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ESSAIS SUR LE CYCLE ÉCONOMIQUE ET LA CROISSANCE SCHUMPÉTÉRIENNE

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RÉSUMÉ

Cette thèse comprend trois chapitres sous forme d'articles, chacun apportant une contribution significative au domaine de la macroéconomie en intégrant une analyse du cycle économique avec la théorie de la croissance endogène.

Dans le Chapitre 1,nous développons un modèle hybride combinant un modèle de croissance endogène schumpétérien avec un modèle de rigidité nominale néo-keynésien, en nous concentrant sur l'interaction entre la croissance tendancielle d'une économie et ses fluctuations cycliques. Pour des fins de tractabilité, une structure de brevets est imposée au modèle dans lequel les droits de propriété intellectuelle d'une technologie expirent après une période. En conséquence, seuls deux niveaux technologiques coexistent à tout moment dans le secteur intermédiaire. Ainsi le modèle de croissance endogène offre une compréhension plus nuancée de la dynamique économique, en particulier en ce qui concerne les fluctuations du cycle économique. Le modèle met l'accent sur l'importance de la $R\&D$ et des effets de retombées, offrant un aperçu des impacts de la politique monétaire, de l'efficacité des investissements et des avancées technologiques. Une analyse de décomposition de la variance souligne l'importance du choc de spillover dans la variabilité des investissements en $R\&D$ et en capital physique. L'étude contribue à la littérature en démontrant le pouvoir explicatif amélioré du modèle de croissance endogène dans l'analyse des cycles économiques et suggère de nouvelles recherches pour affiner ses capacités prédictives.

Le Chapitre 2, "Croissance schumpétérienne, rigidités des prix et cycles économiques", prolonge la discussion sur la croissance schumpétérienne en examinant l'impact des rigidités des prix et des salaires sur les cycles économiques. Il s'appuie sur le modèle du Chapitre 1 en s'affranchissant du système de brevets. Cet ajout permet d'enrichir le modèle en permettant à un continuum de technologies de coexister dans le secteur intermédiaire et donc d'avoir une intéraction plus riche entre la technologie et la rigidité des prix. Les prix sont fonctions d'une technologie, à travers le coût marginal, qui varie dans le temps et à travers le secteur intermédiaire. Le chapitre analyse les caractéristiques communes d'un modèle DSGE néo-keynésien avec des ajouts schumpétériens, étudiant les implications de ces modèles sur les niveaux de prix agrégés, les taux de salaire, la production et les contraintes de ressources. La calibration du modèle est alignée sur les données empiriques, capturant à la fois les éléments néo-keynésiens et schumpétériens. L'assouplissement de la contrainte sur les brevets permet une interaction beaucoup plus riche entre le progrès technologique et les rigidités nominales. Nous constatons que notre modèle reproduit les caractéristiques clés du cycle économique grâce à une calibration standard, capturant efficacement les dynamiques liées à la production, la consommation et les investissements tant dans le capital physique que dans la RD. Cette calibration équilibre le compromis entre précision et temps de traitement. De plus, nous observons que les changements dans la probabilité d'innovation à l'état stable, tout en maintenant constant le taux de croissance global de la production réelle, affectent significativement les réponses dynamiques des variables macroéconomiques. Une probabilité d'innovation plus élevée, qui tend à représenter des avancées technologiques plus modestes, conduit à des retombées technologiques plus larges et à une réduction des rigidités des prix nominaux, à mesure que de nouvelles entreprises établissant des prix optimaux émergent.Nous trouvons également que différentes combinaisons de probabilités d'innovation à l'état stable et d'étendues de retombées de connaissances, même en maintenant un taux de croissance constant pour la frontière technologique, ont des implications profondes pour le bien-être. Spécifiquement, un scénario avec une probabilité d'innovation à l'état stable plus élevée entraîne un niveau de bien-être équivalent à la consommation nettement supérieur. Par exemple, une économie avec une probabilité d'innovation trimestrielle de 23%, par rapport à 15%, produit un impact sur le bien-être $4,7\%$ plus élevé, en tenant

compte des interactions dynamiques. Cela souligne le rôle significatif de la probabilité d'innovation et des retombées de connaissances dans la formation du bien-être économique.

Dans le Chapitre 3, nous présentons une analyse empirique qui s'appuie sur les fondements théoriques établis dans les chapitres précédents. Ce chapitre introduit un taux positif d'inflation tendancielle et les mécanismes d'indexation des prix et des salaires dans le modèle. L'utilisation de l'inférence bayésienne pour l'estimation des paramètres est une contribution méthodologique significative, renforçant la robustesse empirique du chapitre. Les résultats du chapitre, qui valident les modèles théoriques, éclairent la dynamique complexe entre l'innovation, la croissance et les cycles économiques, en particulier sous l'influence de l'inflation tendancielle positive et des mécanismes d'indexation. L'estimation bayésienne dans ce modèle économique néo-keynésien révèle plusieurs résultats clés : les paramètres de Calvo pour la rigidité des prix et des salaires suggèrent une durée moyenne de contrat de 2,5 trimestres. Une forte indexation des prix et des salaires, ainsi qu'une élasticité-prix de la demande indiquant une majoration de 16% par rapport à la tarification concurrentielle, sont observées. L'inflation est inférieure aux 2% anticipés, attribuée à l'impact de la destruction créatrice dans le modèle. La politique monétaire montre une forte réaction à la croissance de la production, avec moins de lissage que prévu. Des paramètres de croissance endogène et une forte persistance dans les paramètres de choc sont notés, avec une suggestion d'étendre la durée des investissements en $R\&D$ pour un modèle plus précis de la persistance économique. Le chapitre conclut en suggérant des avenues potentielles pour des recherches futures, notamment dans l'affinement et l'expansion du cadre empirique utilisé dans l'étude.

Mots-clé: Croissance schumpétérienne, cycle économique, modèles néokeynesiens, rigidités de prix, rigidités de salaires, innovation, croissance, équilibre général, estimation bayésienne, inflation tendancielle, coûts de l'inflation, dynamique de la croissnce, dynamique de l'inflation.

INTRODUCTION

La croissance économique, tout comme les cycles économiques, caractérisés par des fluctuations à court terme de l'activité économique autour d'une tendance à long terme, sont au cœur de l'analyse macroéconomique et ont fait l'objet de nombreuses avancées depuis la Théorie générale de Keynes (2009). L'étude des cycles économiques a donné lieu à la modélisation dans un cadre d'équilibre général dynamique stochastique, avec microfondements, en y intégrant divers types de chocs et de friction. La recherche sur les déterminants de la croissance économique s'est également transformée depuis le modèle de Solow (1956), qui postule un progrès technologique exogène, aux modèles de croissance endogène reposant sur le développement et la diffusion de nouvelles connaissances et de nouvelles technologies.

Pourtant, en conformité avec la pratique adoptée depuis la synthèse néoclassique, des premiers modèles de cycles économiques réels aux modèles macroéconomiques nouveaux keynésiens plus récents, les modèles d'équilibre général dynamique stochastique du cycle économique supposent généralement une croissance exogène d'état stationnaire (e.g., Smets & Wouters (2007), et Justiniano et al. (2010)). Cette hypothèse est certainement commode parce qu'elle permet d'exprimer les variables macroéconomiques en termes de déviation par rapport à une tendance exogène à long terme, ce qui nous permet de centrer strictement une étude sur les fluctuations cycliques. Toutefois, la dissociation des cycles économiques de la tendance de croissance de long terme fait l'hypothèse que des déterminants endogènes de l'évolution de l'économie à long terme ne sont pas pertinents, du moins en première approximation, pour les fluctuations économiques à court terme.

Par contre, une étude empirique comme celle de Comin & Gertler (2006) suggère un lien plus étroit entre les fluctuations économiques à court terme et les moteurs de croissance à long terme, dont la $R\&D$, alors que Fatas (2000) et Barlevy (2007) ont trouvé que les investissements dans l'innovation sont sensibles aux chocs de politique monétaire, ce qui montre que les politiques macroéconomiques peuvent influencer les activités de $R\&D$.

Malgré cela, très peu d'études ont tenté d'intégrer les caractéristiques de la croissance endogène dans les modèles économiques du cycle. Par exemple, Nuño (2011) a étudié un modèle particulier de cycles réels avec croissance schumpétérienne sans rigidités et une fonction de production linéaire. Le modèle d'Annicchiarico & Rossi (2013) considère quant à lui une économie nouvelle keynésienne avec rigidité de prix à la Calvo et des externalités de connaissances avec des rendements d'échelle croissants comme source de croissance endogène. Amano et al. (2012) ont introduit la croissance endogène par le biais d'innovations horizontales dans la variété des biens intermédiaires dans une économie à prix et à salaires échelonnés.

Alors que selon Stiglitz (2018), « la pensée macroéconomique et la synthèse des courants [de pensée] font justement ressortir une vision généralement englobante des enjeux et une analyse intégrée des angles complémentaires de l'analyse macroéconomique » , il nous apparaît qu'une meilleure et pleine compréhension des expansions et contractions périodiques milite en faveur de la prise en compte explicite des facteurs endogènes internes qui déterminent la trajectoire de croissance sous-jacente de l'économie. D'ailleurs, il critique sévèrement les modèles d'équilibre général dynamique stochastique pour leur manque de perspicacité sur les déterminants à moyen et à long terme de la croissance, notamment le rythme de l'innovation et l'accumulation du capital humain. Il souligne ainsi la nécessité d'élaborer des modèles plus complets intégrant les mécanismes de croissance, en particulier ceux pilotés par le progrès technologique et la transmission des connaissances.

La littérature sur la croissance endogène a considéré deux types d'innovation qui présentent des approches distinctes, mais complémentaires de la dynamique de l'innovation comme moteur de croissance : l'innovation horizontale et l'innovation verticale. La première, initialement mise de l'avant par Romer (1990), s'intéresse notamment à l'élargissement de la gamme de choix offerts aux consommateurs. Alors que la croissance économique est stimulée par la diversité parmi des produits et services qui incitent à explorer de nouveaux marchés. La seconde, élaborée par le texte fondateur d'Aghion & Howitt (1992), met de l'avant le rôle clé joué par le principe schumpétérien de destruction créatrice selon lequel le remplacement des technologies obsolètes par des versions plus avancées mène à l'amélioration de la qualité et l'efficacité technologiques des produits et services existants.

Dans cette thèse, nous avons choisi de privilégier l'approche schumpétérienne dans notre intégration de la croissance endogène dans un modèle du cycle économique. L'innovation verticale présente un intérêt particulier dans la mesure où elle permet de générer des progrès technologiques et favorise une concurrence dynamique, deux éléments qui sont particulièrement importants dans les secteurs où la qualité et l'efficacité des produits constituent des facteurs clés de succès. D'ailleurs, poussées par le désir de survie et de rester compétitives, les entreprises tendent à se surpasser pour rester à la pointe de la technologie ou pour gagner une place. L'innovation verticale favorise ainsi une amélioration continue de la productivité et une meilleure utilisation des ressources.

En abordant des questions fondamentales pour la compréhension et la gestion des économies modernes, l'intégration de l'analyse des cycles économiques avec la théorie de la croissance endogène schumpétérienne vise à contribuer à établir une vision plus holistique et nuancée de la dynamique économique.

Premièrement, des variantes d'un modèle intégré du cycle avec croissance schumpétérienne développées dans cette thèse proposent une représentation plus réaliste des processus économiques, alors que les modèles traditionnels, qui traitent souvent la croissance et les cycles comme séparés, échappent des aspects profondément entrelacés de la réalité complexe de la dynamique économique.

Deuxièmement, cette approche intégrée entraîne vraisemblablement des conséquences importantes pour la politique économique. Mieux comprendre l'interaction entre les fluctuations à court terme et la croissance à long terme est essentiel pour les décideurs politiques. Par exemple, les mesures visant à stimuler l'économie en récession peuvent être plus efficaces si elles tiennent compte de leur impact sur la croissance à long terme, notamment par l'innovation. De plus, les politiques visant une croissance à long terme doivent être conscientes des effets cycliques à court terme pour éviter d'exacerber la volatilité économique.

Troisièmement, à une époque où les technologies évoluent rapidement et où la mondialisation est accrue, l'intersection entre le cycle et la croissance économique est devenue plus complexe et plus importante à comprendre.

