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Higgs-mediated FCNC in Supersymmetic Models with Large $\tan\beta$.

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Abstract

In the supersymmetric models with nontrivial flavour structure in the soft-breaking sector the exchange of neutral Higgses mediates $\Delta F = 2$ transitions. This mechanism is studied for $\Delta S, \Delta B = 2$ processes and for a generic form of the soft-breaking terms. We find that Higgs-mediated FCNC amplitudes increase very rapidly with $\tan\beta$ and can exceed $SUSY$ box contribution by up to two orders of magnitude when $\tan\beta \sim m_t/m_b$.

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1 Introduction

The supersymmetric version of Standard Model around the electroweak scale has been a subject of very elaborate theoretical investigations during the past two decades [1]. In this article we study Flavour Changing Neutral Current (FCNC) processes associated with Higgs exchange as the consequence of the supersymmetric threshold corrections to the Yukawa interaction in a generic $SUSY$ model.

The interest to the supersymmetric models with large $\tan\beta$ comes from the simple idea to attribute the huge difference between the observed masses m_b and m_t to the possible hierarchy in the Higgs vevs, $v_u/v_d \sim m_t/m_b$. This means that the Yukawa couplings of the top and bottom quarks can be comparable or even equal, $y_b = y_t$. Supersymmetric models with large $\tan\beta$ predict significant contributions to the low energy observables [2] such as the anomalous magnetic moment of the muon, $b \to s\gamma$ branching ratio and other processes with linear dependence of $\tan\beta$. The radiative corrections $\Delta m_{d,s,b}$ to the masses of the Down-type quarks are also significant and may account for up to 100% of the observable masses [3].

As to the potential influence of the soft-breaking sector on the mass splitting in neutral K and B mesons, it is commonly attributed to the box diagram with the superpartners inside and nontrivial flavour structure of the scalar quark mass matrices [4, 5]. The box diagrams provide very important but not the only source of the non-SM contributions to the $\Delta F = 2$ amplitudes. Another class of contributions is related with the supersymmetric corrections to the Higgs potential. These corrections change the familiar Yukawa interaction generated by the superpotential making it similar to generic two-Higgs doublet type of interaction with the presence of H_u^* and H_d^* fields. It is clear that this two-doublet model will possess FCNC mediated by neutral Higgses *if* the squark sector has nontrivial flavour structure.

How large these Higgs-mediated FCNC amplitudes could be? This can be understood qualitatively in a SUSY theory with a generic form of squark mass matrices. In this case a simple estimate of $\Delta F = 2$ amplitudes in the Down quark sector induced by box diagrams gives $(\alpha_s)^2 m_{sq}^{-2}$ times the flavour changing piece in the soft-breaking sector $\Delta m_{ij}^2/m_{sq}^2$ which we take here to be of the order 1. Here m_{sq} is a typical soft-breaking mass. On the other hand, the tree level amplitudes mediated by Higgses behave as $y_b^2 m_{higgs}^{-2}$ times some power of mixing angles between the quark mass basis dictated by the superpotential and the physical basis which includes v_u corrections. These angles can be estimated as

$\Delta m_b/m_b \sim (\alpha_s/3\pi)\mu m_{sq}^{-1}(v_u/v_d)$ and FCNC amplitudes mediated by Higgses are given by the expression:

$$FCNC_{Higgs} \sim \frac{y_b^2}{m_{higgs}^2}\left[\frac{\alpha_s \tan\beta}{3\pi}\frac{\mu}{m_{sq}}\right]^n \tag{1}$$

with $n = 2$ for $\Delta B = 2$ and $n = 4$ for $\Delta S = 2$ processes. To compare it with the box-induced FCNC amplitude we take $\mu \sim m_{higgs} \sim m_{sq}$ and $y_b \sim \tan\beta/60$. As a result, we obtain the following relation between two FCNC mechanisms:

$$\frac{FCNC_{Higgs}}{FCNC_{box}} \sim \left[\frac{\tan\beta}{60\alpha_s}\right]^2\left[\frac{\alpha_s \tan\beta}{3\pi}\right]^n. \tag{2}$$

If $\tan\beta$ is sufficiently large, $FCNC_{Higgs}$ provides the dominant contribution to $\Delta F = 2$ processes in the Down sector and the estimates of the critical values for $\tan\beta$ are:

$$(\tan\beta)_{cr} \sim \sqrt{180\pi} \sim 25 \text{ and } \frac{FCNC_{Higgs}}{FCNC_{box}} \sim \left[\frac{\tan\beta}{25}\right]^4 \text{ for } \Delta B = 2, \tag{3}$$

$$(\tan\beta)_{cr} \sim \left(540\pi^2\alpha_s^{-1}\right)^{1/3} \sim 35 \text{ and } \frac{FCNC_{Higgs}}{FCNC_{box}} \sim \left[\frac{\tan\beta}{35}\right]^6 \text{ for } \Delta S = 2. \tag{4}$$

Although these critical values appear to be quite large, the strong growth of $FCNC_{Higgs}$ with $\tan\beta$ makes the Higgs-mediated amplitudes one to two orders of magnitude larger than the $SUSY$ box contribution when $\tan\beta \sim 60$. The growth with $\tan\beta$ stabilizes at some point and for $\tan\beta \to \infty$ (purely radiative mechanism for M_d) reaches a certain saturation limit.

These qualitative arguments make us believe that the problem of FCNC in the large $\tan\beta$ regime deserves special consideration. In the next section we will construct $\Delta F = 2$ FCNC amplitudes mediated by Higgs exchange and put the limits on the flavour changing terms in the scalar quark mass matrices. This will be followed by the constraints on the minimal supergravity model, supersymmetric $SO(10)$ GUT and supersymmetric left-right models imposed by the Higgs-mediated FCNC.

2 Higgs-mediated FCNC and the limits on the soft-breaking sector

The superpotential of the minimal supersymmetric standard model (MSSM),

$$W = \epsilon_{ij}[-Q^i H_2^j \mathbf{Y}_u^{(0)} U + Q^i H_1^j \mathbf{Y}_d^{(0)} D + L^i H_1^j \mathbf{Y}_e^{(0)} E + \mu H_1^i H_2^j], \tag{5}$$

contains the same number of free dimensionless parameters as the Yukawa sector of the standard model. All the interactions of the Higgs particles with fermions conserve flavour. For simplicity, we choose the basis where $\mathbf{Y}_d^{(0)}$ is taken in the diagonal form, $\mathbf{Y}_d^{(0)} = diag(y_d^{(0)}; y_s^{(0)}; y_b^{(0)})$.

