PHYSICAL REVIEW D **89,** 073019 (2014)

Partial $\mu - \tau$ textures and leptogenesis

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(Received 21 November 2013; published 22 April 2014)

Motivated by the recent results from Daya Bay, Reno and Double Chooz Collaborations, we study the consequences of small departures from exact $\mu - \tau$ symmetry in the neutrino sector, to accommodate a nonvanishing value of the element V_{e3} from the leptonic mixing matrix. Within the seesaw framework, we identify simple patterns of Dirac mass matrices that lead to approximate $\mu - \tau$ symmetric neutrino mass matrices, which are consistent with the neutrino oscillation data and lead to nonvanishing mixing angle V_{e3} as well as precise predictions for the CP-violating phases. We also show that there is a transparent link between neutrino mixing angles and seesaw parameters, which we further explore within the context of leptogenesis as well as double beta decay phenomenology.

DOI: 10.1103/PhysRevD.89.073019 PACS numbers: 14.60.Pq, 11.30.Hv

I. INTRODUCTION

Neutrinos are among the most elusive particles of the standard model (SM) as they mainly interact through weak processes. Nevertheless, a clear picture of the structure of the lepton sector has emerged thanks to the many succesful neutrino and collider experiments over the past decades. The leptonic mixing angles, contrary to the quark mixing angles are large. In fact, the very recent results from T2K [1], Double Chooz [2], RENO [3] and Daya Bay [4] Collaborations confirm that even the smallest of the observed mixing angles, θ_{13}, of the neutrino mixing matrix is not that small.

We start this work with the observation that the data from neutrino oscillations seem to show an approximate symmetry between the second and third lepton families, also referred to as $\mu - \tau$ symmetry [5,6] (see also [7]). Exact $\mu - \tau$ symmetry when implemented at the level of the Majorana neutrino mass matrix S_ν, leads to the following relations between its elements, namely $S_{12} = S_{13}$ and $S_{22} = S_{33}$. This special texture of S_ν as well as different types of corrections to it have been studied largely in the literature [8]. Exact $\mu - \tau$ implemented in the charged lepton basis is also known to lead, among other possibilities, to a vanishing mixing angle V_{e3} and a maximal atmospheric mixing angle $|V_{\mu 3}| = \frac{1}{\sqrt{2}}$.

We would like to put forward some simple deviations from exact $\mu - \tau$ textures for S_ν in the context of the simple seesaw mechanism [9], and we call these partial $\mu - \tau$ textures. To do so we follow a bottom-up approach and construct textures for the Dirac neutrino mass matrix M_D in the limit in which we relax one of the two previous relations coming from the exact $\mu - \tau$ symmetry. Our main goal is to investigate if a small deviation from exact $\mu - \tau$ symmetry is sufficient to generate the whole mixing structure in the lepton sector, including $\mathcal{CP}$ violation, consistent with the existing experimental data on neutrino oscillations.

We also require that the elements of the light neutrino mass matrix S_ν and the Dirac neutrino mass matrix M_D to be independent. As a consequence, we obtain a few allowed simple textures for the Dirac neutrino mass matrix M_D which in turn leads to simple textures for the light neutrino mass matrix. Among the few possibilities allowed, we single out a simple texture and study fully its phenomenological consequences. In particular the chosen texture prefers an inverted spectrum for the three active neutrinos and predicts the value of the Dirac CP-violating phase δ_D. The impact of such type of textures on leptogenesis and neutrinoless double beta decay will then be considered as well as the associated relationship between low energy and high energy CP-violating parameters [10].

II. PARTIAL $\mu - \tau$ SEESAW

We consider the most simple and popular mechanism for generating tiny neutrino masses, namely the seesaw mechanism [9]. In the case of Dirac neutrinos, the analysis is exactly the same as quarks. However for the general case of Majorana neutrinos, one obtains at low energies an effective mass matrix for the light left-handed Majorana which is complex symmetric related to the Dirac mass matrix, M_D, as

$$S_\nu = -M_D M_R^{-1} M_D^T. \tag{1}$$

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We will work in the basis where the Majorana neutrino mass matrix M_R is a diagonal matrix. So we can parametrize its inverse as $M_R^{-1} = \frac{1}{M_1}\,\mathrm{diag}(1, R_{12}, R_{13})$, with the Majorana hierarchy ratios defined as $R_{12} = M_1/M_2$ and $R_{13} = M_1/M_3$.

To study the consequences of any symmetry implemented at the Lagrangian level in the leptonic sector, it is instructive to construct a Dirac mass matrix M_D which leads naturally to a simple partial $\mu-\tau$ symmetric light neutrinos mass matrix S_ν. In general, M_D is an arbitrary complex matrix:

$$M_D = \begin{pmatrix} a & b & c \\ d & e & f \\ g & h & k \end{pmatrix}. \tag{2}$$

This gives us an S_ν of the form

$$S_\nu = -\frac{1}{M_1}\begin{pmatrix} a^2 + R_{12}b^2 + R_{13}c^2 & ad + R_{12}be + R_{13}cf & ag + R_{12}bh + R_{13}ck \\ ad + R_{12}be + R_{13}cf & d^2 + R_{12}e^2 + R_{13}f^2 & dg + R_{12}eh + R_{13}fk \\ ag + R_{12}bh + R_{13}ck & dg + R_{12}eh + R_{13}fk & g^2 + R_{12}h^2 + R_{13}k^2 \end{pmatrix}. \tag{3}$$