Un élément central de cette thèse réside dans le rôle crucial de la valeur des entreprises dans la dynamique de l'innovation et des cycles économiques. La capacité des firmes à anticiper les bénéfices futurs de leurs innovations influence directement leurs décisions d'investissement en RD, en agissant comme un moteur essentiel des avancées technologiques. Cette valeur reflète les retombées économiques potentielles des innovations jusqu'à ce qu'elles soient supplantées par des technologies concurrentes, intégrant des facteurs tels que les rigidités des prix et la concurrence au sein du marché. En capturant les interactions entre les dynamiques technologiques, les rigidités nominales et les profits escomptés, cette analyse place la valeur de l'entreprise au cœur de l'étude des fluctuations à court terme et de leur connexion avec la croissance à long terme, établissant ainsi un pont entre les cycles économiques et les moteurs endogènes de la croissance.

Cette thèse comporte trois chapitres sous forme d'article. Le premier chapitre présente un premier modèle hybride intégrant à la fois les principes de la théorie de la croissance endogène schumpétérienne, les rigidités nominales néo-keynésiennes et leur interaction, avec deux niveaux technologiques distincts dans le secteur de la production intermédiaire opérant via l'existence d'un système de brevets. Le deuxième chapitre élargit l'étude de l'interaction entre la croissance schumpétérienne et l'impact des rigidités des prix et des salaires sur les cycles économiques, et ouvre la voie à une interaction plus complexe et diversifiée entre la technologie et la rigidité des prix. Il explore également l'impact de variations dans la probabilité d'innovation à l'état stationnaire sur les réponses macroéconomiques des variables et le bien-être économique. Enfin, alors que les modèles des deux premiers chapitres s'appuient sur un étalonnage des paramètres clés du modèle, inspiré de la littérature et de statistiques clefs, le modèle du troisième chapitre procède à l'estimation bayésienne d'une version étendue du chapitre deux et réalise une première vérification empirique d'un modèle du cycle économique avec croissance schumpétérienne.

CHAPTER 1

BRIDGING GROWTH AND CYCLES: SCHUMPETERIAN INSIGHTS IN NEW-KEYNESIAN FRAMEWORKS

ABSTRACT

Despite the development of contemporary endogenous growth models, the widely used DSGE models predominantly rely on exogenous neoclassical long-term growth and focus solely on fluctuations around that trend growth. In contrast, we propose a model of economic fluctuations incorporating New Keynesian nominal rigidities and common shocks, integrating the features of a Schumpeterian endogenous growth model to capture the interactions between an economy's trend growth and its cyclical fluctuations. The innovation process is embedded within the intermediate production sector, generating a dynamic not commonly observed in New Keynesian models while maintaining nominal wage and price stickiness, and examining the impact of productivity, spillover, and monetary shocks.

We demonstrate that endogenous decisions to invest in R&D have implications that influence the likelihood of innovating and pushing the technological frontier, while adding a significant transmission channel. The implications of the model on business cycle characteristics (such as volatility, co-movements, and persistence in real variables and inflation) are emphasized. Furthermore, considering the mechanics of innovation provides support and microfoundations for the monopolistic competition de facto introduced in New Keynesian models. Lastly, our hybrid model highlights and addresses new challenges at the modelling and simulation stages, including the interaction between prices and the innovation process and the addition of new parameters to capture the dynamics of endogenous growth.

KEY WORDS: Schumpeterian endogenous growth; Business cycles; New Keynesian dynamic stochastic general equilibrium (DSGE) model.

JEL CODE: E32, E52, O31, O33, O42

1.1 Introduction

Investing in research and development (R&D) is a critical driver of innovation, which lies at the heart of a knowledge-based economy and provides an endogenous foundation for secular growth. This concept is supported by the second generation of growth models developed by Romer (1986, 1990), Lucas (1988), Rebelo (1991), and Aghion & Howitt (1992), among others. However, most general equilibrium models in the literature tend to overlook this aspect in relation to business cycles, which may not be inconsequential.

For instance, Comin & Gertler (2006) demonstrate that R&D effects are indeed significant, not only for long-term growth as previously believed, but also at business cycle frequencies. By excluding the innovation process from modern general equilibrium models, these models are deprived of a propagation mechanism. Moreover, Barlevy (2007) and Fatas (2000) both illustrate that investments in innovation are sensitive to monetary policy shocks. Introducing a labor and capital augmenting innovation process could partially endogenize technology. In combination with the monetary policy effect on R&D, this may further reduce the contribution to business cycles attributed to the neutral technology shock.

Yet, until recently, only a limited number of studies attempted to merge a business cycle model with features of a Schumpeterian growth model. From early real business cycle (RBC) models to more recent New-Keynesian (NK) macroeconomic models, the typical dynamic stochastic general equilibrium (DSGE) model has been constructed around a classical exogenous growth model. Seminal articles, such as Smets & Wouters (2007) and Justiniano et al. (2010), feature calibrated exogenous growth. Meanwhile, Nuño (2011) introduces Schumpeterian innovation in an RBC model, albeit employing specific functional forms and no nominal rigidity. Then, in a NK model with Calvo staggered prices and wages, as well as endogenous growth through non-rival access to knowledge, Annicchiarico et al. (2011) analyze the relationship between monetary volatility, growth, nominal rigidities, and the persistence of monetary shocks on a Taylor monetary policy rule. Amano et al. (2012) investigates the implications of endogenous growth with horizontal innovations in the variety of intermediate goods for the welfare costs of inflation in NK economies with nominal rigidities modelled as Taylor (1980) staggered price and wage contracts. Annicchiarico & Rossi (2013) explore optimal monetary policy in an NK economy characterized by Calvo staggered prices, which incorporates endogenous growth induced by knowledge externalities with increasing returns-to-scale. Annicchiarico & Pelloni (2014) examine how nominal rigidities influence uncertainty on long-term growth when prices and wages are preset with a one-period lag, assuming constant returns to the level of technology in the innovation activity and a model with only labor as an endogenous input that is divided between producing output or R&D. To evaluate the sources of productivity slowdown following the Great Recession, Anzoategui et al. (2017) construct and estimate a DSGE model with staggered Calvo contracts driving sluggish adjustments of wages and final-good prices, which also features endogenous growth *via* the expanding variety of intermediate goods resulting from public learning-by-doing in the R&D process and an endogenous pace of technology adoption.

In this paper, we argue that incorporating Schumpeterian growth features is crucial for several reasons. First, the endogenous decision to invest in R&D affects the likelihood of advancing the technological frontier and influences the entry and exit of firms. Consequently, the Schumpeterian dimension of our model, which includes Harrod-neutral technical progress in the production function for goods and a decreasing return-to-scale innovation production function, adds a relevant transmission channel for understanding economic fluctuations and the impact of both real and monetary disturbances. In particular, we consider its implications for the volatility, comovements, and persistence of real variables, since this is a prerequisite before examining the policy implications of such a model in future work. Second, this dimension provides some microfoundations to monopolistic competition that has been introduced *de facto* in NK models, as differing levels of technological advancement in the intermediate sector justify existing market power. Third, our hybrid model highlights and addresses new challenges at the modelling and simulation stages when considering the implications of price rigidities on R&D investments. This follows from the sluggish dynamics of prices directly impact the expected discounted value derived from innovations.

Therefore, our objective is to jointly account for endogenous growth through creative destruction and business cycles in an extended NK model. The proposed model consists of the following groups of agents: households, final good producers, employment agency, intermediate good producers, entrepreneurs/innovators and a monetary authority. Forward-looking households maximize their expected utility concerning their sequence of budget constraints by making optimal decisions regarding their time-paths for consumption, labor, utilization of physical capital, private investment, and net bond holdings. Final good producers operate in a perfectly competitive market and use intermediate goods as input. An employment agency aggregates households' specialized labor into homogeneous labor utilized by intermediate good producers. These intermediate good producers operate in a monopolistically competitive setting that allows them to set prices. Entrepreneurs/innovators within the intermediate sector invest final goods to increase their odds of pushing the technological frontier, allowing an intermediate good producer that implements the new technology to replace the incumbent producer in their respective intermediate sector. Finally, prices and wages are subjected to nominal rigidities through contracts *à la* Calvo (1983). Hence, sluggish price adjustments interact directly with the innovation process, as the discounted expected value of investing in R&D influences the rate of innovation over business cycles.

While monetary policy follows a Taylor rule up to a stochastic deviation, we also examine the impacts of the following shocks: a transitory technological shock, a knowledge-spillover shock, and a marginal efficiency of investment shock.

By incorporating an explicit innovation process and its interaction with the producers of intermediate goods that serve as inputs to the production of the final consumption good, we draw attention to an additional propagation mechanism for shocks. As expected, investment in R&D is sensitive to monetary policy shocks. A positive shock increases investments in R&D, which in turn positively affect the marginal productivities of labor and capital. This ultimately influences the growth rates and the levels of macroeconomic prices and quantities, including factor demand and wages.

In Section 2 of this paper, we delve into a comprehensive analysis of the model's attributes. Initially, we explore elements commonly found in contemporary New Keynesian Dynamic Stochastic General Equilibrium (NK-DSGE) frameworks, adapting them to suit the dynamics specific to firms engaged in innovation. Subsequently, we shift our focus to critical features influenced by endogenous Schumpeterian mechanisms associated with innovation processes. This section also includes a discussion on the aggregation process and the overall equilibrium of the model. Section 3 delineates the model's steady-state properties, while Section 4 addresses the calibration techniques and identifies various disturbances influencing our economic model. In Section 5, we present the model's impulse response functions and statistical moments, advancing towards a thorough business cycle analysis. The paper concludes in Section 6 with a summary of key insights and potential directions for future research.

1.2 The model

In this section, we present the environment and the problems faced by various types of agents. First, we describe the characteristics of the final good producer, the employment agency, and the households' problems. These are largely similar to the standard setup found in the modern dynamic stochastic general equilibrium literature. As needed, we introduce features arising from the existence of innovating firms ultimately owned by households. Second, we focus on the specificities brought by Schumpeterian considerations for innovators and intermediate goods producers, particularly with sluggish adjustment in prices. Finally, we address the aggregation issues and present the monetary authority's policy function.

The final consumption good is produced by a representative firm that operates in a perfectly competitive setting and that aggregates a continuum of intermediate goods according to a specific production function:

$$Y_t = \left(\int_0^1 Y_t(i)^{\frac{\epsilon-1}{\epsilon}} di \right)^{\frac{\epsilon}{\epsilon-1}} , \tag{1.1}$$

where ϵ is the elasticity of substitution between intermediate goods.

The final-good producer takes as given the price of its final output and the prices of the intermediate goods. Its profit maximization problem yields the demand for the i-th intermediate good as a negative function of its relative price:

$$Y_t(i) = \left(\frac{P_t(i)}{P_t} \right)^{-\epsilon} Y_t . \tag{1.2}$$

In our model economy, a continuum of households possesses different skills and offers specialized labor that gives them some degree of market power in setting wages. A representative employment agency aggregates specialized labor and turns it into the combined labor input employed by the intermediate firms:

$$L_t = \left(\int_0^1 L_t(j)^{\frac{\gamma-1}{\gamma}} dj \right)^{\frac{\gamma}{\gamma-1}} , \tag{1.3}$$

where γ is the elasticity of substitution between labor types.

Operating in perfect competition, the employment agency maximizes its profits with respect to specialized labor while taking as given the aggregate wage rate and the prevailing labor compensation specific to each labor type:

$$L_t(j) = \left(\frac{W_t(j)}{W_t}\right)^{-\gamma} L_t \, . \tag{1.4}$$

A type j household faces a budget constraint for each period $t+s$, with $s = \{0, 1, ...\}$, of the form:

$$P_{t+s}C_{t+s} + P_{t+s}I_{t+s} + P_{t+s}a(u_{t+s})\bar{K}_{t+s} + \frac{B_{t+s}}{1+r_{t+s}} \leq W_{t+s}(j)L_{t+s}(j) + q_{t+s}u_{t+s}\bar{K}_{t+s} + B_{t+s-1} + D_{t+s} \, . \tag{1.5}$$

The household's nominal after-tax sources of funds arise from its labor income, its nominal payments received from supplying capital services to intermediate firms, the nominal face value of the net discount bond holdings carried from the previous period, and the nominal dividends received from its ownership of shares in the intermediate production sector that operates in monopolistic competition. The household spends its income on consumption, investment, to bare the cost $a(u_t)$ of increasing capacity utilization of physical capital and to save through his holding of bonds.