The soft-breaking sector can influence flavour physics. Among different scalar masses, the soft-breaking sector has the squark mass terms

$$\tilde{U}^\dagger \mathbf{M}_U^2 \tilde{U} + \tilde{D}^\dagger \mathbf{M}_D^2 \tilde{D} + \tilde{Q}^\dagger \mathbf{M}_Q^2 \tilde{Q}; \tag{6}$$

and the trilinear terms

$$\epsilon_{ij} \left(-\tilde{Q}^i H_2^j \mathbf{A}_u \tilde{U} + \tilde{Q}^i H_1^j \mathbf{A}_d \tilde{D} \right) \; + H.c.; \tag{7}$$

as the possible sources of flavour transitions.

Below the supersymmetric threshold the Yukawa interaction of the quarks with the Higgs fields has the generic form of the two-Higgs doublet model:

$$\begin{aligned} -\mathcal{L}_Y \;\; = \;\; & \overline{U}_R Q_L H_u (\mathbf{Y}_u^{(0)} + \mathbf{Y}_u^{(1)}) + \overline{U}_R Q_L H_d^* \mathcal{Y}_u + \overline{D}_R Q_L H_d (\mathbf{Y}_d^{(0)} + \mathbf{Y}_d^{(1)}) \\ & + \overline{D}_R Q_L H_u^* \mathcal{Y}_d + h.c., \end{aligned} \tag{8}$$

and $SU(2)$ indices are suppressed here.

The diagrams generating $\mathcal{Y}_d$ are shown in Fig. 1. The radiative corrections are of the order of the tree level contribution if $\tan\beta$ is large and $\mu \sim m_{sq} \sim m_\lambda$ [3]. As a result, the equality of Yukawa couplings for top and bottom quarks, $y_b^{(0)} = y_t^{(0)}$, does not fix $\tan\beta$, making it rather complicated function of $\mu,\ m_{sq}$, m_λ and the relative phase between the tree-level contribution and the radiative correction piece [6]. It is clear that the same diagrams generate the corrections to the mixing angles if flavour can be changed on the squark line.

To find out wether Higgs particles mediate FCNC, we need to know the quark mass matrices,

$$\begin{aligned} \sqrt{2} M_u \;\; &= \;\; (\mathbf{Y}_u^{(0)} + \mathbf{Y}_u^{(1)}) v_u + \mathcal{Y}_u v_d \\ \sqrt{2} M_d \;\; &= \;\; (\mathbf{Y}_d^{(0)} + \mathbf{Y}_d^{(1)}) v_d + \mathcal{Y}_d v_u \simeq \mathbf{Y}_d^{(0)} v_d + \mathcal{Y}_d v, \end{aligned} \tag{9}$$

and the decomposition of the neutral Higgs fields into the physical eigenstates [7]:

$$\begin{aligned} H_u^0 \;\; &= \;\; v_u + H^0 \sin\alpha + h^0 \cos\alpha + iA^0 \cos\beta + iG^0 \sin\beta \\ H_d^0 \;\; &= \;\; v_d + H^0 \cos\alpha - h^0 \sin\alpha + iA^0 \sin\beta - iG^0 \cos\beta. \end{aligned} \tag{10}$$

In the formula for M_d (9) we have neglected the radiative correction piece proportional to v_d and put v_u equal to the SM Higgs vev v.

The angles α and β here are connected as follows:

$$\sin 2\alpha = -\sin 2\beta \frac{m_H^2 + m_h^2}{m_H^2 - m_h^2}; \;\; \cos 2\alpha = -\cos 2\beta \frac{m_A^2 - m_Z^2}{m_H^2 - m_h^2}. \tag{11}$$

The presence or absence of Higgs-mediated FCNC in the Down quark sector depends on the particular forms of the matrices $M_d, \mathbf{Y}_d^{(0)}, \mathbf{Y}_d^{(1)}$ and $\mathcal{Y}_d$. But even before analyzing possible forms of the mass matrices and Yukawa couplings, we are able to present four-fermion operators which can produce leading effects. These are operators which have the square of the initial Yukawa coupling of the $b^{(0)}$-quark, the eigenstate of the Yukawa coupling $\mathbf{Y}_d^{(0)}$ from the superpotential:

$$\begin{aligned} -\mathcal{L}_{4f} \;=\; & \overline{b}_L^{(0)} b_R^{(0)} \overline{b}_R^{(0)} b_L^{(0)} |y_b^{(0)}|^2 \left(\frac{\cos^2\alpha}{m_H^2} + \frac{\sin^2\beta}{m_A^2} + \frac{\sin^2\alpha}{m_h^2} \right) \\ & + \frac{1}{2} \overline{b}_L^{(0)} b_R^{(0)} \overline{b}_L^{(0)} b_R^{(0)} |y_b^{(0)}|^2 \left(\frac{\cos^2\alpha}{m_H^2} - \frac{\sin^2\beta}{m_A^2} + \frac{\sin^2\alpha}{m_h^2} \right) \\ & + \frac{1}{2} \overline{b}_R^{(0)} b_L^{(0)} \overline{b}_R^{(0)} b_L^{(0)} |y_b^{(0)}|^2 \left(\frac{\cos^2\alpha}{m_H^2} - \frac{\sin^2\beta}{m_A^2} + \frac{\sin^2\alpha}{m_h^2} \right) \end{aligned} \tag{12}$$

In the large $\tan\beta$ regime, $\sin^2\alpha \simeq \tan^{-2}\beta$; $m_H^2 \simeq m_A^2$ and therefore the relevant part of this interaction is:

$$\begin{aligned} -\mathcal{L}_{4f} \;=\; & \overline{b}_L^{(0)} b_R^{(0)} \overline{b}_R^{(0)} b_L^{(0)} |y_b^{(0)}|^2 \frac{2}{m_A^2} \\ & + \frac{1}{2} \left(\overline{b}_L^{(0)} b_R^{(0)} \overline{b}_L^{(0)} b_R^{(0)} + \overline{b}_R^{(0)} b_L^{(0)} \overline{b}_R^{(0)} b_L^{(0)} \right) |y_b^{(0)}|^2 \frac{1}{m_h^2 \tan^2\beta}. \end{aligned} \tag{13}$$

Here we have assumed that $m_H,\ m_A \gg m_Z,\ m_h$ and neglected further $1/\tan^2\beta$ and m_h^2/m_H^2 suppressed terms.