An exact $\mu-\tau$ texture happens when $S_{22} = S_{33}$ and $S_{12} = S_{13}$. This texture is known to have the A_4 and D_4 symmetry groups to be their possible underlying family symmetries [11,12]. We therefore evaluate the differences between the elements of S_ν, $(S_{12} - S_{13})$, $(S_{22} - S_{33})$ as well as $(S_{22} - S_{23})$:

$$S_{12} - S_{13} = \frac{1}{M_1}[a(g-d) + R_{12}b(h-e) + R_{13}c(k-f)] \tag{4}$$

$$S_{22} - S_{33} = \frac{1}{M_1}[(g^2-d^2) + R_{12}(h^2-e^2) + R_{13}(k^2-f^2)] \tag{5}$$

$$S_{23} - S_{22} = \frac{1}{M_1}[d(d-g) + R_{12}e(e-h) + R_{13}f(f-k)]. \tag{6}$$

From these equations we note that if we want to reproduce the neutrino mass matrix S_ν in the limit of exact $\mu-\tau$ without forcing relations between the elements of the Dirac mass matrix M_D and those of the heavy Majorana neutrino mass M_R, then we must have the second row of M_D to be equal to its third row, i.e.

$$g = d, \qquad h = e \quad \text{and} \quad k = f. \tag{7}$$

However this strong limit forces the determinant of M_D to vanish which in turn forces the determinant of S_ν to vanish also. This means that at least one of the eigenvalues of S_ν must vanish. This can also be understood from Eq. (6) which shows that the relations from Eq. (7) will produce additional constraints on the symmetric neutrino mass matrix, quite stronger than $\mu-\tau$ symmetry, namely $S_{12} = S_{13}$ and $S_{22} = S_{33} = S_{23}$. The possibility of vanishing eigenvalues is allowed by the data and has been studied by many authors [13–15]. Since this limit constrains strongly our parameter space, we prefer to avoid it and remain as general as possible.

We will therefore consider small deviations from exact $\mu-\tau$ in this seesaw context. In particular we would like to put forward minimal textures for the Dirac mass matrix M_D which maintain at least one of the two $\mu-\tau$ constraints on S_ν, i.e. either $S_{12} = S_{13}$ is kept, with $S_{33} \neq S_{22}$, or $S_{22} = S_{33}$ is maintained with now $S_{13} \neq S_{12}$. We call this type of setup "partial $\mu-\tau$" as it maintains at least one of the original $\mu-\tau$ constraints on the elements of the neutrino mass matrix. In the following, we will only consider the "partial $\mu-\tau$" case $S_{22} = S_{33}$ and $S_{13} \neq S_{12}$, for a specific texture. A complete study of all possible cases with many more examples will be presented elsewhere.

III. PARTIAL $\mu-\tau$ WITH $S_{22} = S_{33}$ AND $S_{11} + S_{12} = S_{22} + S_{23}$

By inspection of Eqs. (4) and (5) we note that to produce the desired deviation from $\mu-\tau$, we have three natural textures which we dub texture I, texture II, and texture III, respectively. Each texture is associated with one of the eigenvalues of M_R^{-1}, such that the breaking of exact $\mu-\tau$ symmetry is proportional to 1 for texture I, to R_{12} for texture II, and to R_{13} for texture III:

$$M_D = \begin{pmatrix} a & b & c \\ d & e & f \\ -d & e & f \end{pmatrix}, \qquad M_D = \begin{pmatrix} a & b & c \\ d & e & f \\ d & -e & f \end{pmatrix},$$
$$M_D = \begin{pmatrix} a & b & c \\ d & e & f \\ d & e & -f \end{pmatrix}. \tag{8}$$

Note the importance of the minus signs which break the degeneracy of some of the entries, allowing the vanishing of $(S_{22} - S_{33})$, but not that of $(S_{13} - S_{12})$. Of course we are interested in small deviations from $\mu-\tau$ symmetry and this approach allows us to control these with the Majorana mass hierarchy parameters R_{12} or R_{13}. In light of the recent results from T2K [1], Double Chooz [2], RENO [3] and Daya Bay [4] Collaborations pointing out to a large θ_{13}, texture I being the largest, by definition, therefore becomes the natural starting point of our study. So we will concentrate our attention to it in what follows. If furthermore we implement the tribimaximal [16] condition, namely, $S_{11}+S_{12}=S_{22}+S_{23}$, an interesting texture emerges for the Dirac mass matrix M_D which in turn gives us a very simple form for S_ν. Now by avoiding relations between the elements of the Dirac mass matrix M_D and those of the heavy Majorana neutrino mass M_R, we obtain two interesting patterns for M_D which satisfy $\mathrm{Det}(M_D)\neq 0$. Taking into account the above features, for instance, we obtain for the texture I the following allowed two patterns:

$$M^I_{D_1}=\begin{pmatrix} a & b & c \\ -a & -\frac{b}{2} & c \\ a & -\frac{b}{2} & c \end{pmatrix} \tag{9}$$

$$M^I_{D_2}=\begin{pmatrix} a & b & c \\ -a & b & -\frac{c}{2} \\ a & b & -\frac{c}{2} \end{pmatrix}. \tag{10}$$

IV. EXAMPLE CASE STUDY: TEXTURE I

We now concentrate on the phenomenology of the first special texture emerging from texture I. In particular, we start with the following texture,

$$M^I_{D_1}=\begin{pmatrix} a & b & c \\ -a & -\frac{b}{2} & c \\ a & -\frac{b}{2} & c \end{pmatrix}. \tag{11}$$