We, therefore, need to assess the dividends D_{t+s} stemming from the economic rent, as implied in part by investments in R&D. Using aggregate labor and capital, a firm i, belonging to a continuum defined over the interval $[0, 1]$, produces intermediate good i in a monopolistically competitive market, thus generating positive economic profits:

$$\Pi_{i,t+s} = P_{t+s}(i)\, Y_{t+s}(i) - w_{t+s}\, L_{t+s}(i) - r_{t+s}\, K_{t+s}(i) \, , \tag{1.6}$$

These profits are in turn paid as dividends among households.

The investment in R&D has to be accounted for in each period, while being treated as a sunk cost afterwards, since it is irrelevant whether or not an innovator is successful ex post. We will use two examples to illustrate how we account for the innovation process in the representative household's budget constraint.

For instance, having invested $P_t X_t$ to reach the frontier, an unsuccessful innovator generates no profits:

Figure 1.1 A failed innovator's timeline for cashflows

Time period	*t*	*t+1*	*t+2*	*t+3*	*t+4*	*t+5*
Cash flow	$-P_tX_t$	0	0	0	0	0

In this case, the household's budget constraint needs only to include his initial investment. The *ex-post* value of engaging in the innovation process is $-P_tX_t$.

By comparison, an innovator who has invested P_tX_t in R&D, which turns into a successful endeavour allows him to collect each period monopoly profits π_t for τ periods, until it is replaced by a new innovator. Figure 1.2 illustrates the corresponding flow timeline.

Figure 1.2 A successful innovator's timeline for cashflows

Time period	*t*	*t+1*	*t+2*	*t+3*	$t+\tau$	$t+\tau+1$
Cash flow	$-P_tX_t$	π_{t+1}	π_{t+2}	π_{t+3}	$\pi_{t+\tau}$	0

Here, the initial investment as well as future profits should be included in their respective budget constraints. It is important, however, to highlight that the profits included in the timeline above do not exclusively result from the innovation process. Indeed, an intermediate firm is already generating profits prior to an innovator taking over. Hence, the profits generated by the intermediate firm after the takeover include both monopoly profits and innovation profits.

Accordingly, the overall dividends paid to households are therefore defined as:

$$D_t = \int_0^1 \Big[\Pi_{i,t} - P_t\, X_t(i) \Big]\, di \,. \tag{1.7}$$

Utility maximization

Similarly to Christiano et al. (2005), household j maximizes its expected discounted utility function over its planning horizon with respect to its sequence of its budget constraints for each period, while taking into account the law of movement of physical capital. Its preferences for consumption embed habit formation with an intensity parameter $h > 0$, which generates some additional intrinsic dynamics and persistence on both the demand and supply sides of the economy following various shocks. The subjective discount factor is $0 < \beta < 1$, the parameter $\theta > 0$ induces disutility of labour, and the parameter $\nu \geq 0$ implies that the Frisch elasticity of labour supply is $1/\nu$. Furthermore, we assume that the household incurs some cost of adjusting investment $S\big(I_t/I_{t-1}\big) = (\kappa/2)\big(I_t/I_{t-1} - g_t\big)^2$, that is an increasing concave function of the growth rate of investment. It also faces an efficiency of investment shock μ_t which follows an AR(1) process.

The representative household must decide, for $s = \{0, 1, ...\}$, how much to consume C_{t+s}, how many hours $L_{t+s}(j)$ to work, how much capacity to use u_t, how much physical capital they want next period $\bar{K}_{t+s+1}$, how much to invest I_t in physical capital and the size of their net bond holdings B_{t+s}, by solving the following optimization problem, where E^j_{t+s} is the expectation operator conditioned on known information as of the beginning of period $t + s$.

$$\max_{C_{t+s}, L_{t+s}(j), u_{t+s}, I_{t+s}, B_{t+s}} E^j_t \sum_{s=0}^{\infty} \beta^s \left(ln\big(C_{t+s} - h\, C_{t+s-1}\big) - \theta\, \frac{L_{t+s}(j)^{1+\nu}}{1+\nu} \right) \tag{1.8}$$

subject to

$$P_{t+s}\, C_{t+s} + P_{t+s}\, I_{t+s} + P_{t+s}\, a(u_t)\bar{K}_{t+s} + \frac{B_{t+s}}{1+r_t} \leq W_{t+s}(j)\, L_{t+s}(j) + q_{t+s}\, u_{t+s}\bar{K}_{t+s} + B_{t+s-1} + D_{t+s}\ , \tag{1.9}$$

$$\bar{K}_{t+s+1} = \left[1 - S\left(\frac{I_{t+s}}{I_{t+s-1}} \right) \right] \mu_t I_{t+s} + (1-\delta)\bar{K}_{t+s}\ , \tag{1.10}$$

$$ln\mu_t = \rho_\mu ln\mu_{t-1} + \epsilon_{\mu,t}, \tag{1.11}$$

$$K_{t+s} = u_{t+s}\bar{K}_{t+s}\ . \tag{1.12}$$

In addition, based on the assumption that a household j possesses specialized skills that underlie some market power over its wage rate, we also consider the existence of wage rigidities modelled with Calvo contract arrangements. In this context, a constant proportion $1-\xi_w$ is allowed to reoptimize their wage in each period. Consequently, household j sets its wage rate to maximize its expected utility, weighted by the probability ξ_w of not being allowed to reoptimize with respect to wages, subject to the labor demand function. $\Lambda_{t+s} istheLagrangemultiplierofthe$t+s$budgetconstraintinthehousehold'sutilitymaximizationproblem.Thewageo$

$$\max_{W_{t+s}(j)} E_t^j \left(\sum_{s=0}^{\infty} \xi_w^s \, \beta^s \Big(-\theta \, \frac{L_{t+s}^{1+\nu}}{1+\nu} \Big) + \Lambda_{t+s} \, W_{t+s}(j) \, L_{t+s}(j) \right) \tag{1.13}$$

subject to

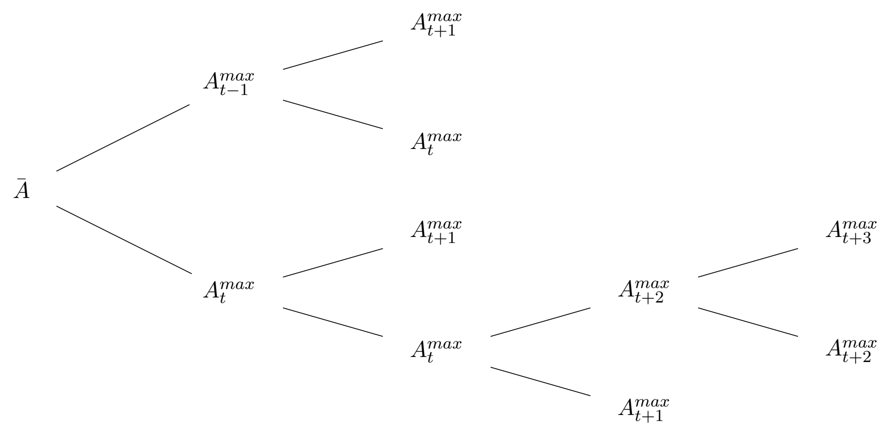
$$L_{t+s}(j) = \left(\frac{W_t(j)}{W_{t+s}} \right)^{-\gamma} L_{t+s} \, , \tag{1.14}$$

where γ is type-j labour demand elasticity to the relative wage.

1.2.1 The Schumpeterian add-ons in a New-Keynesian DSGE model and their implications

The Schumpeterian growth paradigm closely resembles those presented by Aghion & Howitt (1992). However, the presence of price stickiness introduces an additional layer of complexity not found in typical Schumpeterian growth models or in business cycle models with Schum- peterian features that do not incorporate nominal stickiness, such as Nuño (2011). Specifically, since a firm's investments in R&D depend on the discounted expected profits resulting from successful innovation, the monopolistic rent is also contingent on the prices a firm will be allowed to set (with some probability of fixed prices and some probability of adjusted prices). In the Schumpeterian growth framework, the presence of price stickiness can significantly influence the incentive structure for firms to invest in R&D. As firms base their investment decisions on the discounted expected profits from successful innovation, the ability to set and adjust prices directly impacts their expected profits and, consequently, their willingness to invest in R&D.

Figure 1.3 The technological advancement tree



1.2.1.1 The patent structures

A crucial component of endogenous growth models lies in the trajectory of technological advancement. We incorporate a patent system where an innovating firm holds exclusive rights to its innovation for a single period. Following this period, patents expire, allowing competitors to adopt the technology without incurring any costs in the subsequent period. Consequently, each period may see an intermediate producer classified as either lagging or advanced in terms of technology. Figure 1.3 provides a visual representation of this technological advancement hierarchy, assuming that the firm's initial level of technology was $\bar{A}$.

In our model, innovation is drastic, meaning that an innovating firm achieves a technological advantage so significant that neither the previous monopolist nor a competitive fringe can re-enter the market as a viable competitor after the innovation occurs. This implies that the new monopolist does not need to engage in limit-pricing strategies to deter entry, as no other firm can produce a close substitute within the patent duration.

We abstract from step-by-step technological progress, which would involve both Schumpeterian and escape-competition effects, typically characterized by firms competing closely in technology levels, either leapfrogging one another or coexisting in a leader-follower structure. Instead, we assume that RD costs are prohibitively high for developing a perfectly substitutable knockoff of an existing intermediate good. This ensures that each innovation leads to a decisive technological leap rather than incremental improvements that would otherwise foster step-by-step competition. In this framework, the drastic nature of innovation emerges naturally, as no firm can challenge the monopolist within the given period.

This assumption will play an important role when analyzing both the future expected profits of intermediate firms and the expected value of a firm to the innovator.

1.2.1.2 The optimal reset price

Operating within a monopolistically competitive market, intermediate firms possess market power derived from both their diversification and the technology employed in production. Additionally, prices are determined through Calvo contracts and set to maximize expected profits, contingent upon not being permitted to reoptimize. Calvo contracts remain binding, irrespective of the monopoly's leadership (i.e., even if an entrepreneur succeeds and takes over an intermediate sector, they remain bound by the Calvo contract).

Therefore, with an initial level of technological advancement $A_t(i)$, an intermediate firm at date t confronts the following constrained minimization of their costs:

$$\min_{K_t(i),L_t(i)} W_t L_t(i) + q_t K_t(i) \tag{1.15}$$

subject to

$$Y_t(i) = Z_t\, A_t(i)^{1-\alpha}\, K_t(i)^{\alpha}\, L_t(i)^{1-\alpha}\,, \tag{1.16}$$

with Z_t neutral technology shock defined as:

$$lnZ_t = \rho_z lnZ_{t-1} + \epsilon_{z,t}, \tag{1.17}$$

and

$$A_t(i) = \begin{cases} A_{t-1}^{max} & \text{with probability } n_{t-1} \\ A_{t-2}^{max} & \text{with probability } 1 - n_{t-1}\ . \end{cases} \tag{1.18}$$

where n_{t-1} represents the probability that an innovation resulting from investment in R&D during period $t-1$ elevated the technology level to A_{t-1}^{max} for the intermediate firm operating at date t, classifying it as an advanced firm. Conversely, there is a $1 - n_{t-1}$ probability that the intermediate firm remains lagging, while continuing to utilize the previous technology level A_{t-2}^{max}.

In conventional New-Keynesian models, all firms operate at the same level of technological advancement, leading to the establishment of a uniform optimal reset price. However, in our framework, two distinct levels of technological advancement coexist at any given time, as depicted in Figure 1.3. As a result, there are two optimal reset prices: a lagging optimal reset price and an advanced optimal reset price.

Subsequently, given their respective marginal costs, intermediate firms maximize their profits concerning their price $P_t(i)$:

$$\max_{P_t(i)} E_t \Bigg\{ \Big(P_t(i) - MC_t\big(A_t(i)\big) \Big)\, Y_t + \sum_{s=1}^{\infty} \xi_p^s\, \beta^s \frac{\Lambda_{t+s}}{\Lambda_t} \Bigg\{ n_{t+s-1} \Big(P_t(i) - EMC_{t+s} \Big)\, Y_{t+s}(i) \Bigg\} \Bigg\} \tag{1.19}$$

subject to

$$Y_{t+s}(i) = \left(\frac{P_t(i)}{P_{t+s}} \right)^{-\epsilon} Y_{t+s}\ , \tag{1.20}$$

and

$$EMC_{t+s} = n_{t+s-1}\, MC_{t+s}(A_{t+s-1}^{max}) + (1 - n_{t+s-1})\, MC_{t+s}(A_{t+s-2}^{max})\ , \tag{1.21}$$

It is important to recognize that the initial technology level is known and may vary among firms, while future technology levels depend on future investments in R&D. This is why we employ the expected marginal cost of production to account for the possibility that the intermediate firm may be technologically advanced or lagging, as illustrated in Figure 1.3.