The diagonalization of the Down-quark mass matrix (9) by the bi-unitary transformation

$$\sqrt{2} M_d^{diag} = U^\dagger (\mathbf{Y}_d^{(0)} v_d + \mathcal{Y}_d v) V, \tag{14}$$

sets the physical (mass) basis for the Down quarks. Matrix V rotates left-handed fields and, therefore, renormalizes initial Kobayashi-Maskawa matrix $V_{KM}^{(0)}$ provided by the non-commutativity of $\mathbf{Y}_u^{(0)}$ and $\mathbf{Y}_d^{(0)}$ in the superpotential,

$$V_{KM} = V_{KM}^{(0)} V. \tag{15}$$

This by itself can be considered as a certain constraint on V if we assume that no fine tuning occurs in the product of $V_{KM}^{(0)}$ and V so that $|V_{31}| < |(V_{KM})_{td}|$; $|V_{32}| < |(V_{KM})_{ts}|$, etc. As to the right-handed rotation matrix U, it appears absolutely arbitrary and its mixing angles can be limited only through possible FCNC contributions associated with them. The same bi-unitary transformation brings the off-diagonal entries into the four-fermion interaction (14):

$$
\begin{aligned}
-\mathcal{L}_{4f} &= \overline{D}_{Li}D_{Rj}\overline{D}_{Rk}D_{Ll}|y_b^{(0)}|^2V_{3i}^*U_{3j}U_{3k}^*V_{3l}\frac{2}{m_A^2} \\
&+\frac{1}{2}\overline{D}_{Li}D_{Rj}\overline{D}_{Lk}D_{Rl}|y_b^{(0)}|^2V_{3i}^*U_{3j}V_{3k}^*U_{3l}\ \frac{1}{m_h^2\tan^2\beta} \\
&+\frac{1}{2}\overline{D}_{Ri}D_{Lj}\overline{D}_{Rk}D_{Ll}|y_b^{(0)}|^2U_{3i}^*V_{3j}U_{3k}^*V_{3l}\frac{1}{m_h^2\tan^2\beta}
\end{aligned}
\qquad (16)
$$

This interaction allows us to calculate the Higgs exchange contribution to all $\Delta F = 2$ processes in the quark sector. To simplify things further we take m_H^2 and m_A^2 in the ballpark of 500 GeV, so that $m_Z^2 \ll m_H^2$, $m_A^2 \ll m_h^2 \tan^2\beta$ and the first term in Eq. (16) is presumably the dominant. We neglect also possible interference of interaction (16) with supersymmetric and SM box diagrams. Performing standard QCD running of $\overline{q}_Lq_R\overline{q}_Rq_L$ operator from 500 GeV down to the scale of 5 GeV and 1 GeV and taking matrix elements in vacuum insertion approximation, we arrive at the following formulae for mass splitting in the neutral B-mesons, K-mesons and ϵ_K parameter:

$$
\Delta m_B \simeq 1.5\eta_B|y_b^{(0)}|^2\frac{m_Bf_B^2}{m_H^2}|V_{31}^*U_{33}V_{33}U_{31}^*|, \qquad (17)
$$

$$
\Delta m_K = \eta_K|y_b^{(0)}|^2\frac{m_Kf_K^2}{m_H^2}\left(\frac{m_K}{m_s+m_d}\right)^2|V_{31}^*U_{32}V_{32}U_{31}^*|, \qquad (18)
$$

$$
\epsilon_K = \eta_K|y_b^{(0)}|^2\frac{m_Kf_K^2}{\sqrt{2}\Delta m_Km_H^2}\left(\frac{m_K}{m_s+m_d}\right)^2\mathrm{Im}\,(V_{31}^*U_{32}V_{32}U_{31}^*)\,. \qquad (19)
$$

Here $\eta_K \simeq 5$ and $\eta_B \simeq 2.5$ are the QCD renormalization coefficients and the Yukawa coupling $y_b^{(0)}$ is taken at the scale of 500 GeV (At one-loop accuracy, the operator $(\bar{q}_Lq_R)(\bar{q}_Rq_L)$ does not mix with other possible operators). The decay constant are normalized in such a way that $f_K = 160$ MeV and $f_B = 200$ MeV. The comparison with experimental data yields the constraints on the combination of the off-diagonal elements of V and U, Higgs mass and Yukawa coupling $y_b^{(0)}$. These constraints are summarized in Table 1.

Table 1.

Δm_B	$\left(\frac{500\text{ GeV}}{m_H}\right)^2 \lvert y_b^{(0)}\rvert^2 \lvert V_{31}^* U_{33} V_{33} U_{31}^*\rvert < 1.1\cdot 10^{-7}$
Δm_K	$\left(\frac{500\text{ GeV}}{m_H}\right)^2 \lvert y_b^{(0)}\rvert^2 \lvert V_{31}^* U_{32} V_{32} U_{31}^*\rvert < 1.2\cdot 10^{-9}$
ϵ_K	$\left(\frac{500\text{ GeV}}{m_H}\right)^2 \lvert y_b^{(0)}\rvert^2 \mathrm{Im}\,(V_{31}^* U_{32} V_{32} U_{31}^*) < 3.5\cdot 10^{-12}$

If $y_b^{(0)} \sim y_t^{(0)} \simeq 1$ (large $\tan\beta$ regime), it implies very strong constraints on the off-diagonal elements of V and U. These constraints are 'oblique', i.e. the specifics of the soft-breaking sector in different supersymmetric models enters only through V_{13}, U_{23}^*, etc. They are strongly violated if we take $|y_b^{(0)}| \simeq 1$, $|V_{31}| \sim |U_{31}| \sim |V_{ts}|$ and $|V_{32}| \sim |U_{32}| \sim |V_{td}|$. It shows that unlike the eigenvalues of the mass matrices, the mixing angles should not receive more than 10% renormalization from the threshold corrections if both V and U matrices are nontrivial. The numbers on the r.h.s. in Table 1 coincide with the numbers quoted usually as the limits on the "superweak interaction" [8]. Moreover, the tightest constraint from ϵ_K suggests the possibility of having nearly real V_{KM} with CP-violation in the Kaon sector coming from the small phases of the order 10^{-2} in V and U.