Now we put forward a minimal texture for M_D with the additional requirements of nonvanishing elements $(M_D^\dagger M_D)_{12}$ (or $(M_D^\dagger M_D)_{13}$) and $(M_D^\dagger M_D)_{11}$, necessary for successful leptogenesis as well as nonvanishing determinant of M_D. The goal is to keep the parameter content as minimal as possible while keeping the main features motivated by the partial $\mu-\tau$ ansatz in order to fully describe the neutrino masses, neutrino mixing and $\mathcal{CP}$ violation, as well as the additional possibility of leptogenesis. Taking into account all of this, we further simplify the previous texture by setting $c=b=m_D$ so that in the basis where M_R is diagonal, we have the following texture (and redefining $z=\frac{a}{b}$),

$$M^I_D=m_D\begin{pmatrix} z & 1 & 1 \\ -z & -\frac{1}{2} & 1 \\ z & -\frac{1}{2} & 1 \end{pmatrix}, \tag{12}$$

where m_D sets the Dirac mass scale and its phase is a global unphysical phase. With this parametrization, the resulting light neutrino mass matrix S_ν is given by

$$S_\nu=-\frac{2}{3}\tilde{m}_\nu\begin{pmatrix} \varepsilon+\frac{(3+\eta_M)}{2} & -\varepsilon+\frac{\eta_M}{2} & \varepsilon+\frac{\eta_M}{2} \\ -\varepsilon+\frac{\eta_M}{2} & \varepsilon+\frac{(3+2\eta_M)}{4} & -\varepsilon+\frac{(3+2\eta_M)}{4} \\ \varepsilon+\frac{\eta_M}{2} & -\varepsilon+\frac{(3+2\eta_M)}{4} & \varepsilon+\frac{(3+2\eta_M)}{4} \end{pmatrix}, \tag{13}$$

where we have introduced the parameters ε and η_M defined by

$$\varepsilon=\frac{M_2}{M_1}z^2 \quad \text{and} \quad M_2=\frac{M_3}{2}(1+\eta_M). \tag{14}$$

Both parameters will prove to be important in this ansatz, and they both depend on the hierarchy between two heavy Majorana masses. In particular the parameter η_M denotes the deviation from the special relationship $M_2=\frac{M_3}{2}$ between the two heaviest Majorana neutrino masses. Large deviations from that special relationship will produce physical neutrino mass splittings too large to be phenomenologically acceptable.

We have also defined the light neutrino mass scale $\tilde{m}_\nu$ as

$$\tilde{m}_\nu=\frac{3}{2}\frac{m_D^2}{M_2}, \tag{15}$$

exemplifying the seesaw mechanism at work, since m_D is an electroweak scale mass parameter and M_2 is a heavy Majorana mass of intermediate scale.

The matrix S_ν is diagonalized as

$$U_\nu^\dagger S_\nu U_\nu^*=D_\nu, \tag{16}$$

where

$$U_\nu=P_L V_{CKM} P_R. \tag{17}$$

P_L and P_R are diagonal phase matrices and V_{CKM} [17] is a *CKM*-like mixing matrix with one phase and three angles which can be parametrized as

$$V_{CKM\text{-Like}}=\begin{pmatrix} \times & |V_{e2}| & |V_{e3}|e^{-i\delta_D} \\ \times & \times & |V_{\mu 3}| \\ \times & \times & \times \end{pmatrix}. \tag{18}$$

The phases in P_L can be rotated away in the charged current basis, and the ones in $P_R=\mathrm{diag}(1,e^{i\alpha},e^{i\beta})$ describe Majorana $\mathcal{CP}$-violating phases. The V_{PMNS} [18] mixing matrix is then given by

$$V_{PMNS} = V^{CKM\text{-Like}} P_R. \tag{19}$$

We can now compute the determinant of S_ν in our ansatz, and obtain the simple exact relation,

$$|m_1||m_2||m_3| = \frac{4}{3}|\tilde{m}_\nu|^3(1+\eta_M)|\varepsilon|. \tag{20}$$

With it, we obtain approximate analytical expressions for the mixing angles in the neutrino sector for small enough values of $|\varepsilon|$ and η_M. In particular, we find that

$$V_{e3} = -\frac{2\sqrt{2}}{3}|\varepsilon|e^{-i\theta_\varepsilon} + \mathcal{O}(|\varepsilon|^2), \tag{21}$$

and so, at this expansion order, we can trade the parameter $|\varepsilon|$ by the mixing angle $|V_{e3}|$, and its phase $\theta_\varepsilon = \mathrm{Arg}(z^2)$ is identified as the Dirac phase δ_D, i.e. $\delta_D \simeq \theta_\varepsilon$. We can now express the rest of the mixing entries as expansions in powers of $|V_{e3}|$ and η_M. We find

$$|V_{\mu 3}|^2 \simeq \frac{1}{2} - \frac{1}{2}|V_{e3}|^2 + \mathcal{O}(\eta_M|V_{e3}|, |V_{e3}|^3) \tag{22}$$

and

$$|V_{e2}|^2 \simeq \frac{1}{2} + \frac{1}{r}\left[\frac{|V_{e3}|}{\sqrt{2}}\cos\delta_D + \frac{5}{4}|V_{e3}|^2 - \frac{\eta_M}{3}\right] + \mathcal{O}(\eta_M|V_{e3}|, |V_{e3}|^3), \tag{23}$$

where we have introduced the neutrino mass hierarchy parameter r given by

$$r = \frac{\Delta m_{21}^2}{\Delta m_{13}^2} = \frac{|m_2|^2 - |m_1|^2}{|m_1|^2 - |m_3|^2}. \tag{24}$$

As expected, the value of the atmospheric mixing angle is not far from the exact $\mu-\tau$ symmetry value $|V_{\mu 3}^0|^2 = \frac{1}{2}$ with the deviation being suppressed by the smallness of $|V_{e3}|^2$. Also note that its value must lie in the first octant, i.e. the correction is negative. We show in Fig. 1 the numerical dependence of $|V_{\mu 3}|$ as a function of $|V_{e3}|$, allowing the Dirac phase δ_D to take any value and limiting the possible values of η_M. The simple analytical approximation of Eq. (22) is also shown as a dotted curve and it proves to be a very good approximation when the values of η_M are small, which as we will shortly see happens to be a phenomenological requirement.