Thus, it can be shown that the optimal reset price is a function of initial technology $A_t(i)$:

$$P_t^{\#}(i) = \frac{\epsilon}{\epsilon - 1} \frac{MC_t\big(A_t(i)\big)\, P_t^{\epsilon}\, Y_t + \sum_{s=1}^{\infty} \xi_p^s\, \beta^s\, \frac{\Lambda_{t+s}}{\Lambda_t}\, P_{t+s}^{\epsilon}\, Y_{t+s}\, EMC_{t+s}}{\sum_{s=0}^{\infty} \xi_p^s\, \beta^s\, \frac{\Lambda_{t+s}}{\Lambda_t}\, P_{t+s}^{\epsilon}\, Y_{t+s}} \,. \tag{1.22}$$

Given the patent system introduced in the previous subsection, at any given time, there will only be two optimal reset prices. Technologically advanced firms will set their prices at:

$$P_{A,t}^{\#} = \frac{\epsilon}{\epsilon - 1} \frac{MC_t(A_{t-1}^{max})\, P_t^{\epsilon}\, Y_t + \sum_{s=1}^{\infty} \xi_p^s\, \beta^s\, \frac{\Lambda_{t+s}}{\Lambda_t}\, P_{t+s}^{\epsilon}\, Y_{t+s}\, EMC_{t+s}}{\sum_{s=0}^{\infty} \xi_p^s\, \beta^s\, \frac{\Lambda_{t+s}}{\Lambda_t}\, P_{t+s}^{\epsilon}\, Y_{t+s}} \,, \tag{1.23}$$

while lagging firms will set their price at:

$$P_{L,t}^{\#} = \frac{\epsilon}{\epsilon - 1} \frac{MC_t(A_{t-2}^{max})\, P_t^{\epsilon}\, Y_t + \sum_{s=1}^{\infty} \xi_p^s\, \beta^s\, \frac{\Lambda_{t+s}}{\Lambda_t}\, P_{t+s}^{\epsilon}\, Y_{t+s}\, EMC_{t+s}}{\sum_{s=0}^{\infty} \xi_p^s\, \beta^s\, \frac{\Lambda_{t+s}}{\Lambda_t}\, P_{t+s}^{\epsilon}\, Y_{t+s}} \,. \tag{1.24}$$

1.2.1.3 The innovation process

R&D activities are conducted by entrepreneurs or innovators. If their efforts result in an innovation, the implementation of this technology by an intermediate goods producer grants additional market power by producing an improved version of the intermediate good, as it reaches the new technological frontier. Thus, the innovation process occurs within the intermediate sector and pushes the technological frontier forward.

Entrepreneurs invest a certain amount of final goods to maximize the probability of innovating. External researchers or a new successful innovator can supplant or "leapfrog" an incumbent entrepreneur. However, the prospects of innovation are uncertain, as the probability to innovate is n_t, and that of not making a discovery is $1 - n_t$. Yet, n_t is endogenous, as it is linked to the intensity of R&D effort $\frac{X_t}{\omega, A_t^{max}}$, where X_t represents the real amount of final goods invested in R&D, A_t^{max} denotes the prevailing state of technology or the technological frontier prior to new innovations, and $\omega > 1$ indicates the extent of productivity improvement derived from the innovation. Specifically, when ωA_t^{max} is larger, a given amount of resources X_t devoted to R&D corresponds to a lower level of research intensity. This feature is meant to reflect the increasing complexity of further progress. Finally, the innovation production function is assumed to exhibit diminishing marginal returns, with $\eta > 0$:

$$n_t = \left(\frac{X_t}{\omega \, A_t^{max}} \right)^{1/(1+\eta)} \quad . \tag{1.25}$$

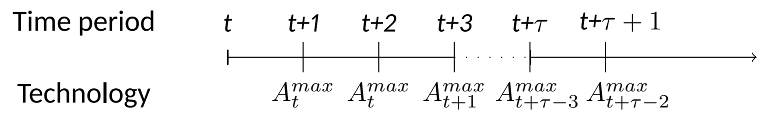
For convenience and compatibility with complete markets, we assume that entrepreneurs invest in a diversified form of R&D. In essence, this means that a successful entrepreneur does not know in which sector they may end up. Even though, *ex ante*, an entrepreneur is unaware of the sector in which he will be innovating, he can evaluate the discounted expected profits from a potential discovery using the individual prices of the continuum of intermediate goods. Hence, to choose the optimal amount of final good to be invested in R&D that maximizes expected discounted profits, an entrepreneur seeks to maximize his expected discounted profit, where $E_t V_{t+1}(A_t^{max})$ represents the expected discounted value of future profits contingent on the entrepreneur remaining at the helm of the monopoly:

$$\max_{X_t} \beta \frac{\Lambda_{t+1}}{\Lambda_t} n_t \, E_t V_{t+1}(A_t^{max}) - P_t \, X_t, \tag{1.26}$$

If successful, the innovator will collect monopoly profits as long as no further innovation occurs in their sector.

Given the diversification assumed for the entrepreneurs, all will invest the same amount of final goods in R&D. Consequently, if an innovation occurs, the technology advances to the frontier, and the expected value

Figure 1.4 The timeline for technology



of the intermediate firm will be the same. In accordance with the problem in equation (1.26), the optimal investment in R&D is given by

$$X_t = \beta \, \frac{\Lambda_{t+1}}{\Lambda_t} \, n_t \, \frac{E_t V_{t+1}(A_t^{max})}{P_t} \; . \tag{1.27}$$

To complete the solution of equation (1.27), we still need to write explicitly the expected value of the firm to the entrepreneur. This happens to be more challenging than it may look at first, as it relies on the answers to three questions:

– Which path will technology follow?

– Which path will prices follow?

– What happens if the innovator is not allowed to reset its price at its optimal value when he takes over?

The first challenge is ensuring that the assessment of future profits follows the correct technological path. For example, as illustrated in figure 1.4, an innovator may reach the frontier A_t^{max} in $t+1$, hold a patent on this innovation, and remain there in $t+2$. Subsequently, the patent on the $t+2$ frontier, A_{t+1}^{max}, expires in $t+3$ and can therefore be adopted by everyone. The adoption of the new technology is automatic, as a more advanced technology decreases the marginal cost of operation, which leads every intermediate good producer to adopt it.

The second challenge arises from price rigidities, as they play a crucial role in determining future profits, since they condition both the profit margin and the conditional demand for that specific intermediate good. Profits will differ based on whether the innovator is allowed to reoptimize. In addition, all intermediate firms face an identical probability ξ_p of not being allowed to reoptimize their price in a Calvo-contract setup. If this contingency occurs, the monopolist is stuck at charging a certain price that may not be the optimal reset one.

Let us consider the relevant cases to fully characterize the value of the firm, to an entrepreneur/innovator, as a function of the path of prices.Four possible scenarios can be thought as covering all the possible time paths of prices to be considered when assessing the value of investing in R&D. In case 1, an innovating producer for the i^{th} good is allowed to set the intermediate good's optimal price upon taking over, with that price prevailing for the remainder of his tenure. In the second case, an innovating producer faces a sticky prevailing price even with the new technology for his whole tenure at the helm of the monopoly. In the third case, an incumbent producer operating at date t, charges the prevailing price inherited from the previous period, yet some time in the future as long as he is in operation, he will be subject to the Calvo probability for resetting or not his price to the level of that associated with the latest lagging technology, still in operation.

There is a fourth possible case that has no bearing on the calculation, since it does not contribute to the value of an innovation from R&D.Indeed, when an alternate innovation sometimes occurs in the future, the incumbent is thus evicted, so the time path of prices is no longer relevant for the current investment in R&D.

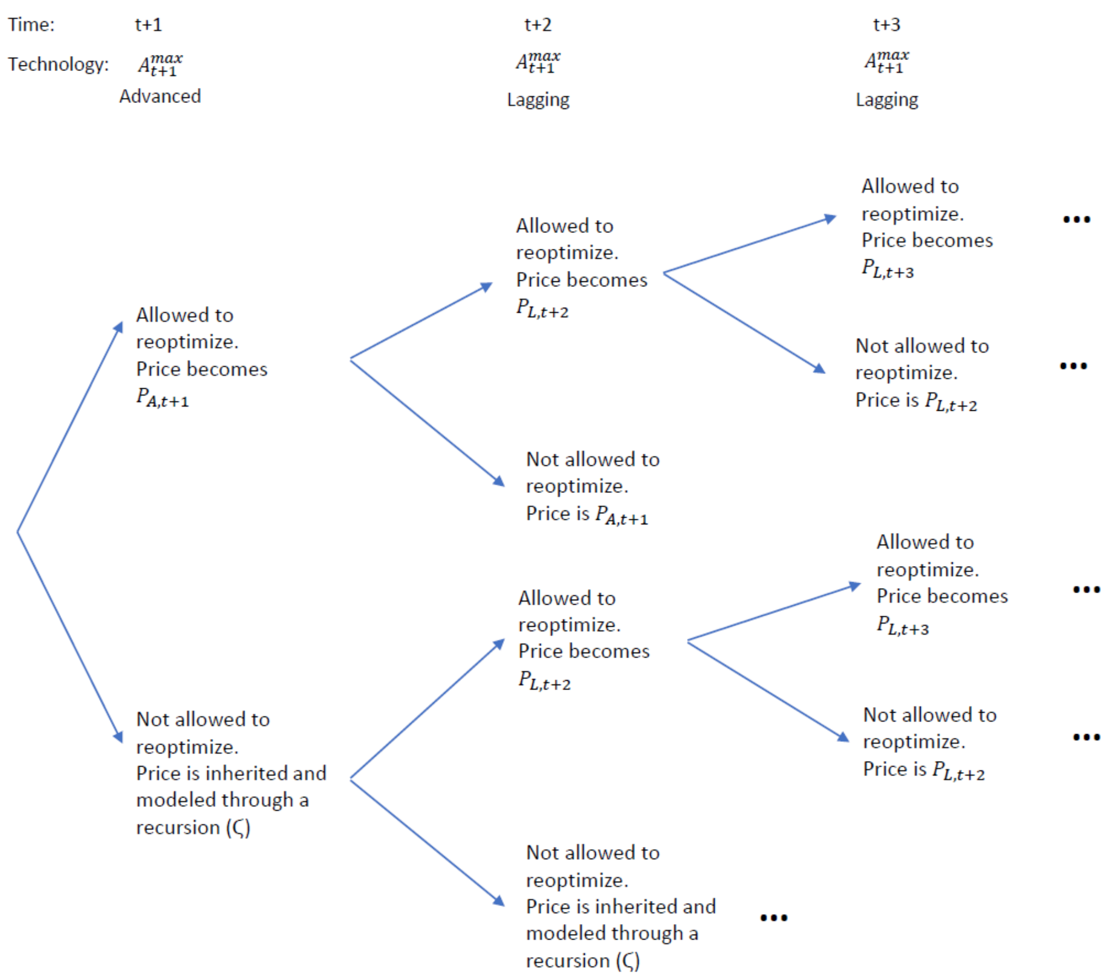
Figure 1.5 illustrates all relevant scenarios.

Case #1: Initial optimal price setting by date-t innovating producer, with sticky price thereafter for the remaining of his tenure.

If the innovating producer is allowed to reoptimize its price upon taking over, he will choose the advanced optimal reset price. The technological path followed will be that of Figure 1.4. The price for the first period will be $P_{A,t+1}$ at $t+1$, as the monopolist is allowed to reoptimize, for subsequent periods prices and will yield the following profits:

$$\Psi_{A,t+1} = P_{A,t+1}^{1-\epsilon}(Y_{t+1} + \frac{X_{1,t+2}}{\Lambda_{t+1}}) - P_{A,t+1}^{-\epsilon}(MC_{t+1}(A_t^{max})y_{t+1} + \frac{X_{2,t+2}}{\Lambda_{t+1}}), \tag{1.28}$$

Figure 1.5 Price tree for a successful innovator



where $X_{1,t+2}$ and $X_{2,t+2}$ are two recursive auxilliary variables. They represent the discounted future revenues or costs, respectively, and account only for the cases where the innovator, now a monopolist, remains at the helm of the monopoly and is not allowed to reset his price. Algebraically, that is:

$$X_{1,t+2} = \xi_p \beta \Lambda_{t+2}(1 - n_{t+1})Y_{t+2} + \xi_p \beta (1 - n_{t+1})X_{1,t+3}, \tag{1.29}$$

$$X_{2,t+2} = \xi_p \beta \Lambda_{t+2}(1 - n_{t+1})MC_{t+2}(A_t^{max})Y_{t+2} + \xi_p \beta (1 - n_{t+1})X_{2,t+3} \tag{1.30}$$

Case #2 Sticky prevailing price even with the new technology for the whole tenure of date-t innovating producer.