For the generic form of the soft-breaking sector, the limits from Table 1 can be converted into the limits on the off-diagonal entries in the Down squark mass matrices $\mathbf{M}_Q^2$ and $\mathbf{M}_D^2$. Treating these entries as the mass insertions on the scalar quark lines and taking also $(\mathbf{M}_Q^2)_{ii} \sim (\mathbf{M}_D^2)_{ii} \sim m_\lambda^2 = m^2$, we are able to calculate V_{ij} and U_{ij},

$$\begin{aligned} V_{3i} &= \frac{1}{3}\frac{\alpha_s v}{3\pi(v_d + (\mu/m)(\alpha_s/3\pi)v)}\frac{\mu}{m}\frac{(\mathbf{M}_Q^2)_{3i}}{m^2} \\ U_{3i} &= \frac{1}{3}\frac{\alpha_s v}{3\pi(v_d + (\mu/m)(\alpha_s/3\pi)v)}\frac{\mu}{m}\frac{(\mathbf{M}_D^2)_{3i}}{m^2} \end{aligned} \tag{20}$$

Here we keep only gluino exchange contribution and neglect diagrams with charginos and neutralinos. The combination $v_d + (\mu/m)(\alpha_s/3\pi)v$ is the 33 element of the mass matrix $\sqrt{2}M_d$, and its moduli squared corresponds to the observable $2m_b^2$ at the scale m. This allows us to modify Eq. (20) so that the combinations of the mixing angles, relevant for $\Delta B = 2$ and $\Delta S = 2$ processes, look as follows:

$$\begin{aligned} |V_{31}U_{31}| &= \left| y_b^{(0)}\frac{1}{3}\frac{\alpha_s}{3\pi y_{SM}}\frac{\mu}{m}\right|^2 \left|\frac{(\mathbf{M}_Q^2)_{31}}{m^2}\frac{(\mathbf{M}_D^2)_{31}}{m^2}\right| \\ V_{31}^* U_{32} V_{32} U_{31}^* &= \left| y_b^{(0)}\frac{1}{3}\frac{\alpha_s}{3\pi y_{SM}}\right|^4 \left(\frac{\mu}{m}\right)^4 \frac{(\mathbf{M}_Q^2)_{31}}{m^2}\frac{(\mathbf{M}_D^2)_{31}}{m^2}\frac{(\mathbf{M}_Q^2)_{32}}{m^2}\frac{(\mathbf{M}_D^2)_{32}}{m^2}. \end{aligned} \tag{21}$$

Here y_{SM} is the SM Yukawa coupling of the b-quark, $y_{SM}(m) = \sqrt{2}m_b(m)/v$. It compensates the smallness coming from the high power of the loop factor, $\alpha_s/3\pi$. We summarize the constraints on the $SUSY$ parameter space for the case of the generic form of the soft-breaking in Table 2.

Table 2.

Δm_B	$\left(\frac{500\ \text{GeV}}{m_H}\right)^2 \lvert y_b^{(0)}\rvert^4 \left\lvert\frac{\mu}{m}\right\rvert^2 \left\lvert\frac{(\mathbf{M}^2_{dL})_{31}}{m^2}\frac{(\mathbf{M}^2_{dR})_{31}}{m^2}\right\rvert < 1.4\cdot 10^{-6}$
Δm_K	$\left(\frac{500\ \text{GeV}}{m_H}\right)^2 \lvert y_b^{(0)}\rvert^6 \left\lvert\frac{\mu}{m}\right\rvert^4 \left\lvert\frac{(\mathbf{M}^2_{Q})_{31}}{m^2}\frac{(\mathbf{M}^2_{D})_{31}}{m^2}\frac{(\mathbf{M}^2_{Q})_{32}}{m^2}\frac{(\mathbf{M}^2_{D})_{32}}{m^2}\right\rvert < 2.0\cdot 10^{-7}$
ϵ_K	$\left(\frac{500\ \text{GeV}}{m_H}\right)^2 \lvert y_b^{(0)}\rvert^6 \mathrm{Im}\left(\left(\frac{\mu}{m}\right)^4 \frac{(\mathbf{M}^2_{Q})_{31}}{m^2}\frac{(\mathbf{M}^2_{D})_{31}}{m^2}\frac{(\mathbf{M}^2_{Q})_{32}}{m^2}\frac{(\mathbf{M}^2_{D})_{32}}{m^2}\right) < 5.5\cdot 10^{-10}$

We take m_λ as the real parameter and therefore we have to include possible phase of μ in the ϵ_K-constraint. These constraints look somehow relaxed as compared to those from Table 1. The reason for that is the presence of additional combinatorial 1/3 in Eqs. (20) as compared to the $\alpha_s/(3\pi)$ factor, characterizing the renormalization of eigenvalues. This combinatorial factor arises from the $n = 1$ order of mass insertions in the squark line and leads to one order of magnitude suppression for Δm_B and two order of magnitudes for Δm_K and ϵ_K. The fourth and sixth power of $y_b^{(0)}$ in Table 2 at low and intermediate $\tan\beta$ correspond to the $\tan^4\beta$ and $\tan^6\beta$ growth of Higgs-mediated FCNC amplitudes claimed earlier in the Introduction.

The constraints on the soft-breaking terms quoted in Table 2 are very sensitive to the value of $y_b^{(0)}$ and μ/m ratio. Nevertheless, they provide valuable limits on the squark flavour sector in the case of large $\tan\beta$, complementary and sometimes much stronger than those from the box diagram if $y_b^{(0)} \sim \mu/m \sim 1$ [5]. The exception are the limits on $(\mathbf{M}^2_Q)_{12}$, $(\mathbf{M}^2_D)_{12}$ entries in the squark mass matrices. The limits on these entries provided by Higgs exchange are relaxed by some power of the ratio $y_s^{(0)}/y_b^{(0)}$and we do not quote them here.