The physical neutrino masses predicted by the setup are such that $|m_1|^2 \sim |m_2|^2 \sim |\tilde{m}_\nu|^2$ and

$$|m_3|^2 \simeq 2|V_{e3}|^2|\tilde{m}_\nu|^2, \tag{25}$$

so that the spectrum corresponds to an inverted mass hierarchy spectrum, and the lightness of the lightest

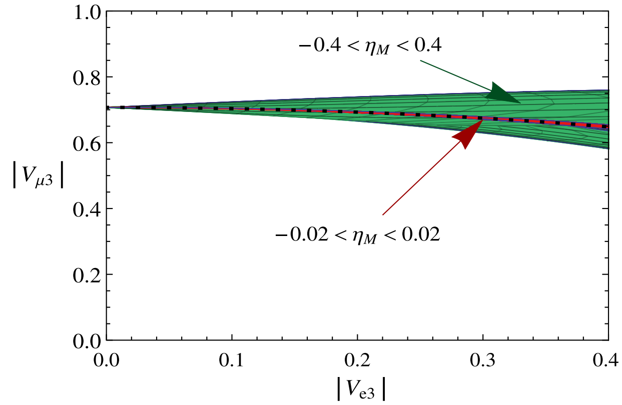

FIG. 1 (color online). Parametric plot of $|V_{\mu 3}|$ with respect to $|V_{e3}|$ varying η_M from $-0.4 < \eta_M < 0.4$ in the large triangular shaded area, and $-0.02 < \eta_M < 0.02$ in the central band marked by dots (the region where acceptable Δm_{21}^2 can be obtained [see Fig. 2)]. The dotted curve is the approximate expression obtained in Eq. (22). The phase $\theta_\varepsilon \simeq \delta_D$ is here allowed the whole range from 0 to 2π, although its value fixes $|V_{e2}|$ (see also Fig. 2).

neutrino ν_3 is explained by the smallness of $|V_{e3}|$. The solar neutrino mass $\Delta m_{21}^2 = |m_2|^2 - |m_1|^2$ is also small, but its expression is a complicated admixture of terms of similar order in η_M, $|V_{e3}|\cos\delta_D$ and $|V_{e3}|^2$.

From Eq. (23) it might seem that for very small η_M and $|V_{e3}|$ the value of $|V_{e2}|^2$ approaches $\frac{1}{2}$. This is not so, since the value of r depends itself on η_M and $|V_{e3}|$. The limiting values for $|V_{e2}|^2$ are

$$\lim_{\eta_M \to 0}|V_{e2}|^2 = 1 \quad \text{or} \quad 0, \tag{26}$$

$$\lim_{|V_{e3}| \to 0}|V_{e2}|^2 = \frac{1}{3} \quad (\eta_M > 0) \tag{27}$$

$$\lim_{|V_{e3}| \to 0}|V_{e2}|^2 = \frac{2}{3} \quad (\eta_M < 0), \tag{28}$$

where the choice of 1 or 0 in the first limit depends on a flip of masses $|m_1|$ and $|m_2|$ controlled by the value of δ_D. The experimentally preferred value of $|V_{e2}|$ is closest to the limit of Eq. (27), meaning that the model naturally produces it when $|V_{e3}|$ is sufficiently small and when η_M is positive. In that limit we have also

$$\lim_{|V_{e3}| \to 0} r = 2|\eta_M|, \tag{29}$$

where $r = \frac{\Delta m_{21}^2}{\Delta m_{13}^2}$, and in that situation we see that the value of η_M (which parametrizes the deviation from the relationship $M_2 = \frac{M_3}{2}$) directly fixes the hierarchy measured between

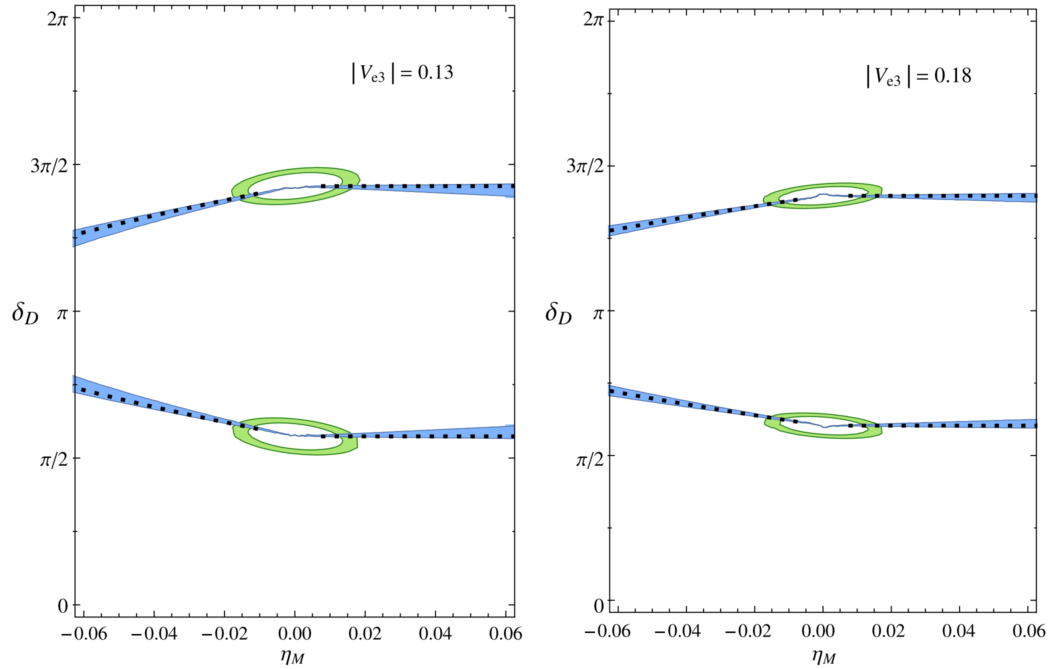

FIG. 2 (color online). Contours in the plane (δ_D, η_M) [where η_M is such that $M_2 = \frac{M_3}{2}(1+\eta_M)$] showing the regions where $0.509 < |V_{e2}| < 0.582$ (blue bands) and where the neutrino mass ratio $r = \frac{\Delta m^2_{21}}{\Delta m^2_{13}}$ is such that $0.0264 < r < 0.036$ (green ellipses). In the left panel we fix $|V_{e3}| = 0.13$ and in the right panel $|V_{e3}| = 0.18$. In both panels, the dotted lines represent the approximation of Eqs. (30) and (31).

