

# Clarification of the Water Saturation Represented on Ice Crystal Growth Diagrams

MÉDÉRIC ST-PIERRE AND JULIE M. THÉRIAULT

*Centre pour l'Etude et la Simulation du Climat à l'Echelle Régionale, Département des Sciences de la Terre et de l'Atmosphère, Université du Québec à Montréal, Montreal, Quebec, Canada*

(Manuscript received 12 December 2014, in final form 1 April 2015)

## ABSTRACT

It has been known for a long time that the shape of ice crystals depends on both the air temperature and the relative humidity of the environment. The relationships among these factors have been summarized in classification diagrams and are intensively referred to in the cloud physics literature. To put in perspective the atmospheric conditions in which the different ice crystal habits grow with respect to the level of saturation in the atmosphere, the vapor density excess and supersaturation with respect to ice at liquid water saturation have been included on those diagrams as a function of air temperature. Over the years, the definition of the water saturation included in those types of diagrams has been misdefined. The goal of this study is to show that an error has been introduced in the definition of the excess of water vapor with respect to ice.

## 1. Introduction

Depending on the atmospheric conditions, ice crystals grow in different shapes such as dendrites, columns, and plates. Nakaya (1954) was the first to relate the shape of ice crystals with the environmental temperature and the supersaturation with respect to ice of the atmosphere. The findings suggested that the type of ice crystal depends mainly on the environmental temperature. These results were obtained in laboratory using a convective-type environment to produce tiny cloud droplets so that the total water density consisted of both liquid droplets and vapor. Shortly after the first crystal growth diagram was developed, Marshall and Langeben (1954) hypothesized that the shape of the ice crystals was also strongly driven by the supersaturated conditions. To test this hypothesis, Kobayashi (1957) used a diffusion chamber without the presence of water droplets. In this case, the ambient water density is only the water vapor density. The results were represented in terms of saturated water vapor density excess with respect to ice.

These results, among other findings, led to two versions of ice crystal classification diagrams, which were

summarized first by Kobayashi (1961) and are shown in Fig. 1. To put in perspective the laboratory findings with respect to the level of saturation in the atmosphere, the vapor density excess and the supersaturation with respect to ice at liquid water saturation, each as a function of air temperature  $T_{\text{air}}$ , were included on those diagrams. The temperature of a growing ice crystal by deposition is, however, generally warmer than the air temperature. Because the air temperature is the abscissa on the classification diagram, one may assume that the atmospheric water conditions representing the saturation vapor pressure correspond to air temperature, which is different than the temperature of the ice crystal  $T_{\text{ice}}$ .

This study aims to clarify the definition of the water saturation curves included on the ice crystal classification diagrams. In particular, we aim to clarify whether the air or the ice crystal temperature was used to compute the supersaturation with respect to ice and the excess of saturated vapor density with respect to ice. Note that the goal of the paper is not related to the uncertainty associated with the temperature and supersaturation thresholds measured in laboratory. Many studies such as Bailey and Hallett (2009), Fletcher (1973), Knight (2012), Pruppacher and Klett (1997), and Curry and Webster (1999) have referred to these ice crystal growth diagrams where that curve is identified as the water saturation or excess of water density with respect to ice without clearly defining it.

---

*Corresponding author address:* Julie M. Thériault, Department of Earth and Atmospheric Sciences, Université du Québec à Montréal, P.O. Box 8888, Montreal QC H3C 3P8, Canada.  
E-mail: theriault.julie@uqam.ca