In the situation where the innovating producer is not allowed to reoptimize upon taking over, he will inherit the price set by the latest Calvo contract. Under the diversified R&D hypothesis, there is a straightforward solution to this problem. If the innovator does not know in which sector he will end up, he has to take into account all possible prices set in the past in the continuum of intermediate sectors:

$$\Psi_{I,t+1} = \zeta_{1,t+1}^{1-\epsilon}(Y_{t+1} + \frac{X_{1,t+2}}{\Lambda_{t+1}}) - \zeta_{2,t+1}^{-\epsilon}(MC_{t+1}(A_t^{max})Y_{t+1} + \frac{X_{2,t+2}}{\Lambda_{t+1}}), \tag{1.31}$$

where $\zeta_{1,t+1}$ and $\zeta_{2,t+2}$ are two auxilliary variables, which through recursion account for the previously set Calvo prices:

$$\zeta_{1,t+1}^{1-\epsilon} = (1 - \xi_p)\left(n_{t-1}(P_{A,t})^{1-\epsilon} + (1 - n_{t-1})(P_{L,t})^{1-\epsilon}\right) + \xi_p(\zeta_{1,t})^{1-\epsilon}, \tag{1.32}$$

$$\zeta_{2,t+1}^{-\epsilon} = (1 - \xi_p)\left(n_{t-1}(P_{A,t})^{-\epsilon} + (1 - n_{t-1})(P_{L,t})^{-\epsilon}\right) + \xi_p(\zeta_{2,t})^{-\epsilon}. \tag{1.33}$$

The difference with equation 1.28 is that the innovator does not know ex ante the inherited price. Hence the adjustments made with $\zeta_{1,t+1}$ and $\zeta_{2,t+1}$

Case #3: after $t+2$, can be allowed to reoptimize or not but he can only reoptimize at the lagging price $P_{L,t+s+1}$, with $= 0, 1, ...$

After $t+2$, the only price that can be set by the monopolist is the lagging optimal reset price given that if the technology advances in his sector, he will be eliminated and his profits will be 0. However, unlike the previous two scenarios, the lagging optimal reset price will change with time. To characterize it properly, we must use a double recursion:

$$\Psi_{t+2} = (1-\xi_p)(X_{3,t+2} - X_{4,t+2}) + \beta\frac{\Lambda_{t+3}}{\Lambda_{t+2}}(1-n_{t+1})\Psi_{t+3}, \tag{1.34}$$

where

$$X_{3,t+2} = P_{L,t+1}^{1-\epsilon}(1-n_{t+1})Y_{t+2} + \xi_p\beta\frac{\Lambda_{t+3}}{\Lambda_{t+2}}\left(\frac{P_{L,t+2}}{P_{L,t+3}}\right)^{1-\epsilon}(1-n_{t+1})X_{3,t+3}, \tag{1.35}$$

and

$$X_{4,t+2} = P_{L,t+1}^{-\epsilon}(1-n_{t+1})MC_{L,t+2}Y_{t+2} + \xi_p\beta\frac{\Lambda_{t+3}}{\Lambda_{t+2}}\left(\frac{P_{L,t+2}}{P_{L,t+3}}\right)^{-\epsilon}(1-n_{t+1})X_{4,t+3}. \tag{1.36}$$

$X_{3,t+2}$ and $X_{4,t+2}$ are the sum of, respectively, the revenue and the costs, generated by the monopoly if the firm is allowed to reoptimize at $t+2$ and does not reoptimize thereafter. The double recursion ensures that Ψ_{t+2} accounts for all the possible reoptimization after $t+3$.

When all of these scenarios are put together, we can finally write the value of the firm as a combination of all three relevant cases:

$$V_{t+1} = (1-\xi_p)\Psi_{A,t+1} + \xi_p\Psi_{I,t+1} + \beta\frac{\Lambda_{t+2}}{\Lambda_{t+1}}\Psi_{t+2} \tag{1.37}$$

where $\Psi_{A,t+1}$ is associated with the scenario where the innovating producer is allowed to reoptimize upon taking over the sector and is not allowed to reoptimize, $\Psi_{I,t+1}$ for the innovating producer inheriting a price and not being allowed to reoptimize and Ψ_{t+2} for the innovating producer being allowed to reoptimize or not, yet, if possible, only at the lagging price $P_{L,t+s+1}$, with $= 0, 1, ...$

1.2.2 Spillovers and the technological growth rate

When a firm introduces a novel technology, it not only enriches the pool of knowledge available to other entities but also sets in motion a chain reaction of enhanced productivity. This knowledge spillover, facilitated through various channels, like publications, patents, or employee mobility, enables other firms to access and apply this newfound knowledge. As a result, these firms have experienced a surge in technology spillovers, marked by improved operational processes and increased outputs. This dynamic of knowledge transfer and the consequent productivity gains exemplify the significant role spillovers play in fostering economic growth and innovation across multiple firms, transcending the boundaries of specific sectors or industries.

The gross growth rate of the economy g_t is defined as:

$$A_t^{\text{max}} = g_t A_{t-1}^{\text{max}} = (1 + \sigma_t n_{t-1}) A_{t-1}^{\text{max}}. \tag{1.38}$$

It is thus a function of the probability of innovating n_{t-1} and the technology spillover σ_t:

$$ln_t = (1 - \rho_\sigma)\, ln\sigma_{ss} + \rho_\sigma\, ln\sigma_{t-1} + \epsilon_{\sigma,t} \tag{1.39}$$

where σ_{ss} is the steady state value of the spillover effect and is going to be calibrated to target the historical average growth rate of TFP.

1.2.3 The aggregate resource constraint

The aggregation of the budget constraint over the continuum of households yields the aggregate resource. The aggregate output can be used either for consumption, investment in physical capital, altering the utilization of capital or investing in R&D, algebraically, that is:

$$C_t + I_t + a(u_t)\widetilde{K}_t + X_t = Y_t\,. \tag{1.40}$$

1.2.4 The specification of monetary policy

The central bank's policy function is modelled as a Taylor-type rule. It captures the monetary policy interest rate decision, which is influenced by a combination of past interest rate, current inflation, and output growth. It shows that the interest rate is partly a function of its previous period's value, indicating a degree of policy inertia. Additionally, it adjusts in response to the deviation of current inflation from its target and in output growth relative to trend growth. This adjustment is moderated by specific policy response coefficients and is further influenced by an exogenous monetary policy shock, ensuring that the policy rate adapts dynamically to economic conditions. It sets the interest rate according to the following equation:The central bank's policy function is modelled as a Taylor-type reaction function, as it sets the nominal interest rate according to the following equation:

$$\frac{1+R_t}{1+R} = \left(\frac{1+R_{t-1}}{1+R}\right)^{\rho_R} \left[\left(\frac{\pi_t}{\pi}\right)^{\alpha_\pi} \left(\frac{Y_t}{Y_{t-1}}\,g^{-1}\right)^{\alpha_y}\right]^{1-\rho_R} m_t\,, \tag{1.41}$$

where m_t is a monetary policy shock defined as:

$$ln\, m_t = \rho_m\, ln\, m_{t-1} + \epsilon_{m,t}. \tag{1.42}$$

The parameter ρ_R represents the degree of smoothing of interest rate changes, as the monetary authority aims to avoid overly large shifts concerning its one-period-lag value, R_{t-1}, and adjusts it somewhat gradually following demand and technology shocks. The parameters α_π and α_y are the monetary authority's weights attached to deviations from its inflation target, π, and its output growth trend.

1.2.4.1 The aggregate economy and its general equilibrium

In the proposed model, we observe fluctuations in various economic elements, such as output, consumption, physical capital, and investments in both physical capital and R&D. These fluctuations align with a balanced growth path, attributed to the endogenous progression of technology. Prior to simulating the model around the steady state, it is essential to detrend these variables. Notably, the investment in R&D, represented by X_t, correlates with the state of the technological frontier, as outlined in equation (1.27). In this context, detrending is performed using the technological frontier as a benchmark. Subsequently, we calculate the steady state for the model once it has been detrended. The final step involves approximating the model log-linearly around its steady state.

1.3 The calibration of the parameters and the characteristics of the various shocks

1.3.1 The standard parameters

Table 1.1 presents the values used for the calibration of key parameters in the model.

Parameters such as the capital share in the production function, denoted by α, the discount rate β, and the depreciation rate of physical capital δ, along with the framework for monetary policy, are aligned with conventional values identified in existing literature. The model sets the steady-state gross trend inflation, symbolized by π, at 1, implying a zero inflation rate at the steady state. Additionally, it is posited that there is full capacity utilization in the steady state, indicated by $u = 1$. The model incorporates market power in the markets for labor and intermediate goods, leading to the adoption of wage and price markups approximately 20%, a standard established in the work of Christiano et al. (2005). This markup translates to an elasticity of substitution of 6, both among different types of intermediate goods and across various categories of labor.

The Calvo parameters in our model, ξ_p and ξ_w, which determine the frequency of price and wage adjustments, respectively, are calibrated to align with empirical findings from microeconomic data. Both parameters are assigned a value of 0.5, reflecting the average contract duration of two quarters for both prices and wages. This calibration draws upon Bils & Klenow (2004) and is consistent with the priors used by Smets & Wouters (2007).

Regarding the inverse Frisch elasticity of labor supply, denoted by ν, this parameter is critical in determining the response of hours worked to wage changes, while holding the marginal utility of wealth constant. The labor disutility is influenced by the preference parameter θ. Both ν and θ are carefully chosen to ensure that the steady-state worked hours approximate $L = .3$. The selected value for the Frisch elasticity balances be-

Table 1.1 Key parameters

Parameter	**Value**	**Meaning**
α	1/3	Share of capital in the intermediate goods production function
β	.99	The households subjective discount rate
δ	.025	Depreciation rate of physical capital
α_π	2	Taylor rule's inflation gap coefficient
α_y	.2	Taylor rule' output gap coefficient
ρ_R	.8	Taylor rule interest smoothing parameter
u	1	Steady state capacity utilization rate
ϵ	6	Elasticity of substitution of intermediate goods
γ	6	Elasticity of substitution of labor
ξ_p	.5	Calvo probability for prices
ξ_w	.5	Calvo probability for wages
θ	5	Disutility of labour parameter
ν	1	Utility parameter that determines the Frisch elasticity of labour, ($\frac{1}{\nu}$)

tween macroeconomic estimates, typically ranging between 2 and 4, and microeconomic estimates, which are generally below 0.5. This selection is informed by the work of Peterman (2016), which highlights the sensitivity of Frisch elasticity to different estimation methodologies. For our model, we adopt $\theta = 5$ and $\nu = 1$, ensuring an appropriate balance in the labor supply response within the model's framework.

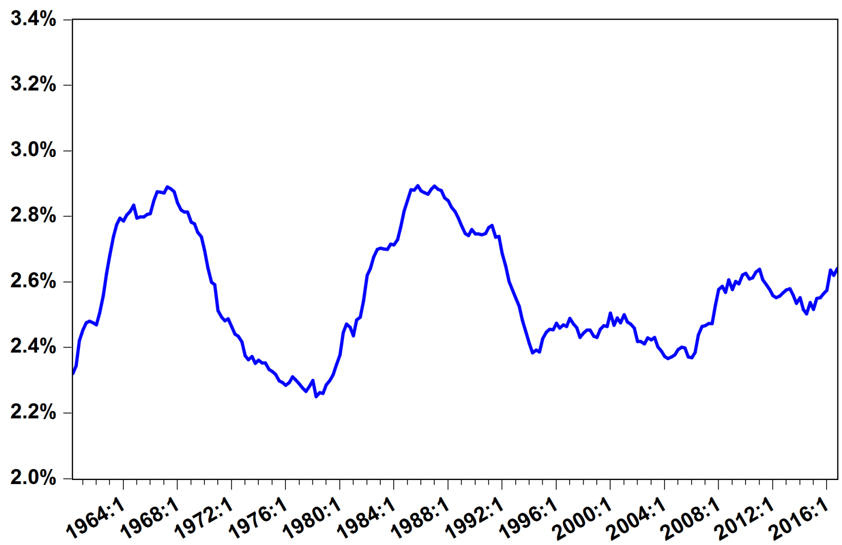
1.3.2 The parameters pertaining to the Schumpeterian features of the model

The calibration of Schumpeterian models is a relatively new area of research in economics. To calibrate our model, we rely on statistical moments related to research and development and technological advancement.