the neutrino mass differences, given by $r_{\rm exp} = \frac{\Delta m^2_{21\,\rm exp}}{\Delta m^2_{13\,\rm exp}} \simeq 0.03$ (which would require that $|\eta_M| \sim 0.015$).

Of course, $|V_{e3}|$ does not seem to be so small according to the recent reactor neutrino experiments results, with a value sitting around $|V_{e3}| \sim 0.15$ according to global analysis fits [19–21]. For these larger values of $|V_{e3}|$, the parameters η_M, $|V_{e3}|^2$ and/or $(|V_{e3}|\cos\delta_D)$ can be of the same order and the (nice) tight prediction of $|V_{e2}|$ is lost, as it can now take almost any value. In Fig. 2 we show the regions allowed by the experimental bounds on $|V_{e2}|$ (the blue bands) and r (the green ellipses), in terms of the Dirac phase δ_D and the Majorana mass parameter η_M. The viable regions (the intersections) are quite restricted and point towards small $\eta_M \sim \pm 0.015$ and pretty well constrained values of δ_D. This fact pushes us to try and make further approximate analytical predictions in order to obtain a simple expression for the viable values of δ_D in this ansatz. Since we observe in Fig. 2 that in the viable region of parameter space $r \simeq 2|\eta_M|$, we will use this approximation in Eq. (23) and enforce the tribimaximal value $|V^{tb}_{e2}|^2 = \frac{1}{3}$ as a first-order approximation. We obtain the following constraints on the value of the CP-violating phase δ_D,

$$\cos\delta_D \simeq -\frac{5}{2\sqrt{2}}|V_{e3}| \quad (\eta_M > 0) \tag{30}$$

$$\cos\delta_D \simeq -\frac{5}{2\sqrt{2}}|V_{e3}| - \frac{2}{3}\frac{\eta_M}{|V_{e3}|} \quad (\eta_M < 0). \tag{31}$$

These approximations appear in Fig. 2 in the form of dotted curves, and it is apparent that they fit the numerical results extremely well. This tight prediction of the Dirac phase δ_D as a function of $|V_{e3}|$ (along with the prediction of an inverted spectrum) is a most important element of the ansatz as it can be easily falsified as new neutrino data and global fits further tighten the bounds on leptonic CP violation.

Finally, we compute the rephasing invariant quantity defined as $J = \mathrm{Im}\{V_{e2}V_{\mu3}V^*_{e3}V^*_{\mu2}\}$, which is a measure $\mathcal{CP}$ violation. In our Ansatz it is given by

$$J \simeq \frac{1}{3\sqrt{2}}|V_{e3}|\sin(\delta_D) \tag{32}$$

where we have used $2|\eta_M| \simeq r$ which is observed to fit nicely in the neighborhood of the tribimaximal texture.

V. LEPTOGENESIS AND NEUTRINOLESS DOUBLE BETA DECAY

Now, we will discuss leptogenesis in the present model. For that we will assume that in the early universe, the heavy Majorana neutrinos, N_i, were produced via scattering processes and reached thermal equilibrium at temperatures higher than the seesaw scale. Since the mass term N_iN_i violates the total lepton number by two units, the out-of-equilibrium decay of the right-handed (RH) neutrinos[1] into

[1]We will work in the basis where the mass matrix M_R is a diagonal matrix.

the standard model leptons can be a natural source of lepton asymmetry [22]. The CP asymmetry due to the decay of N_i into a lepton with flavor α reads

$$\epsilon_i^\alpha = \frac{1}{8\pi v^2} \sum_{j\neq i} \frac{\mathrm{Im}[(m_D^+ m_D)_{ij}(m_D^+)_{i\alpha}(m_D)_{\alpha j}]}{(m_D^+ m_D)_{ii}} F(M_i, M_j), \tag{33}$$

where $F(M_i, M_j)$ is the function containing the one-loop vertex and self-energy corrections [23]. For heavy neutrinos far from almost degenerate its expression is given by

$$F(M_i,M_j) = \frac{M_j}{M_i}\left[\frac{M_i^2}{M_i^2 - M_j^2} + 1 - \left(1 + \frac{M_j^2}{M_i^2}\right)\ln\left(1 + \frac{M_i^2}{M_j^2}\right)\right]. \tag{34}$$