Many specific $SUSY$ models predict certain patterns for the squark mass matrices so that FCNC amplitudes can be calculated in more details and where the comparison with the box diagram contribution can be made.

3 Constraints on the soft-breaking terms in different SUSY models

In what follows we consider the cases of Minimal supergravity model, $SUSY\ LR$ and $SUSY\ SO(10)$ models and so called "effective supersymmetry".

1. Minimal supergravity model

It is customary to assume, at the scale of the breaking, that the following, very restrictive conditions are fulfilled:

$$\mathbf{M}^2_Q = m^2_Q\mathbf{1};\ \ \mathbf{M}^2_D = m^2_D\mathbf{1};\ \ \mathbf{M}^2_U = m^2_U\mathbf{1}\ \ \text{"degeneracy"} \tag{22}$$

$$\mathbf{A}_u = A_u\mathbf{Y}_u;\ \ \mathbf{A}_d = A_d\mathbf{Y}_d\ \ \text{"proportionality"}. \tag{23}$$

These conditions, if held, ensure that the physics of flavour comes entirely from the superpotential. We would refer to this possibility as to the supergravity scenario. Further RG evolution of the soft-breaking parameters, from the scale of the $SUSY$ breaking down to the electroweak scale, induces significant off-diagonal terms in $\mathbf{M}_Q^2$ whereas $\mathbf{M}_D^2$ and $\mathbf{M}_U^2$ stay essentially flavour blind. As a result, no significant right-handed rotation angles can be generated at the threshold, i.e. $|U_{ij}| \sim \delta_{ij}$. The left-handed squark mass matrix is nontrivial, though, and, as a result, 13 and 23 elements of the rotation matrix V are proportional to V_{td} and V_{ts} times the characteristic splitting in left-handed sector induced by RG running, $y_t^2 3\ln(\Lambda/m)/(8\pi^2)$. Thus, the Higgs exchange would induce the operator $\overline{d}_L b_R \overline{d}_L b_R$ proportional to V_{td}^2 which is relevant for the B-meson splitting. Unfortunately, in the case of the Higgs exchange this operator is suppressed by $\tan^{-2}\beta$ (See Eq. (16)) and cannot compete with the box diagram. For the $\Delta S = 2$ processes the degree of suppression is even higher, since the Higgs mediation mechanism involves $\left(y_s^{(0)}/y_b^{(0)}\right)^2$.

2. Effective supersymmetry

The departure from the strict conditions of degeneracy and proportionality may occur in several ways which usually implies significant $SUSY$ contributions to FCNC amplitudes. Here we would skip the theoretical justification for certain choices of the soft-breaking parameters going directly to the phenomenological consequences related with Higgs-mediated FCNC.

We turn now to the "effective supersymmetry" picture [9] which has much less degrees of freedom and where flavour physics can be formulated in a more definitive way. In the squark mass matrices, diagonalized by unitary transformations U and V,

$$\mathbf{M}_Q^2 = \widetilde{V}^\dagger \begin{pmatrix} m_1^2 & 0 & 0 \\ 0 & m_2^2 & 0 \\ 0 & 0 & m_3^2 \end{pmatrix} \widetilde{V}; \quad \mathbf{M}_D^2 = \widetilde{U}^\dagger \begin{pmatrix} m'^2_1 & 0 & 0 \\ 0 & m'^2_2 & 0 \\ 0 & 0 & m'^2_3 \end{pmatrix} \widetilde{U}, \tag{24}$$

the eigenvalues m_1^2, m_2^2, m'^2_1, m'^2_2 are taken to be in the multi-TeV scale and eventually decoupled from the rest of particles. At the same time, the squarks from the third generation are believed to be not heavier than 1 TeV and weakly coupled to the first and second generations of quarks to avoid the excessive fine-tuning in the radiative corrections to the Higgs potential and suppress FCNC contribution to the Kaon mixing. The advantage of this approach in our case is that the same loop integral stands for the renormalization of the Yukawa couplings and mixing angles. Moreover, it is easy to see that matrices $\widetilde{V}$ and $\widetilde{U}$ in Eq. (24) and V and U in Eq. (14) are closely related:

$$|V_{31}U_{31}| = \left| y_b^{(0)} \frac{\alpha_s}{3\pi y_{SM}} \frac{\mu}{m_3} F(m_\lambda/m_3) \right|^2 \left| \widetilde{V}_{31}\widetilde{U}_{31} \right| \simeq 0.7 \left(\frac{y_b^{(0)}\mu}{m_3} \right)^2 \left| \widetilde{V}_{31}\widetilde{U}_{31} \right|$$

$$V_{31}^* U_{32} V_{32} U_{31}^* = \left| y_b^{(0)} \frac{\alpha_s}{3\pi y_{SM}} F(m_\lambda/m_3) \right|^4 \left(\frac{\mu}{m_3} \right)^4 \widetilde{V}_{31}^* \widetilde{U}_{32} \widetilde{V}_{32} \widetilde{U}_{31}^* \simeq 0.5 \left(\frac{y_b^{(0)}\mu}{m_3} \right)^4 \widetilde{V}_{31}^* \widetilde{U}_{32} \widetilde{V}_{32} \widetilde{U}_{31}^*,$$

$$F(x) = \frac{2x}{1-x^2} + \frac{2x^3 \ln x^2}{(1-x^2)^2}, \tag{25}$$

and $F(1)=1$ is substituted in the first and second lines of Eq. (24). These formulae allows us to limit the mixing angles in $\widetilde{V}$ and $\widetilde{U}$ matrices; for $F \simeq 1$ and $y_b^{(0)} \simeq 1$ they are almost as strong as the constraint on V and U from Table 1. Numerically, it is important that the combinatorial suppression from Eq. (21), $(1/3)^2$ and $(1/3)^4$, does not apply here. Table 3 summarizes the limits on the parameter space of the model, provided by $\Delta F = 2$ amplitudes mediated by Higgses