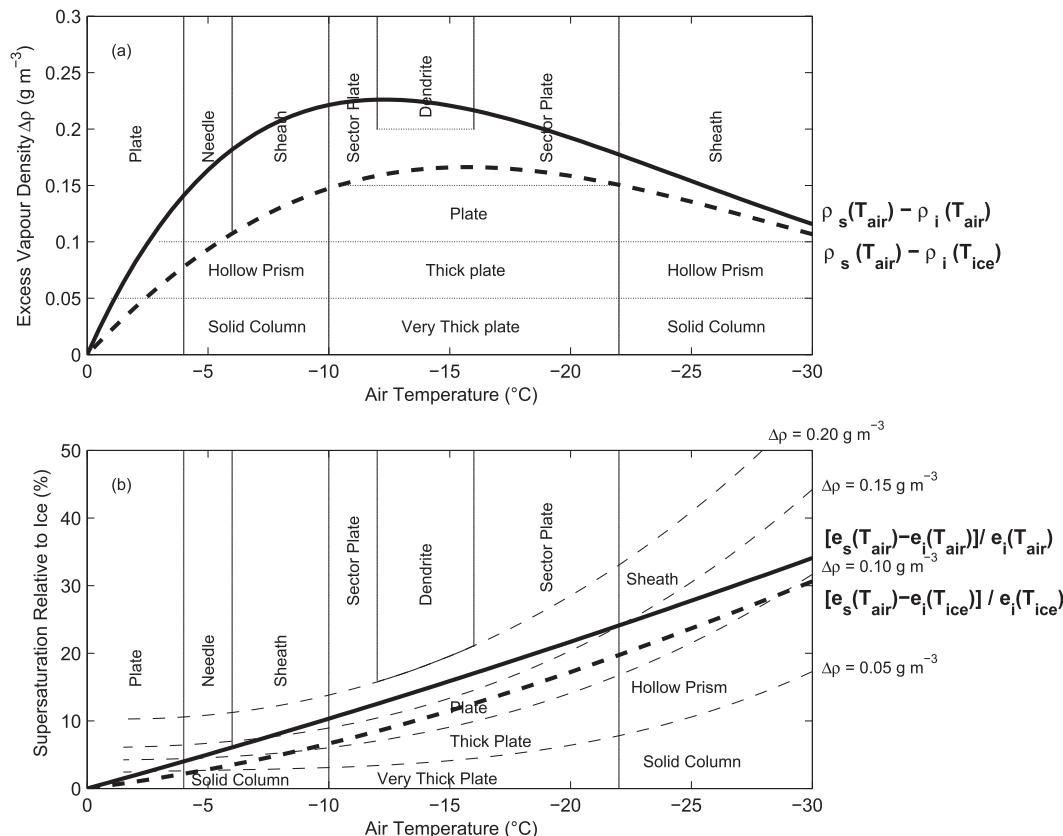


FIG. 1. Classification diagrams of the atmospheric conditions (air temperature and supersaturation) associated with the morphology of ice crystals: (a) the excess of vapor density with respect to ice and (b) the supersaturation with respect to ice. For both panels, it is assumed that the temperature of the ice crystal growing by deposition is the same as the air temperature (thick solid black line) and at a higher temperature than the surroundings (thick dashed black line) [adapted from Kobayashi (1961)].

## 2. Comparison with the Kobayashi (1961) diagram

To verify the definition of the water saturation used on the ice crystal growth diagrams, the excess of saturated vapor density with respect to ice at water equilibrium and water saturation were both computed assuming that the ice crystal is at thermal equilibrium with the environment ( $T_{ice} = T_{air}$ ) and when it is not ( $T_{ice} > T_{air}$ ). Note that an ice crystal is generally warmer than the surrounding environment because of the latent heat release by deposition (appendix). For example, the temperature difference between the environment and the growing ice crystal reaches a maximum of 0.45°C at  $T_{air} = -15^\circ\text{C}$ , which is very small. However, this small temperature difference introduces a 0.06 g m<sup>-3</sup> difference in the saturated vapor density excess with respect to ice, which impacts the maximum value of the water vapor density excess shown in Fig. 1a.

Some differences in the water saturation and vapor density excess curves at both thermal and nonthermal equilibrium are evident when compared with the

Kobayashi (1961) diagrams, as can be seen in Fig. 1. First, the water vapor excess with respect to ice and the water saturation curves found, respectively, in Figs. 1a and 1b are not consistent. To compute the excess of water vapor with respect to ice, the Kobayashi diagram (Fig. 1a) assumed that the ice crystal was not in thermal equilibrium with the environment, which is consistent with the theory proposed by Marshall and Langleben (1954). On the other hand, the other representation of the diagram (Fig. 1b) shows the water saturation when assuming that the ice crystal is at thermal equilibrium. For example, at a temperature of -10°C, the supersaturation with respect to ice at thermal equilibrium is 10%, whereas it is 6% when associated with an ice crystal that is warmer than the surroundings.