As the temperature of the universe cools down to about 100 GeV, sphaleron processes [24] convert the lepton-antilepton asymmetry into a baryon asymmetry [25]. If one takes into account the flavor effects, and assumes that the CP asymmetry is dominated by N_1, then there are three regimes for the generation of the baryon asymmetry [26] (see also [27]):

$$|\eta_B| \simeq \begin{cases} 1\times 10^{-2} \sum\limits_{\alpha=e,\mu,\tau} \epsilon_1^\alpha W(\tilde{m}_1); & (M_1 \geq 10^{12}\ \text{GeV}) \\ 3\times 10^{-3}(\epsilon_1^e + \epsilon_1^\mu) W\left(\frac{417}{589}(\tilde{m}_1^e + \tilde{m}_1^\mu)\right) + \epsilon_1^\tau W\left(\frac{390}{589}(\tilde{m}_1^\tau)\right); & (10^9\ \text{GeV} \leq M_1 \leq 10^{12}\ \text{GeV}) \\ 3\times 10^{-3}\epsilon_1^e W\left(\frac{151}{179}(\tilde{m}_1^e)\right) + \epsilon_1^\mu W\left(\frac{344}{537}(\tilde{m}_1^\mu)\right) + \epsilon_1^\tau W\left(\frac{344}{537}(\tilde{m}_1^\tau)\right); & (M_1 \leq 10^9\ \text{GeV}) \end{cases}, \tag{35}$$

where

$$\tilde{m}_i = \frac{(m_D^+ m_D)_{ii}}{M_i}; \tag{36}$$

$$\tilde{m}_i^\alpha = \frac{(m_D^+)_{i\alpha}(m_D)_{\alpha i}}{M_i}; \qquad \alpha = e, \mu, \tau \tag{37}$$

$$W(x) \simeq \left[\frac{8\times 10^{-3}\ \text{eV}}{x} + \left(\frac{x}{2\times 10^{-4}\ \text{eV}}\right)^{1.16}\right]^{-1}. \tag{38}$$

Note that in the above expressions of $\tilde{m}_i$ and $\tilde{m}_i^\alpha$ there is no summation over repeated indices. The quantity $W(x)$ accounts for the washing out of the total lepton asymmetry due to $\Delta L = 1$ inverse decays. If there is a strong hierarchy between the heavy neutrino masses, i.e. $M_1 \ll M_2 \ll M_3$, the asymmetry is dominated by the out of equilibrium decay of the lightest one, N_1, with $F(M_1, M_{j\neq 1}) \simeq -\frac{3}{2}R_{1j}$. In this case, by using the expressions of the mass matrix $M_D^\dagger M_D$,

$$M_D^\dagger M_D = |m_D|^2 \begin{pmatrix} 3|z|^2 & z^* & z^* \\ z & \frac{3}{2} & 0 \\ z & 0 & 3 \end{pmatrix}, \tag{39}$$

we find that the individual lepton flavor asymmetries are given by

$$\epsilon_1^e \simeq \frac{M_1|\tilde{m}_\nu|(3+\eta_M)\sin(\delta_D)}{48\pi v^2} \tag{40}$$

$$\epsilon_1^\mu = -\epsilon_1^\tau \simeq -\frac{M_1|\tilde{m}_\nu|\eta_M \sin(\delta_D)}{48\pi v^2}. \tag{41}$$

Thus, the high energy CP asymmetry is directly proportional to the CP-violating phase of the effective low energy theory of the neutrino sector. Note that in the present model, $\delta_D \simeq \pi/2$, which allows for the possibility that CP violation could be observed in neutrino (and antineutrino) long baseline oscillation experiments [28–30].

For the case where two of the RH neutrinos, say N_1 and N_2, are almost degenerate, then the function $F(M_i, M_j)$ is dominated by the contribution of the one loop self energy diagram and it is given by [31]

$$F(M_i, M_j) = -\frac{\Delta M_{ij}^2 M_i M_j}{(\Delta M_{ij}^2)^2 + M_i^2 \Gamma_i^2}; \quad i, j = 1, 2. \tag{42}$$

Here $\Delta M_{ij}^2 = (M_j^2 - M_i^2)$ and $\Gamma_i = (m_D^+ m_D)_{ii}/8\pi v^2 M_i$ is the decay width of the ith right-handed neutrino. As a result, the lepton asymmetry produced from the decay of N_1 and N_2 can be considerably enhanced when the mass splitting is of the order of the decay width of $N_{1,2}$. In the strong wash-out regime, the baryon

