (eq. 13).

\[
\text{volume} = \frac{1}{3} \pi h (r_1^2 + r_1 r_2 + r_2^2)
\]

where \(r_1\) and \(r_2\) are, respectively, the radius at the base of the tree and the one at the top of bole (4.5 cm) and where \(h\) represents the height of the bole.

To calculate directly the quantity of nutrient in the bole we need to multiply the results of eq. 12 by the volumetric mass.

**Comparison between the developed model and other published models**

The equations that were developed to estimate the nutrient content of a tree bole assumed that the stem was conical and that wood density did not vary inside the bole with either distance from the tree center or height. These equations are valid for the species and site types for which they were developed and for the range of diameters at the tree base ranging from 10 to 50 cm for aspen, 10 to 45 cm for birch, 10 to 45 cm for white spruce, and 10 to 30 cm for balsam fir.

The estimates of nutrient concentrations over the whole bole provided by the equations were compared with estimates obtained using published equations for trees representing a range of tree sizes found on well-drained clay soil of the Abitibi region. In the literature, two types of equations are found. The first type uses a constant nutrient concentration for all sizes of trees and, therefore, ignores retranslocation (Hendrickson et al. 1987; Jokela et al. 1981). The second type uses different nutrient concentrations according to tree size (Chatarpaul et al. 1985). These results are presented in Figs. 3 and 4. The equations developed in the present study showed a decrease in nutrient concentrations with tree size for all nutrients and all species. This decline is exponential when trees vary from 10 to 20 cm in diameter. Thereafter, the decline is linear. The equations of the second type generally follow this pattern, except for Ca and Mg in aspen as estimated by Chatarpaul et al. (1985), which showed the opposite trend for reasons unknown to us. The concentrations estimated by the equations of the present study are generally lower than those of other studies whether they consider variation in nutrient concentrations with tree size or not. The sampling scheme used in other studies is largely unknown to us. Therefore, it is difficult to discuss the reasons for the difference that we observed. It is clear that average nutrient concentrations using trees of different sizes or sampled at different heights to the bole without proper weighting could yield unrealistically high results.

On small trees, concentrations estimated in the present study are often higher than those estimated using equations based on a single nutrient concentration (Hendrickson et al. 1987) (Figs. 3 and 4). Obviously, this is due to the fact that these equations ignore retranslocation, which is important in large trees. This may also be caused by selecting samples to assess nutrient concentrations from relatively large trees.

The variation of nutrient content with tree size for the four species is presented in Figs. 5 and 6. In general, the trends were the same as those observed with nutrient concentrations. On small trees our estimates of nutrient content are generally equal to or greater than those found in other studies. However, with larger trees, our equations generally predict lower nutrient concentrations.
Fig. 6. Estimated nutrient mass for white spruce and balsam fir of different sizes, estimated by different authors: - - -, Hendrickson et al. (1987), Krumlik (1985) for P in balsam fir, Kimmins (1985) for P in white spruce; ——, this study.

Contents, and the difference between the estimates increases with tree sizes.

Conclusions

The method of estimating nutrient content that considers the continuous variability of nutrient concentrations in the bole generates smaller values for most nutrients especially for large trees. Nutrient loss through biomass removal during forest harvesting may therefore be smaller than what has been estimated in the past. Since whole-tree harvesting is still commonly used, estimates of the nutrient content of branches using the same approach could provide interesting results. This method could also be used to evaluate the importance of internal cycling of elements inside the bole. By comparing stands of different ages, this method should also permit us to adequately estimate changes in net nutrient requirement with forest development.

In practice, the method developed could be used on other sites or with other species. Assuming that there is no vertical variation in nutrient concentrations for samples located within a similar distance from the periphery, as it was observed in the present study, a single disk could be sampled at the base of a tree and analyzed. The parameters A, B, and K of eq. 2 must then be found by nonlinear regression iteration techniques. These parameters could then be written to eq. 11 and multiplied by the tree biomass to obtain the global nutrient content.

Acknowledgements

Financial support was provided by a Natural Sciences and Engineering Research Council of Canada strategic grant and by the Quebec Ministry of Natural Resources. We thank Yves Bergeron and Juergen Bauhus for their comments, Claire Vasseur, Benoit Hamel, and Frédérick Rochon for their help in the field and in the laboratory, and Noël Gascon and Guillaume Therien for their help with the mathematical approach.

References