Figure B.1 displays the share of GDP dedicated to R&D investment in the United States between 1960 and 2016. Over this period, the share has varied between 2.3% and 2.9%, averaging a 2.56% share of GDP. We use this sample average as the steady state to replicate investment in R&D.

To replicate the growth rate of technology, we target the average quarterly growth rate of U.S. Total Factor

Figure 1.6 R&D investment-to-GDP ratio in the United States(1960q2-2016q4)



Productivity (TFP) estimated by Fernald (2014, 2017) at 1.21% annualized. Our model examines technology from two angles: the effects of a transitory technological shock and a Schumpeterian innovation. When innovations occur, they result in a permanent technological shift that permanently pushes the frontier forward. In this regard, the frontier growth rate corresponds to the growth rate of the trend in TFP. Therefore, we target a steady-state growth rate of the frontier of 1.06% annualized, in line with the Hodrick-Prescott trend component of U.S. TFP. Assigning values to the innovation probability and the spillover effect at the steady state enables us to mimic the above steady-state growth rate.

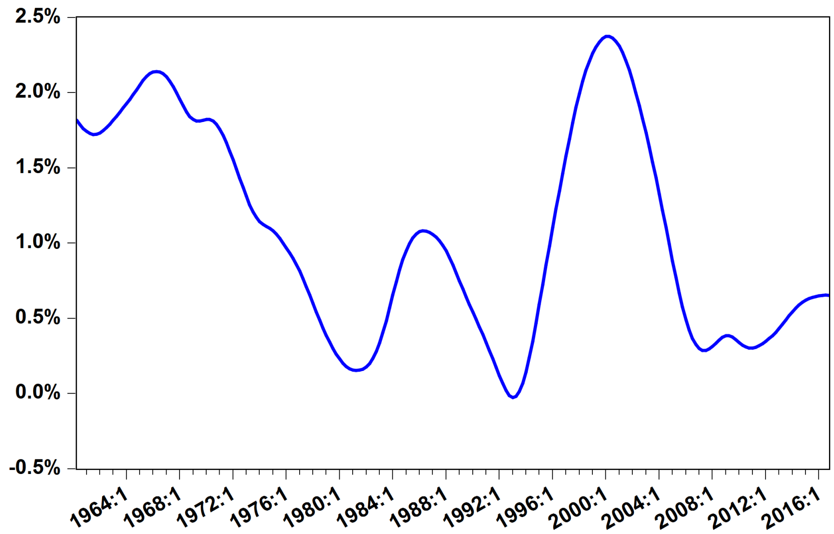
Given the relationship between the frontier growth rate, the spillover effect, and the innovation probability, there is an additional degree of freedom. A variation along one dimension can be offset by a change in another. This problem is not significant when examining steady states, but it deserves further consideration when discussing the volatility and comovements of aggregate variables, as we discuss in the next section.

1.3.3 Persistence and variance of shocks

1.3.3.1 Calibrating the shocks' parameters

A spillover embodies a the positive externality derived from an innovation,since it permanently pushes the technological frontier forward. So, we first apply an HP filter on Fernald (2014, 2017)'s quarterly utilization-adjusted TFP series to extract both the cycle and the trend. Then, we compute the growth rate of the trend that is graphed in Figure 1.7.

Figure 1.7 TFP trend growth in the United States (1960q2-2016q4). Source: Fernald (2017)



For our model to exhibit the persistence exhibited by the trend growth rate of TFP, it requires a somewhat persistent spillover shock. We set the persistence parameter ρ_σ to $.9$, with a volatility $\sigma_\sigma = .01$ as well as $\rho_z = 0.2$ and a variance of $\sigma_z = 0.005$ to match the observed autocorrelation and volatility of TFP.

To calibrate the investment-specific shock that affects the efficiency with which investment is transformed into capital, the persistence of the associated shock is ser to $\rho_\mu = 0.6$ and its variance to $\sigma_\mu = 0.005$. Finally, we calibrate the monetary policy shock that is an innovation to the Taylor rule, as specified in the previous section. The monetary policy shock has a persistence of $\rho_m = 0.6$ and a variance of $\sigma_m = 0.005$.

1.4 The business cycle analysis

The business cycle analysis revolves around the study of various impulse responses [1] and the corresponding variance decomposition analysis. In both cases, we compare the endogenous growth model to the exoge-

[1] The impulse responses are that of real, detrended variables. It shall be noted that they are expressed as deviations from the trend so they should, therefore, be analyzed as such. For instance, a decrease of the output would not necessarily entail a decrease of aggregate production, it may rather represent a smaller rise in output when taking the trend into account

nous growth model. Both models include sticky prices and wages, investment adjustment cost and variable capacity utilization.

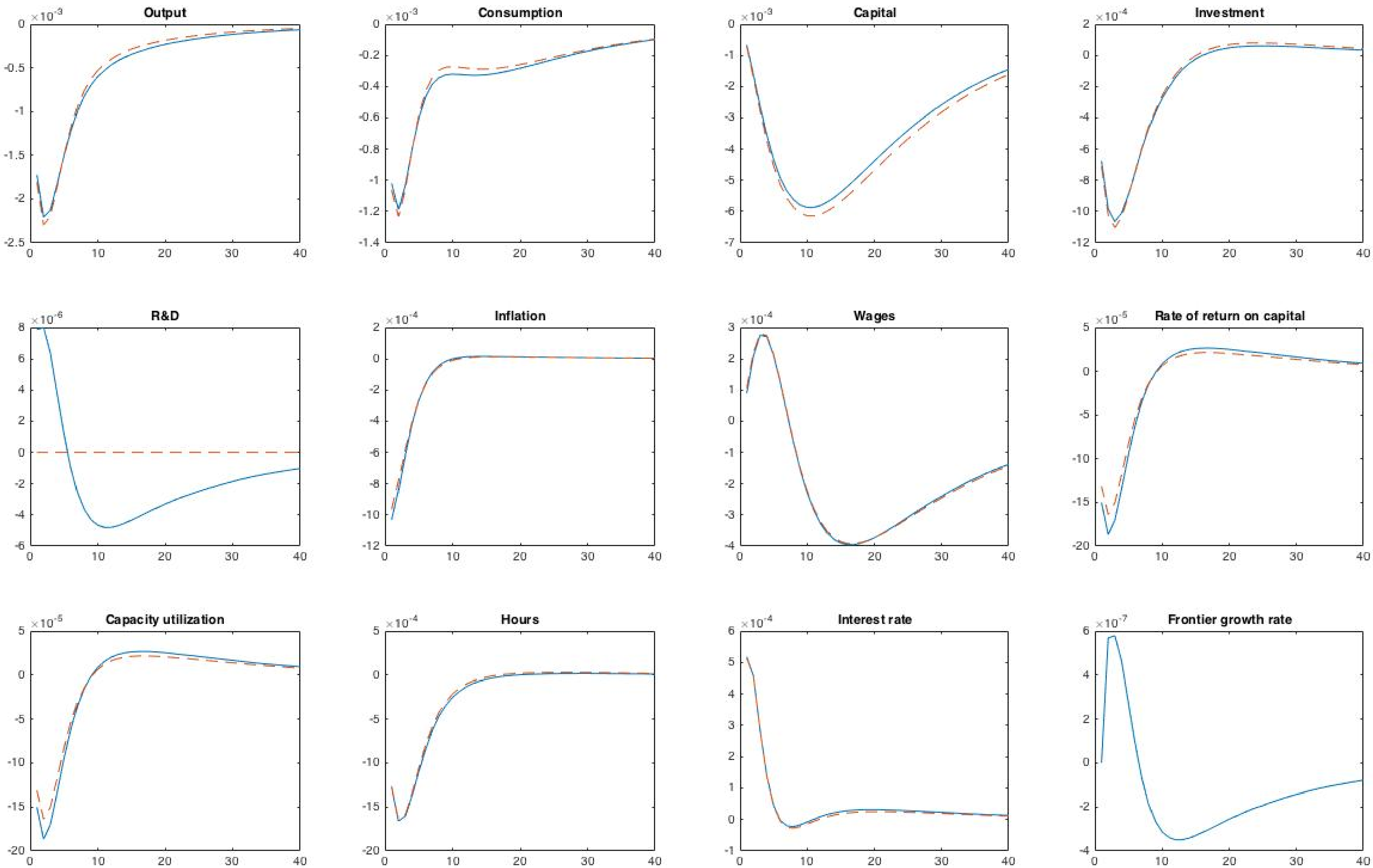
1.4.1 The impulse response functions

As can be seen in Figures 1.8, 1.9 and 1.10, the impulse response functions in the model exhibit similarities with and without endogenous growth for the shocks on either the aggregate productivity, the efficiency of investment, or the monetary shocks. However, slight differences in amplitude can be explained by the additional transmission channel arising from endogenous growth. In the case of the monetary policy shock, an increase of one standard deviation in the interest rate generates decreases in inflation, output, consumption, and investment, as observed in Figure 1.8. The decrease in the rate of return on capital is due to an increase in R&D, which becomes a more attractive alternative in comparison. In turn, this R&D increase pushes up the growth rate of the technological frontier, which explains why the impulse responses under exogenous growth.

Similarly, a positive investment shock as in Figure 1.9 makes the conversion from investment to physical capital more efficient. This leads to an increase in physical capital, output, consumption, and investment before returning to their steady states around ten quarters after the shock. The impulse responses of the technological shock, shown in Figure 1.10, are also in line with the literature. In both cases, the difference in impulse responses with either exogenous or endogenous growth can be traced back to an arbitrage between the rate of return on physical capital and R&D in the latter case.

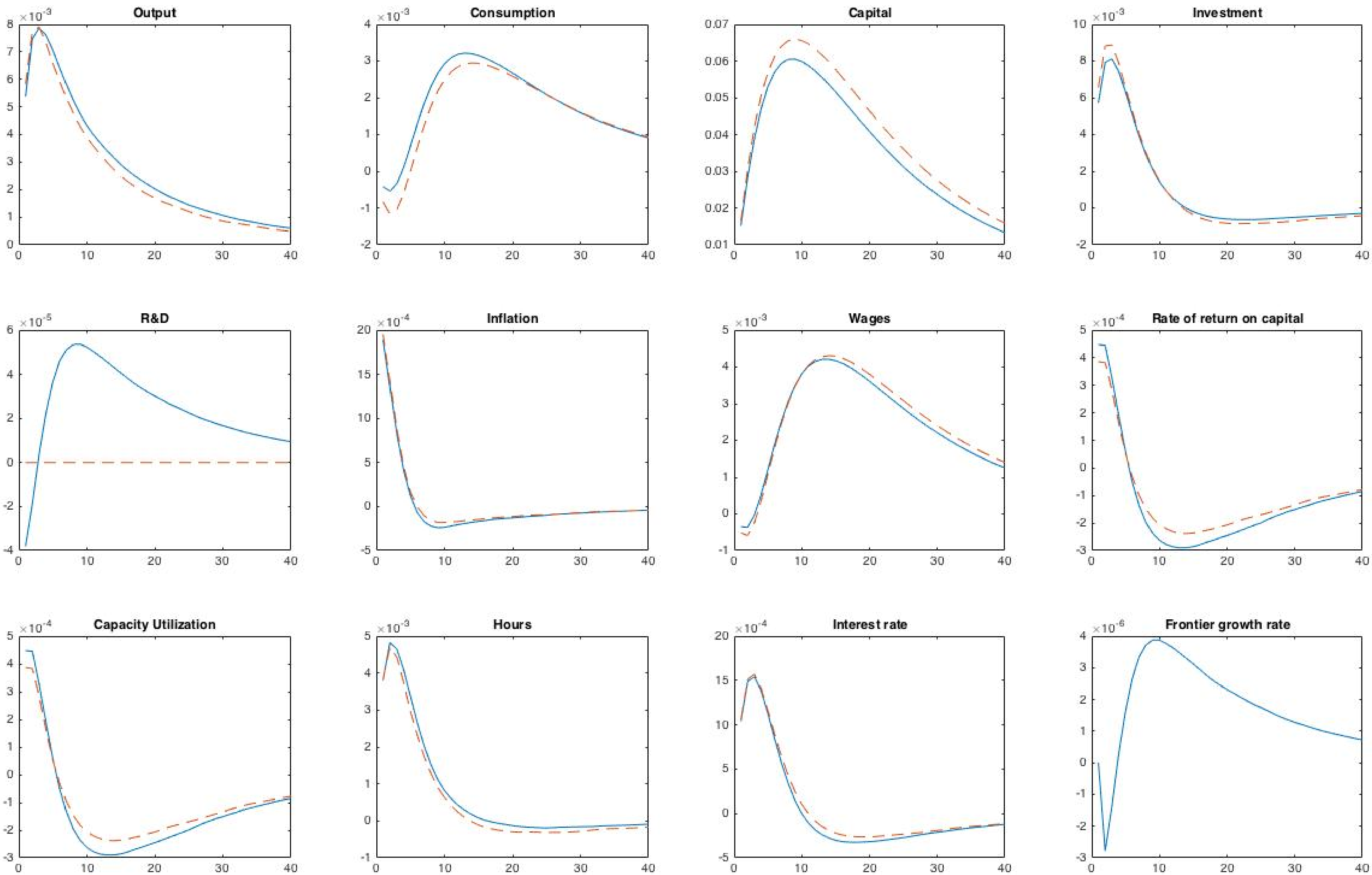
In Figure 1.11, the impulse responses for a spillover shock shown are specific to the endogenous growth model. That is, since the exogenous growth framework does not include such a shock. An increase in spillover increases the growth rate of the frontier, leading to an increase in output and inflation. Given the specification of monetary policy, the interest rate then goes up because of deviations from both output growth and inflation targets. Moreover, the persistent increase in the interest rate increases the discount rate, which depreciates the discounted value of an innovation, driving investment in R&D down.