Table 3

Δm_B	$\left(\frac{500\text{ GeV}}{m_H}\right)^2 \lvert y_b^{(0)}\rvert^4 \left\lvert\frac{\mu}{m_3}\right\rvert^2 \left\lvert \widetilde{V}_{31}\widetilde{U}_{31}\right\rvert < 1.6\cdot 10^{-7}$
Δm_K	$\left(\frac{500\text{ GeV}}{m_H}\right)^2 \lvert y_b^{(0)}\rvert^6 \left\lvert\frac{\mu}{m_3}\right\rvert^4 \left\lvert \widetilde{V}_{31}^*\widetilde{U}_{32}\widetilde{V}_{32}\widetilde{U}_{31}^*\right\rvert < 2.4\cdot 10^{-9}$
ϵ_K	$\left(\frac{500\text{ GeV}}{m_H}\right)^2 \lvert y_b^{(0)}\rvert^6 \mathrm{Im}\left(\left(\frac{\mu}{m_3}\right)^4 \widetilde{V}_{31}^*\widetilde{U}_{32}\widetilde{V}_{32}\widetilde{U}_{31}^*\right) < 7.0\cdot 10^{-12}$

These constraints are taken at $F \simeq 1$. In Fig. 2 we plot F and F^4 as a function of the ratio m_λ/m_3 . It is interesting to note that Higgs-mediated amplitudes can probe the region of the parameter space where gluino is considerably heavier than stop and sbottom. All other observables related with dipole amplitudes drop off very fast with m_λ and do not produce significant constraints in this part of the parameter space. The comparison of the box diagram contribution to the $B-\overline{B}$ mixing with the Higgs-mediated mechanism provides the value of the critical $(\tan\beta)_{cr}$ above which the Higgs exchange dominates:

$$(\tan\beta)_{cr} = 17 \tag{26}$$

The numbers from Table 3 suggest also that the observable value of ϵ_K parameter in this model can be achieved through the small imaginary phases of μ and/or $\widetilde{V}_{ij}$, $\widetilde{U}_{ij}$.

3. Supersymmetric SO(10) and left-right models

Another example of $SUSY$ theory that we would like to consider here are the supersymmetric $SO(10)$ [10, 11] and the left-right models [12]. In $SO(10)$ model, left- and right-handed quark superfields are unified at some scale Λ_{GUT} into the same multiplet. Therefore, above the scale of the unification the RG evolution of $\mathbf{M}_Q^2$ and $\mathbf{M}_D^2$ should be the same and these matrices will be equally affected by large Yukawa couplings of the third generation. Therefore, if $\mathbf{M}_Q^2$ exibits non-trivial flavour structure, so does $\mathbf{M}_D^2$. As a result, the squark degeneracy condition is violated both in the left- and right-handed sector and the characteristic splitting is proportional to $y_t^2\ln(\Lambda_{Plank}^2/\Lambda_{GUT}^2)$. In the basis where $\mathbf{Y}_d^{(0)}$ matrix is diagonal, the off-diagonal entries in the squark mass matrices can be related to the mixing angles of the rotation matrices between two basis given by diagonal $\mathbf{Y}_d^{(0)}$ and diagonal $\mathbf{Y}_u^{(0)}$. Further RG evolution, from Λ_{GUT} to the weak scale, deviates $\mathbf{M}_Q^2$ from $\mathbf{M}_D^2$ but the nontrivial part of $\mathbf{M}_D^2$ survives and generates very important phenomenological consequences. It is easy to see, though, that somehow milder assumption about left-right symmetry in the theory with $SU(2)_L\times SU(2)_R\times U(1)$ gauge group can be responsible for exactly the same phenomenology [14]. The role of Λ_{GUT} in this case is played by Λ_{LR}, the scale where left-right symmetry gets spontaneously broken and heavy right-handed gauge bosons decouple.

To study the Higgs-mediated FCNC amplitudes we need to know the initial form of the Yukawa matrix $\mathbf{Y}_d^{(0)}$ in the basis where $\mathbf{Y}_u^{(0)}$ is diagonal. Two choices seem to be justified: hermitian $\mathbf{Y}_d^{(0)}$ [13] or complex symmetric $\mathbf{Y}_d^{(0)}$ [15]. Depending on this choice, right-handed mixing matrix is either exactly the same as KM matrix or equal to the transposed KM matrix times the diagonal matrix with two new physical phases in it. Now we are ready to calculate relevant combinations of mixing angles in terms of the KM matrix elements V_{td} and V_{ts}:

$$\begin{aligned}
|V_{31}U_{31}| &= \left|y_b^{(0)}\frac{\alpha_s}{3\pi y_{SM}}\frac{\mu}{m_3}G(m_\lambda,m_3,m)\right|^2|V_{td}|^2\simeq 7\cdot 10^{-5}\left(\frac{y_b^{(0)}\mu}{m_3}G(m_\lambda,m_3,m)\right)^2\\
|V_{31}^*U_{32}V_{32}U_{31}^*| &= \left|y_b^{(0)}\frac{\alpha_s}{3\pi y_{SM}}\frac{\mu}{m_3}G(m_\lambda,m_3,m)\right|^4|V_{td}|^2\,|V_{ts}|^2 \qquad (27)\\
&\simeq 8\cdot 10^{-8}\left(\frac{y_b^{(0)}\mu}{m_3}G(m_\lambda,m_3,m)\right)^4,
\end{aligned}$$

where we take $V_{td}\simeq 0.01$ and $V_{ts}\simeq 0.04$. The invariant function $G(m_\lambda,m_3,m)$ still depends on many parameters: rihgt-handed sbottom mass, left-handed sbottom mass, masses of squarks from first and second generations and gluino mass. Substituting (27) into the general limits from Table 1, we obtain the following constraints on the parameter space of the $SO(10)$ and left-right symmetric types of models:

Table 4

Δm_B	$\left(\frac{500\ \text{GeV}}{m_H}\right)^2 \lvert y_b^{(0)}\rvert^4\left(\frac{\mu}{m_3}G\right)^2 < 1.6\cdot 10^{-3}$
Δm_K	$\left(\frac{500\ \text{GeV}}{m_H}\right)^2 \lvert y_b^{(0)}\rvert^6\left(\frac{\mu}{m_3}G\right)^4 < 1.5\cdot 10^{-2}$
ϵ_K	$\left(\frac{500\ \text{GeV}}{m_H}\right)^2 \lvert y_b^{(0)}\rvert^6\left(\frac{\mu}{m_3}G\right)^4\sin(\phi_s-\phi_d) < 4.4\cdot 10^{-4}$

The last row in Table 4 refers to the case of the complex symmetric mass matrices and ϕ_s, ϕ_d are the new physical phases associated with the right-handed mixing matrix. The dependence of these phases is the same as in the case of the box-diagram [11]. The significance of these constraints depends very strongly on G. To illustrate this we take the masses of the first and second generation of squarks to be equal to gluino mass and take also the same mass m_3 for the left- and right-handed sbottom. Then, the function G can be reexpressed in terms of the function F, introduced earlier in Eq. (25),

$$G(m_\lambda, m_3, m) = F(m_\lambda/m_3) - \frac{m_3}{m} F(m_\lambda/m) = F(m/m_3) - \frac{m_3}{m} \tag{28}$$

In Fig. 3 we plot G and G^4 as a function of the ratio m/m_3. As in the previous case, there is a significant sensitivity to the part of the parameter space where gluino and first generation of squarks are significantly heavier than m_3. The comparison with the box diagram is also very much dependent of the m/m_3 ratio. If this ratio is large $m/m_3 > 2$, the critical value of $tan\beta$ is approximately the same as in the case of completely decoupled first and second generations of squarks (26).

4 Conclusions

We have considered in details FCNC processes mediated by Higgs particles in the supersymmetric models with large $\tan\beta$. Large $\tan\beta$ and $\mu \sim m_{sq} \sim m_\lambda$ invoke significant renormalization of M_d, mass matrix for the Down-type of quarks. It is evident that the mixing angles can also acquire additional contributions from the threshold corrections if flavour can be changed in the squark sector. When this renormalization occurs both in left- and right- handed rotation matrices, it generates flavour changing operators $\overline{d}_L b_R \overline{d}_R b_L$ and $\overline{d}_L s_R \overline{d}_R s_L$ mediated by heavy Higgs particles H and A. The coefficients in front of these operators are further enhanced by RG evolution to the low-energy scale and, in the case of neutral kaons, by chiral enhancement factor in the matrix elements.

For the models with $\tan\beta \sim O(1)$ the Higgs-mediated FCNC amplitudes are truly marginal because they appear at two or moore loops. The $SUSY$ contribution to FCNC in this case is given by box diagrams. However, Higgs-mediated FCNC amplitudes strongly depend on $\tan\beta$; they grow as the fourth and sixth power of $\tan\beta$ for $\Delta B = 2$ and $\Delta S = 2$ processes respectively. As a result, Higgs-mediated amplitudes can match $SUSY$ box diagrams when $\tan\beta \sim 20$ and become the dominant mechanism for FCNC when $\tan\beta \sim m_t/m_b$. In latter case, Higgs-mediated FCNC amplitudes provide significant constraints on the off-diagonal elements in the squark mass matrices. Very strong limits on the soft-breaking sector are formulated here in Tables 1 and 2. These limits are complementary to those provided by box diagrams [5]. The constraints from Table 1 are oblique, i.e. all the details about the soft-breaking sector are confined in the rotation angles between the mass basis provided by superpotential and the physical mass basis which includes v_u-dependent radiative corrections.

The significance of this mechanism varies from one model to another and depends mainly on the nontrivial structure of the right-handed squark mass matrix. For the minimal supergravity scenario this matrix is trivial and Higgs exchange mechanism is not important for any value of $\tan\beta$. Significant contributions to Δm_K, Δm_B and ϵ_K can be induced in other types of $SUSY$ models where right-handed squark mass matrices are similar to the left-handed ones.

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References

[1] H.P. Nilles, Phys. Rept. **110** (1984) 1; H.E. Haber and G.L. Kane, Phys. Rept. **117** (1985) 75.

[2] A.E. Nelson and L. Randall, Phys. Lett. **B316** (1993) 516; R. Rattazzi and U. Sarid, Nucl. Phys. **B501** (1997) 297; T. Blazek and S. Raby, hep-ph 9712257; H. Baer *et al.*, hep-ph/9712305.

[3] L. Hall, R. Rattazzi and U. Sarid, Phys. Rev. **D50** (1994) 7048; R. Hempfling, Phys. Rev. **D49** (1994) 6168; M. Carena *et al.* Nucl. Phys. **B426** (1995) 269; T. Blazek, S. Raby and S. Pokorski, Phys. Rev. **D52** (1995) 4151.

[4] M. Dugan, B. Grinstein and L. Hall, Nucl. Phys. **B255** (1985) 413.

[5] F. Gabbiani *et. al.*, Nucl. Phys. **B447** (1996) 321; P.H. Chankowski and S. Pokorski, hep-ph/9703442.

[6] C. Hamzaoui and M. Pospelov, hep-ph/9803354.

[7] J.F. Gunion, H.E. Haber, G.L. Kane and S. Dawson, *The Higgs hunter's guide*, Addison-Wesley, 1990.

[8] L. Wolfenstein, Comments Nucl. Part. Phys. **21** (1994) 275.

[9] A.G. Cohen, D.B. Kaplan and A.E. Nelson Phys. Lett. **B388** (1996) 588; A.G. Cohen, D.B. Kaplan, F. Lepeintre, and A.E. Nelson, Phys. Rev. Lett. **78** (1997) 2300.

[10] S. Dimopoulos, L.J. Hall and S. Raby, Phys. Rev. Lett. **68** (1992) 1984, Phys. Rev. **D45** (1992) 4192; P. Ramond, R.G. Roberts and G.G. Ross, Nucl. Phys. **B406** (1993) 19.

[11] R. Barbieri, L.J. Hall and A. Strumia, Nucl. Phys. **B449** (1995) 437.

[12] R. Kuchimanchi and R.N. Mohapatra, Phys. Rev. Lett. **75** (1995) 3989; C. S. Aulakh *et al*, hep-ph/9712551.