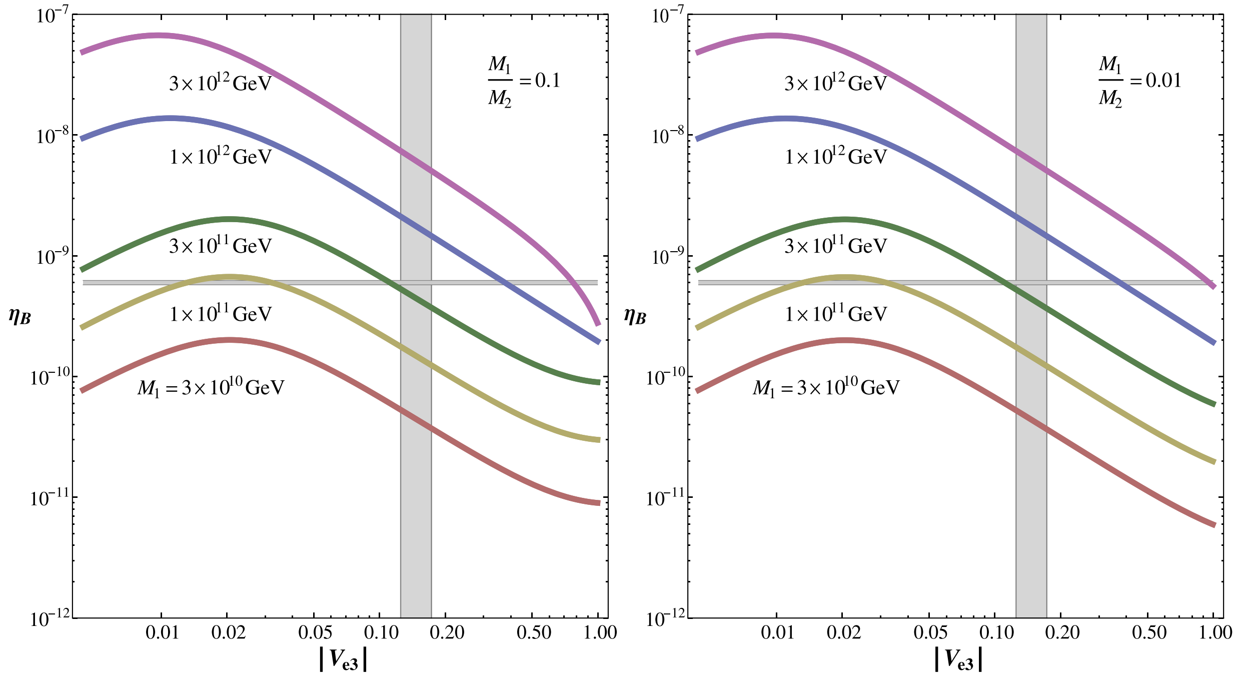

FIG. 3 (color online). Baryon asymmetry produced in our specific scenario as a function of $|V_{e3}|$, in a hierarchical limit for the masses of the two lightest heavy Majorana masses, i.e. $M_1/M_2 = 0.1$ and $M_1/M_2 = 0.01$. The horizontal and vertical bands represent the current experimental bounds on $|\eta_B|$ and $|V_{e3}|$. Interestingly, we observe that the higher the value of $|V_{e3}|$, the higher the required mass of M_1 necessary to generate enough baryon asymmetry.

asymmetry can be estimated using the analytic expression[32,33][2]

$$\eta_B \simeq -2.4\times10^{-2}\sum_{\alpha=e,\mu,\tau}\frac{\sum_{i=1}^{2}\epsilon_i^\alpha}{\sum_{i=1}^{2}K_i^\alpha \ln(25K_i^\alpha)}, \tag{43}$$

where

$$K_i^\alpha = \frac{\Gamma(N_i\to L_\alpha + H^\dagger) + \Gamma(N_i \to \bar{L}_\alpha + H)}{\zeta(3)H_{N_i}} \simeq \left(\frac{\tilde{m}_i^\alpha}{10^{-3}\ \mathrm{eV}}\right) \tag{44}$$

[2]In Eq. (61) in Ref. [34], the expression of the baryon asymmetry for $M_1 \simeq M_2$ and without considering the flavor effect is approximated as

$$\eta_B \simeq -10^{-2}\sum_{\alpha=e,\mu,\tau}(\epsilon_1^\alpha+\epsilon_2^\alpha)\kappa_\alpha(K_1^\alpha+K_2^\alpha),$$

where κ_α is the wash-out factor given by

$$\kappa_\alpha(x)\simeq\frac{2}{(2+4x^{0.13}e^{-2.5/x})x},$$

which is valid in the limit where N_1 and N_2 are almost degenerate [35]. We have checked that the plots of the baryon asymmetry obtained using this expression agree well with the one presented in Fig. 4.

with $H_{N_i} \simeq 1.66\sqrt{g_*}M_i^2/M_{Pl}$ as the Hubble parameter at temperature $T = M_i$, where $M_{\mathrm{Pl}} = 1.2\times10^{19}$ GeV is the Planck mass, and $g_* = 106.75$ is the total number of degrees of freedom. Here the asymmetries ϵ_i^α are calculated using the expression of the function $F(M_i, M_j)$ given in Eq. (42).

We show in Fig. 3 the dependence of the baryon asymmetry on the reactor mixing parameter $|V_{e3}|$ for different values of M_1, ranging from 3×10^{10} to 3×10^{12} GeV with $R_{12} = 0.1$ and $R_{12} = 0.01$ (hierarchical mass limit). We see that successful leptogenesis requires that $M_1 \simeq 3\times10^{11}$ GeV, and also that there is an interesting dependence on $|V_{e3}|$, due to flavor effects, such that smaller values correspond to higher asymmetry. Irrespective of the experimentally allowed values of $|V_{e3}|$, we find that for $M_1 \leq 10^{11}$ GeV, the value of η_B is too small to account for the observed matter-antimatter asymmetry of the universe, due to the strong wash-out effect. In Fig. 4, we make a similar plot for the case of almost degenerate right-handed neutrino spectrum, where we consider $R_{12} = 0.95$ (left panel) and $R_{12} = 0.995$ (right panel). It shows that it is possible to generate a baryon asymmetry in agreement with the observation for M_1 smaller than 10^{11} GeV, thanks to the resonant effect when the masses of N_1 and N_2 are sufficiently close. In that limit, the flavor effects are now different and indeed we observe that the dependence on $|V_{e3}|$ is much milder obtaining basically flat curves, whose heights are increased for values