Figure 1.8 Impulse responses to a one standard deviation monetary policy shock



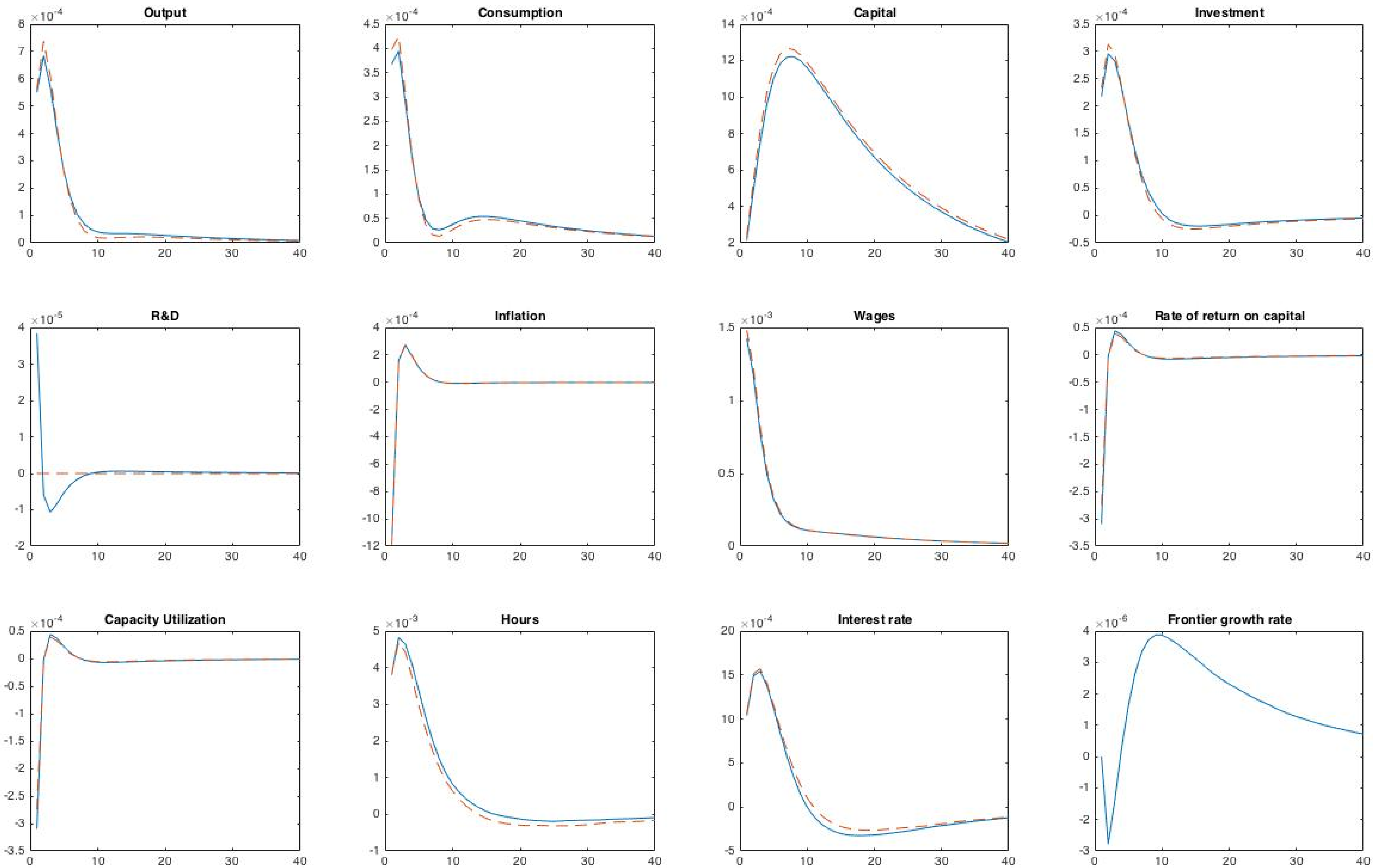
The solid line is the endogenous growth model while the dashed line is the exogenous growth model.

Figure 1.9 Impulse responses to a one standard deviation investment shock



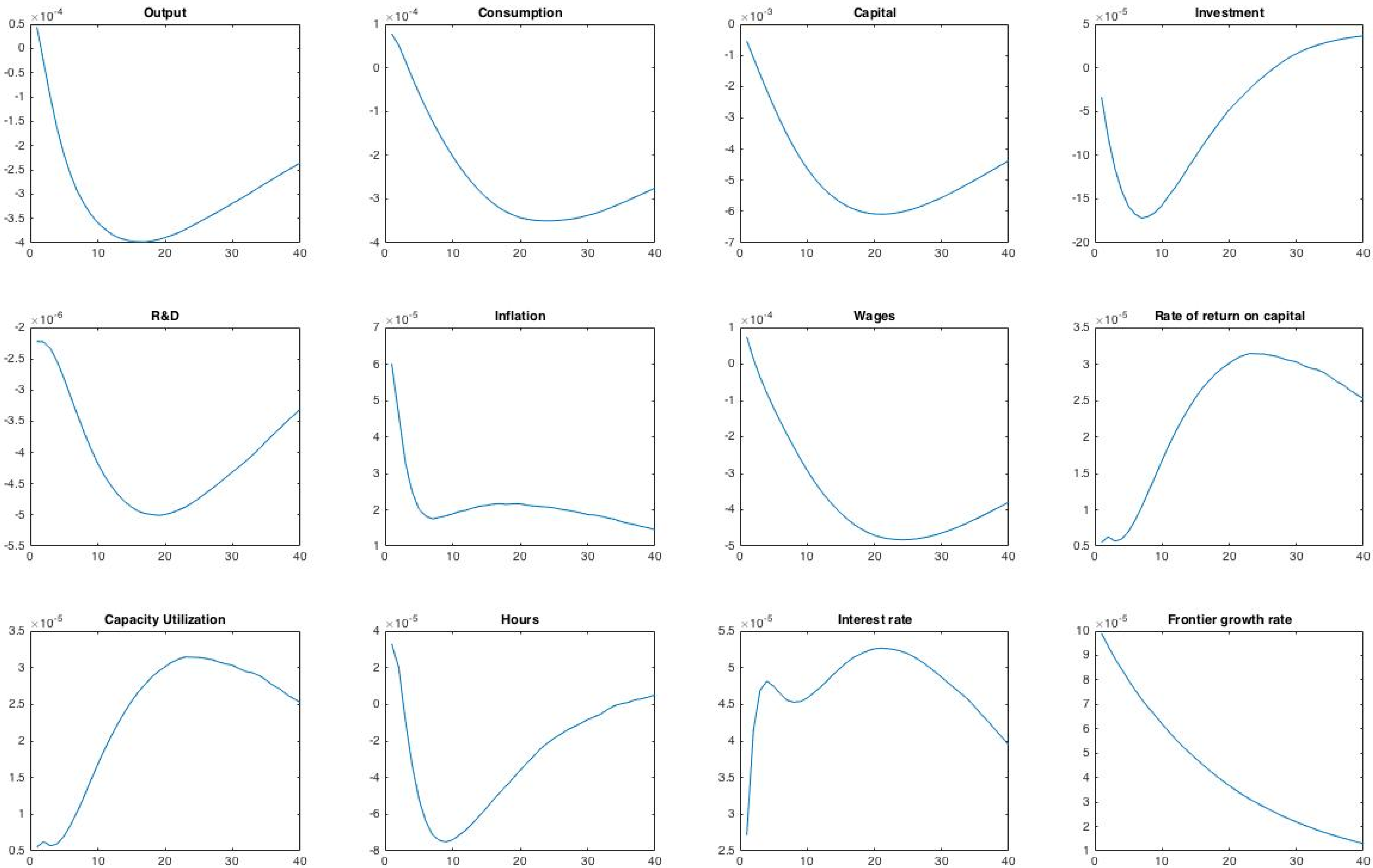
The solid line is the endogenous growth model while the dashed line is the exogenous growth model.

Figure 1.10 Impulse responses to a one standard deviation technology shock



The solid line is the endogenous growth model while the dashed line is the exogenous growth model.

Figure 1.11 Impulse responses to a one standard deviation spillover shock



The solid line is the endogenous growth model while the dashed line is the exogenous growth model.

1.4.2 Variance decomposition

Tables 1.2 presents the variance decomposition, at a 15 quarters horizon , that allows for the analysis of the relative contribution of each shock to the volatility of selected variables. Two conclusions can be drawn from these results. Firstly, the spillover shock, which is specific to the endogenous growth model, explains a significant share of variability. Indeed, the spillover shock accounts for 15% and 24% of the changes in R&D investment and physical capital, respectively. Secondly, the monetary policy shock has an impact on R&D investment, responsible for 19% of its variance. The result is consistent with the findings of Comin & Gertler (2006).

Table 1.2 Variance decomposition at t=15 quarters

	Technology Shock	**Monetary Shock**	**Investment Shock**	**Spillover Shock**
Output	2%	16%	77%	5%
Consumption	2%	11%	80%	7%
Investment	1%	34%	62%	3%
R&D	36%	19%	31%	15%
Physical Capital	0%	40%	35%	24%
Capacity Utilization	19%	7%	72%	2%
Wages	30%	28%	21%	20%
Labor	18%	7%	75%	0%
Rate of return on capital	19%	7%	72%	2%
Interest Rate	12%	14%	70%	4%

Note: The table shows the variance decomposition for different economic variables at t=15 quarters.

The variance decompositions at different horizons, reported in the appendix in tables A.1, A.2, and A.3, confirm the conclusions drawn earlier. They also illustrate the role of spillover in aggregate fluctuations, with the size of its contribution increasing with the horizon due to the inherent longer-term nature of R&D and its effects.

1.4.3 Contemporaneous correlations

We, now, compare contemporaneous correlations of the cyclical components of key variables that were generated by the different models with the corresponding correlations in the data for the 1960Q1 to 2016Q4.

Table 1.3 Key contemporaneous correlations

	$\rho(\hat{Y},\hat{C})$	$\rho(\hat{Y},\hat{I})$	$\rho(\hat{C},\hat{I})$
Data	0.85492	0.75337	0.57955
Exogenous growth	0.39616	0.14439	-0.05816
Endogenous growth	0.8464	0.88807	0.50802

Note: $\hat{Y}$, $\hat{C}$ and $\hat{I}$ are percentage deviations from the trend.

By examining selected correlations from an exogenous growth New Keynesian model, an endogenous growth New Keynesian model, and data from the Federal Reserve Bank of St. Louis F.R.E.D. database. To extract the cyclical component, we apply a logarithmic transformation to the data and utilize the Hodrick-Prescott filter. As the simulated data is already detrended, we use logarithmic differences to compute percentage deviations from the trend.