## 3. Conclusions

To conclude, our calculations using basic thermodynamic concepts show that an error has been introduced

in the Kobayashi diagram that started approximately in 1961. In particular, the vapor excess with respect to ice (referred to as  $\rho_w - \rho_i$ ; Fig. 1a) considers the temperature of the ice crystal when growing by deposition. The implications of these findings are critical when determining the ice crystal habit with respect to the excess of vapor density. The lower values of the excess of vapor density associated with the formation of, for example, dendrites and needles are associated with subsaturated conditions with respect to liquid water. On the other hand, the implication of these findings is minimal in terms of our understanding of ice crystal growth. For example, in cloud microphysics modeling, the depositional growth considers both the calculation of the saturated vapor density with respect to ice at the crystal temperature and the vapor density with respect to liquid water at air temperature. In the derivation of the depositional growth rate (Byers 1965), the only approximation applied is assuming that the ice crystal temperature is approximately the air temperature. This is an acceptable assumption, because the maximum temperature difference is on the order of  $0.1^\circ\text{C}$ .

*Acknowledgments.* The first author (MS) would like to acknowledge the Faculty of Science at UQAM for an undergraduate fellowship. We thank the *Fonds québécois de la recherche sur la nature et les technologies* for funding this research. Special thanks go to Enrico Torlaschi, Eva Monterio, and Eric Girard for their constructive advice.

## APPENDIX

### Calculation of the Ice Crystal Temperature

This section aims to review the basic calculation of the temperature of the stationary ice crystal growing by deposition, which is described in many cloud physics books such as Rogers and Yau (1988) and Pruppacher and Klett (1997). Note that for ice particles above a few hundred microns in diameter, their fall velocity and ventilation would cool the particle surface whereas riming would increase the surface temperature relative to the stationary case (Heymsfield 1982).

To calculate the temperature of the ice crystal at a given air temperature, we used the diffusion theory found in Marshall and Langleben (1954). Under steady-state conditions, there is a balance between the diffusion transfer of water vapor at the surface of the ice crystal and the latent heat released from phase changes. Based on this assumption, a relationship between the air temperature and the ice crystal temperature is obtained:

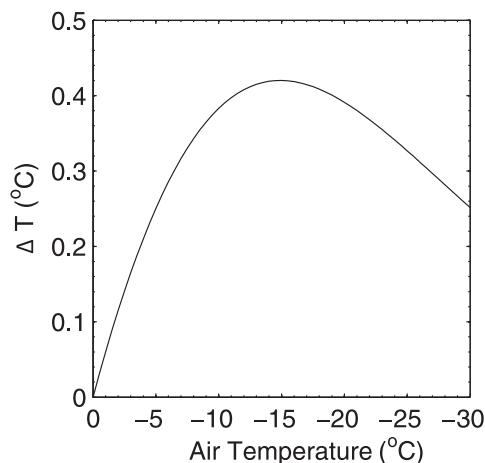


FIG. A1. The temperature difference between the ice crystal and the environment varying with the environmental temperature.

$$\frac{\Delta T}{\rho_i(T_{\text{air}} + \Delta T) - \rho_s(T_{\text{air}})} = -\frac{DL}{K}, \quad (\text{A1})$$

where  $T_{\text{air}}$  is the air temperature,  $\Delta T$  is the temperature difference between the environment and the ice crystal,  $T_{\text{air}} + \Delta T$  is the temperature of the ice crystal,  $\rho_s$  is saturation water vapor density with respect to liquid water,  $\rho_i$  is the saturated vapor density with respect to ice,  $K$  is the coefficient of thermal conductivity,  $D$  is the diffusivity of water vapor in air, and  $L$  is the latent heat of sublimation.