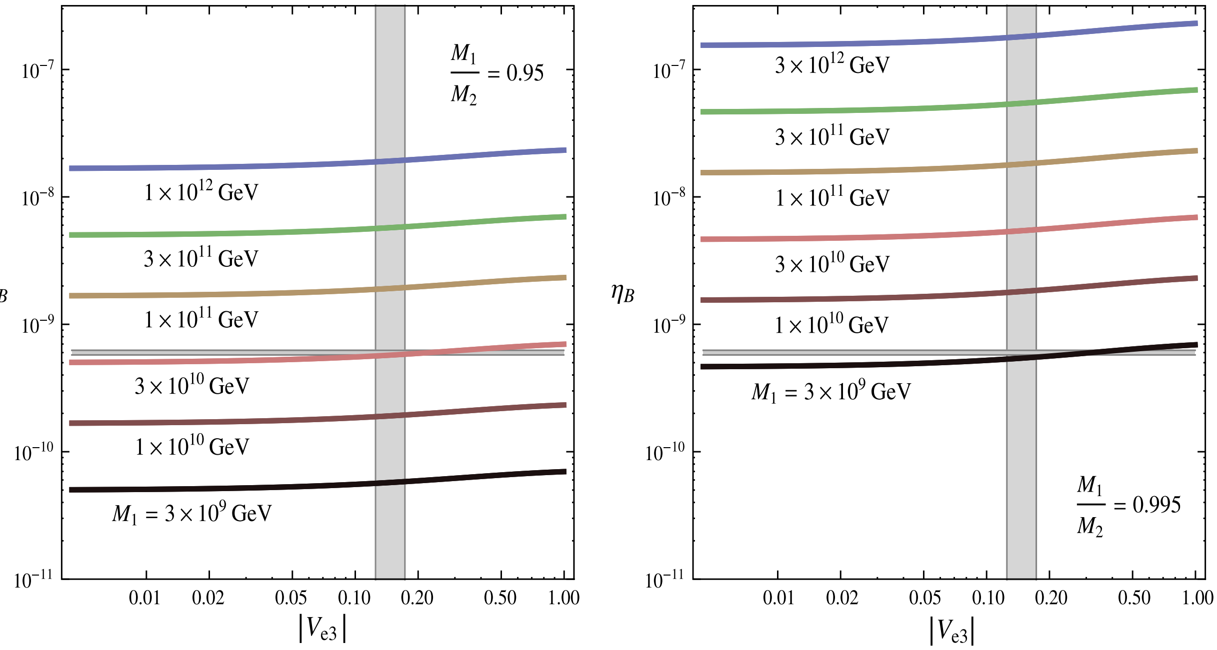

FIG. 4 (color online). Baryon asymmetry produced in our specific scenario as a function of $|V_{e3}|$, in a limit in which the two lightest heavy Majorana masses are nearly degenerate, i.e. $M_1/M_2 = 0.95$ and $M_1/M_2 = 0.995$, thus producing a resonant enhancement of the asymmetry. The horizontal and vertical bands represent the current experimental bounds on $|\eta_B|$ and $|V_{e3}|$. Note that the dependence on $|V_{e3}|$ is much milder than in the nondegenerate case.

of R_{12} closer to 1. For instance, when $R_{12} = 0.95$, a RH neutrino with mass $M_1 \sim 3 \times 10^{10}$ GeV can produce the correct baryon asymmetry. If the degeneracy between M_1 and M_2 is made stronger, as for our choice of $R_{12} = 0.995$, the mass for M_1 is lowered by an order of magnitude to $M_1 \sim 3 \times 10^9$ GeV.

Now, we compute the contribution to the effective mass $m_{\beta\beta}$ which parametrizes the neutrinoless double beta Decay. Note that $m_{\beta\beta} = |S_{11}|$, with S_{11} is given by Eq. (13),

$$m_{\beta\beta}^2 \simeq |\Delta m_{13}^2| \left[1 + \frac{|V_{e3}|\cos(\delta_D)}{\sqrt{2}} + \frac{5|V_{e3}|^2}{4} + \frac{r}{3}\right], \quad (45)$$

where we have used the following expansion for $|\tilde{m}_\nu|^2$ (making use of the approximation $\eta_M \simeq \frac{r}{2}$),

$$|\tilde{m}_\nu|^2 \simeq |\Delta m_{13}^2| \left[1 - \frac{|V_{e3}|\cos(\delta_D)}{\sqrt{2}} + \frac{3|V_{e3}|^2}{4}\right]. \quad (46)$$

Since in this model, the Dirac CP phase is approximately $\pi/2$, we can write

$$m_{\beta\beta} \simeq \sqrt{|\Delta m_{13}^2|}\left(1 + \frac{5|V_{e3}|^2}{8} + \frac{r}{6}\right). \quad (47)$$

Thus, for the mass texture (12), neutrinoless double beta mass parameter is predicted to be $m_{\beta\beta} \simeq 5 \times 10^{-2}$ eV, which is smaller than the current bound by about an order of magnitude. However, experiments such as GERDA, CURO, and MAJORANA with 1 ton.yr exposure will have sensitivity of about 0.03 eV [36], and hence it will be possible to test the above prediction.

VI. CONCLUSION

In this paper we investigated some of the implications of deviating from exact $\mu - \tau$ symmetry assuming that neutrino masses are generated via the seesaw mechanism. A simple parametrization of the Dirac neutrino mass matrix, M_D, with just three parameters, was presented and studied. The scenario is consistent with all neutrino oscillations data and has interesting predictions for some of the observable parameters. We were able to find transparent relations among the different observables of the setup, and in particular the value of the Dirac CP phase happens to be highly constrained as a function of the mixing angle V_{e3}. The dependence of the other mixing angles of the V_{PMNS} mixing matrix in terms of V_{e3} was also obtained. The neutrino masses are also linked directly to the seesaw structure in a very simple way as well as the lepton asymmetry generated out of the decay of the lightest right-handed neutrino. We find that lepton asymmetry is directly proportional to the mixing angle $|V_{e3}|$, which thus has to be nonvanishing to be in agreement with the observed baryon asymmetry of the universe. The Dirac phase happens to be also the relevant phase for leptogenesis, linking low scale CP violation to high scale CP violation in a transparent way. Moreover the predicted value for the Dirac phase (close to $\pi/2$) gives an almost maximal contribution to leptogenesis.

We expect that all the different types of ansatzes that can be considered in our framework of partial $\mu - \tau$ will have similar simple predictions and structures as the one studied here. A thorough investigation is underway and will be the subject of future publication.

ACKNOWLEDGMENTS

One of us (C. H.) would like to thank Zhi-Zhong Xing for useful discussions and acknowledge the support and hospitality of the High Energy Institute in Beijing. C. H. also wishes to thank Michel Lamothe for discussions.

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