[13] R. Mohapatra and A. Razin, Phys. Rev. **D54** (1996) 5835.

[14] M. Pospelov, Phys. Lett. **B391** (1997) 324.

[15] S. Dimopoulos and L.J. Hall, Phys. Lett. **B344** (1995) 185.

Figure captions

Figure 1. The diagrams generating v_u-dependent threshold corrections to $\mathbf{M_d}$.

Figure 2. In the case of decoupled first and second generation of squarks, the invariant functions F and F^4 are plotted against m_λ/m_3 ratio. The Higgs-mediated FCNC amplitudes are sensitive to large m_λ.

Figure 3. $SO(10)$ and left-right $SUSY$ models. G and G^4 are plotted as a function of m/m_3 ratio taking $m = m_\lambda = m_1 = m_2$.

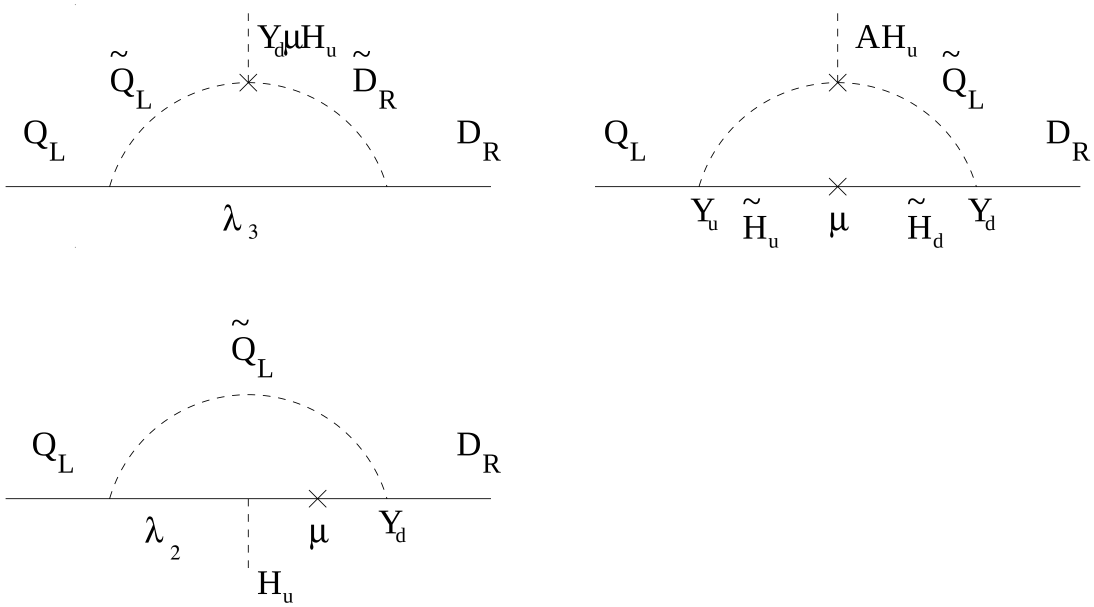

Fig. 1

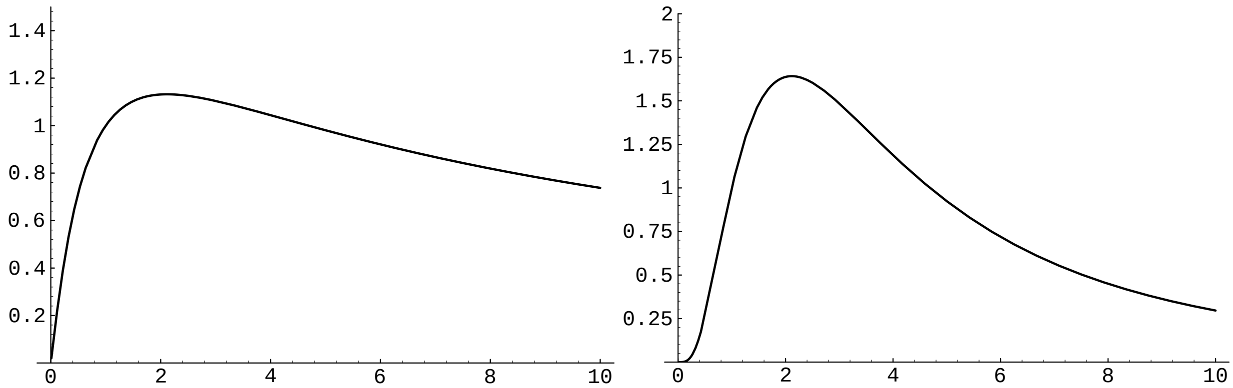

Figure 2

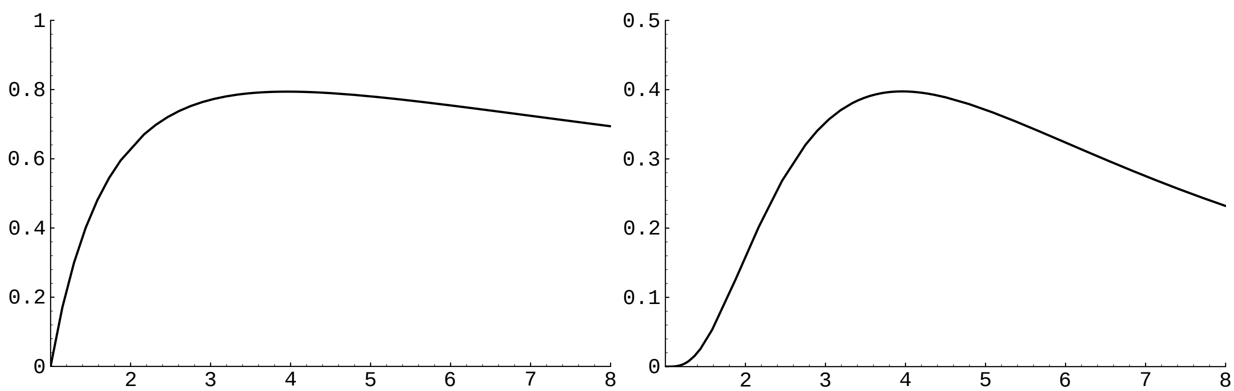

Figure 3