It is possible to solve for  $\Delta T$  at a given  $T_{\text{air}}$  to obtain the temperature of the ice crystal. The temperature difference between the surface of the ice crystal and the environment is given in (Fig. A1). The difference between the environmental temperature increases as temperature decreases to reach a maximum of  $0.45^\circ\text{C}$  at  $T_{\text{air}} = -15^\circ\text{C}$ . Once the temperature of the ice crystal relative to the environmental temperature is obtained, the equation of state is used to calculate the water vapor excess with respect to ice for both air and crystal temperatures with thermal and nonthermal equilibrium, respectively.

Using the ideal gas law, the vapor density excess with respect to ice is

$$\rho_s(T_{\text{air}}) - \rho_i(T_x) = \frac{e_s(T_{\text{air}}) - e_i(T_x)}{R_v T_{\text{air}}}, \quad (\text{A2})$$

where  $x$  is air and ice for air and ice crystal temperature, respectively;  $e_i$  is the saturation vapor pressure with respect to ice; and  $e_s$  is the saturation vapor pressure with respect to liquid water.

Similarly, the supersaturation with respect to ice is computed for the temperatures of both the ice crystal and air using

$$s(T_{\text{air}}, T_x) = \frac{e_s(T_{\text{air}}) - e_i(T_x)}{e_i(T_x)}. \quad (\text{A3})$$

Therefore, Eqs. (A2) and (A3) were used in Figs. 1a and 1b, respectively.

#### REFERENCES

- Bailey, M. P., and J. Hallett, 2009: A comprehensive habit diagram for atmospheric ice crystals: Confirmation from the laboratory, AIRS II, and other field studies. *J. Atmos. Sci.*, **66**, 2888–2899, doi:10.1175/2009JAS2883.1.
- Byers, H. R., 1965: *Elements of Cloud Physics*. University of Chicago Press, 191 pp.
- Curry, J. A., and P. Webster, 1999: *Thermodynamic of Atmosphere and Oceans*. Academic Press, 471 pp.
- Fletcher, N. H., 1973: Dendritic growth of ice crystals. *J. Cryst. Growth*, **20**, 268–272, doi:10.1016/0022-0248(73)90090-0.
- Heymtsfield, A. J., 1982: A comparative study of the rates of development of potential graupel and hail embryos in high plains storms. *J. Atmos. Sci.*, **39**, 2867–2897, doi:10.1175/1520-0469(1982)039<2867:ACSOTR>2.0.CO;2.
- Knight, C. A., 2012: Ice growth from the vapor at  $-5^{\circ}\text{C}$ . *J. Atmos. Sci.*, **69**, 2031–2040, doi:10.1175/JAS-D-11-0287.1.
- Kobayashi, T., 1957: Experimental researches on the snow crystal habit and growth by means of a diffusion cloud chamber. *J. Meteor. Soc. Japan*, **35A**, 38–47.
- , 1961: The growth of snow crystals at low supersaturations. *Philos. Mag.*, **6**, 1363–1370, doi:10.1080/14786436108241231.
- Marshall, J. S., and M. P. Langleben, 1954: A theory of snow-crystal habit and growth. *J. Meteor.*, **11**, 104–120, doi:10.1175/1520-0469(1954)011<0104:ATOSCH>2.0.CO;2.
- Nakaya, U., 1954: *Snow Crystals: Natural and Artificial*. Harvard University Press, 510 pp.
- Pruppacher, H. R., and J. D. Klett, 1997: *Microphysics of Clouds and Precipitation*. Kluwer Academic, 954 pp.
- Rogers, R. R., and M. K. Yau, 1988: *A Short Course in Cloud Physics*. International Series in Natural Philosophy, Butterworth-Heinemann, 304 pp.

Copyright of Journal of the Atmospheric Sciences is the property of American Meteorological Society and its content may not be copied or emailed to multiple sites or posted to a listserv without the copyright holder's express written permission. However, users may print, download, or email articles for individual use.