As seen in Table 1.3 endogenous growth model does a better job to replicate relevant characteristics of business cycles than the exogenous growth model, with respect to key correlations. In particular, for the exogenous growth model, the correlation between detrended consumption and investment is -0.05816. By contrast, our endogenous growth model produces a correlation of 0.50802, which performs significantly better and is much closer to the value of 0.57955 observed in the data.

Table 1.4 Contemporaneous correlations with respect to R&D

	$\rho(\hat{X},\hat{I})$	$\rho(\hat{X},\hat{C})$	$\rho(\hat{Y},\hat{X})$
Data	0.53291	0.51521	0.57955
Endogenous growth	0.61245	0.97237	0.89619

Note: $\hat{Y}$, $\hat{X}$, $\hat{C}$ and $\hat{I}$ are percentage deviations from the trend.

MoreoverTable 1.4 shows selected correlations from an endogenous growth New Keynesian(which includes trend inflation, variable capacity utilization and no price or wage indexation) and data from the Federal Reserve Bank of St. Louis. There is no comparable data that can be generated from the exogenous growth model, since it is devoid of R&D. The model does very well when looking at correlations of investment R&D

and investment in physical capital. It, however, overstates the correlations of R&D investment with output and consumption. We offer some conjectures as to why and how this issue may be resolved in the next section that will warrant further verification.

1.5 Conclusion

In this chapter, we developed a hybrid model that integrates Schumpeterian endogenous growth into a New Keynesian DSGE framework, allowing us to explore the interactions between RD-driven innovation and business cycle fluctuations. The model highlights the role of endogenous technology investment as a transmission channel for economic shocks, particularly in response to monetary policy, productivity, and spillover effects.

A key modelling choice in this chapter was the assumption of one-period patents, which ensures that innovations provide temporary monopoly power before becoming freely accessible to competitors. This assumption simplifies the analysis of innovation dynamics by emphasizing the short-term incentives to invest in RD and their impact on economic fluctuations. It also allows us to maintain analytical tractability while integrating innovation within a DSGE framework with nominal rigidities. Moreover, it provides a direct link between business cycles and growth, as firms must continuously innovate to sustain their market position, reinforcing the pro-cyclicality of RD investment.

However, this approach also has limitations. By restricting the duration of monopoly power, the model abstracts from the long-term strategic behavior of firms, such as the impact of patent length on investment incentives, pricing strategies, and knowledge accumulation. Additionally, the assumption that firms lose exclusivity after one period may overstate the responsiveness of innovation to short-term economic fluctuations, as real-world patents typically grant longer-lasting protection that influences firms' dynamic optimization over multiple periods.

To address these limitations, Chapter 2 extends the framework by introducing infinitely lived patents, allowing for a richer analysis of how long-term market power influences innovation decisions and macroeconomic outcomes. This alternative approach captures how sustained monopolistic rents affect firms' incentives to invest in RD and the broader implications for economic growth and business cycle persistence.

Beyond this extension, several avenues for future research remain. One potential direction is to examine patents with finite but multi-period duration, which would bridge the gap between the extreme cases of one-period and infinitely lived patents. Another promising extension would be to introduce endogenous patent length as a policy variable, analyzing the optimal balance between incentivizing innovation and promoting competition. Finally, further work could investigate the role of RD subsidies or intellectual property rights enforcement in shaping the cyclical properties of innovation-driven growth.

By integrating innovation within a DSGE framework, this chapter provides new insights into the macroeconomic effects of endogenous growth, setting the stage for the more general analysis of patent structures in the following chapter.

CHAPTER 2

SCHUMPETERIAN GROWTH, PRICE RIGIDITIES, AND THE BUSINESS CYCLE*

*Joint with Alain Paquet

ABSTRACT

This research examines the implications of embedding Schumpeterian innovation into the intermediate production sector within a New Keynesian DSGE model with nominal wage and price stickiness. We find that endogenous decisions to invest in $R\&D$ have significant implications for the likelihood of innovating and pushing the technological frontier, while also providing a relevant transmission channel for common shocks that affect business cycles.

The study addresses new theoretical challenges in modelling and simulation, particularly with respect to the interaction between Schumpeterian innovation and price rigidities, as well as between business cycle and growth. Our incorporation of Schumpeterian innovation enables us to consider the implications of knowledge-spillover shocks, an additional dimension not typically found in standard business cycle models.

Our calibration of the model yields key moments and comovements for important macroeconomic variables that are consistent with their observed counterparts. We observe the cyclical impacts of various common shocks as well as knowledge-spillover shocks on macroeconomic variables. We find that the variables' dynamics are not invariant to the parameter calibration of steady-state endogenous growth.

We also investigate the welfare implications of different combinations of steady-state innovation probability and the extent of knowledge spillovers, for the same steady-state growth rate of the economy. Our findings show that, compared to a 15% quarterly innovation probability, a 23% quarterly probability of innovating, accounting for dynamic interactions and consistent with an annual 1.95% of the technological frontier, leads to a 4.7% increase in welfare in consumption-equivalent terms, as it is associated with a lower degree of prevailing price rigidity.

KEY WORDS: Schumpeterian endogenous growth; Innovation; Business cycles; New Keynesian dynamic stochastic general equilibrium (DSGE) model; nominal price rigidity and flexibility.

JEL CODE: E32, E52, O31, O33, O42

2.1 Introduction

Economic growth is widely considered the backbone of any economy. One of its key drivers is investment in Research and Development (R&D), which fuels the creation of knowledge and technological progress, ultimately generating long-term growth (Romer, 1986, 1990; Lucas, 1988; Rebelo, 1991; Aghion & Howitt, 1992). However, most dynamic stochastic general equilibrium (DSGE) models exclude this mechanism, assuming that it is not relevant for the business cycle. Ghironi (2018) and Stiglitz (2018) challenge this notion, arguing that DSGE models require stronger microeconomic foundations, especially concerning endogenous growth. As (Stiglitz, 2018) notes, "DSGE models are, of course, not really a model of medium- to long-term growth: that is determined by factors like the pace of innovation and the accumulation of human capital on which they provide little insight." This underscores the need for a more detailed analysis of technological progress, including investments in basic research and knowledge transmission across firms, to better understand both short-term fluctuations and long-term growth.

Comin & Gertler (2006) demonstrated that R&D has business cycle frequency effects, contradicting the assumption that innovation plays only a long-run role. Similarly, Barlevy (2007) and Fatas (2000) showed that R&D investment responds to monetary policy shocks, indicating that innovation can act as a transmission channel for cyclical fluctuations. Despite these findings, until recently, few studies have attempted to integrate Schumpeterian endogenous growth into business cycle models. Most DSGE models—whether early real business cycle (RBC) models or modern New-Keynesian (NK) macroeconomic frameworks—have been built around an exogenous growth structure (Smets & Wouters, 2007; Justiniano et al., 2010). Some work has incorporated Schumpeterian innovation, such as Nuño (2011), who introduced it into an RBC model but without nominal rigidities, or Amano et al. (2012), who examined horizontal innovations in a staggered-price NK economy. More recent contributions, such as Annicchiarico & Rossi (2013) and Annicchiarico et al. (2011), have explored the implications of endogenous growth for monetary policy and inflation dynamics.

In Chapter 1, we introduced a hybrid NK-DSGE model with Schumpeterian endogenous growth, where innovators were granted one-period patents before their technology became freely accessible to competitors. This assumption simplified the analysis of innovation dynamics and allowed for a tractable representation of the interaction between growth and business cycles. However, it also imposed constraints on firm behavior and market structure, as innovators could only benefit from their discoveries for a single period. While this assumption helped illustrate the cyclical sensitivity of R&D, it did not allow for a full examination of how long-term market power shapes innovation incentives and macroeconomic fluctuations.

In this chapter, we extend the model by introducing infinitely lived patents, allowing firms to retain monopoly power over their technological advancements indefinitely. This modification provides a more general and flexible way to integrate Schumpeterian innovation into an NK-DSGE framework while maintaining solvability. The introduction of persistent patent protection allows us to analyze how monopoly rents from innovation affect firms' R&D investment decisions, price-setting behavior, and aggregate macroeconomic outcomes over time. Furthermore, this change influences the transmission of monetary and technological shocks, as firms with lasting patent protection may respond differently to policy changes than those in a model with short-lived patents.

Beyond its implications for economic growth and business cycles, innovation also plays a crucial role in determining price flexibility. The arrival of new products, often driven by innovation, affects how frequently firms adjust prices, influencing inflation dynamics in an economy with nominal rigidities. Empirical work by Bils & Klenow (2004) highlights that markets with higher product turnover tend to exhibit greater price flexibility. They report that between 1995 and 1997, the U.S. Bureau of Labor Statistics classified 46% of all product substitutions as noncomparable, meaning that the introduction of new products led to significant changes in consumption baskets. Their findings suggest that a 1% increase in the rate of noncomparable substitutions raises the frequency of price changes by 1.25%, independent of market concentration or markup levels. More recently, Goolsbee & Klenow (2018) found that 44% of online sales between 2014 and 2017 were for products that did not exist a year earlier, reinforcing the notion that digital markets further amplify this relationship between innovation and price flexibility.

This empirical evidence motivates a key refinement in our model: new innovating firms gain the ability to set an optimal price upon entry but may later face price stickiness due to nominal rigidities. This captures an essential economic mechanism—innovation does not just advance the technological frontier; it also increases price flexibility by introducing new goods that reset pricing structures. To our knowledge, this is the first model that explicitly formalizes the link between innovation and price flexibility in a New Keynesian framework, enabling us to assess its impact on both inflation persistence and the transmission of shocks.

From a theoretical perspective, this required a meticulous modelling of the interaction between Schumpeterian innovation and price rigidities, as well as a precise articulation of the link between business cycles and growth. In the presence of Calvo-style price setting, firms face constraints on price adjustments, which directly influence expected profits, the incentives to invest in R&D, and the overall pace of innovation. Importantly, our framework endogenously generates a link between innovation and price flexibility, reinforcing the idea that more innovation leads to greater price flexibility in a world with nominal rigidities.

By transitioning from one-period patents to infinitely lived patents and explicitly linking innovation with price flexibility, this chapter moves toward a more general and adaptable framework for integrating Schumpeterian innovation into New Keynesian DSGE models. This approach ensures that the model remains both economically realistic and computationally tractable, while also shedding light on a key empirical regularity—that higher innovation-driven product turnover leads to more frequent price adjustments, influencing both inflation persistence and the transmission of monetary policy shocks.

The remainder of this chapter is structured as follows. Section 2 presents the model, beginning with standard NK-DSGE components and then introducing the key modifications required to incorporate infinitely lived patents and the innovation-price flexibility link. Section 3 discusses the aggregation process and general equilibrium conditions, particularly how to address the trending behavior of technology in the model's steady state. Section 4 covers calibration and characterizes the different disturbances affecting the economy. Section 5 examines statistical moments and their alignment with empirical data. Section 6 presents the impulse response functions and analyzes how innovation influences business cycle dynamics and price flexibility. Section 7 explores the welfare implications of different patent structures, particularly how changes in innovation probability and knowledge spillovers affect long-run growth and inflation dynamics. Finally, Section 8 summarizes our findings and suggests possible extensions, including models with finite but multi-period patent duration or endogenous patent length.

2.2 The model

The framework, consisting of different groups of agents, such as households, final good producers, an employment agency, intermediate good producers, entrepreneurs/innovators, a monetary authority, and the government. Forward-looking households aim to maximize their expected utility by making optimal decisions regarding consumption, labor, physical capital utilization, private investment, and net bond holdings over their budget constraints. Final good producers operate in a perfectly competitive market and utilize intermediate goods as input. The employment agency aggregates the specialized labor of households to generate homogeneous labor used by the intermediate good producers, who operate within a monopolistically competitive environment that allows them to set prices.

In each intermediate sector i, entrepreneurs/innovators invest final goods to increase their chances of pushing the technological frontier. When an intermediate good producer implements a new technology, it takes over the incumbent producer in the same intermediate sector.

We can describe the timing of events within each intermediate sector i as follows

- A time t:
- Step 0: The prevailing productivity level $A_{t-1}(i)$ is inherited from the previous period
- Step 1: In sector i, a randomly chosen entrepreneur invests $X_t(i)$ in $R\&D$
- Step 2: The entrepreneur either succeeds or fails. If successful, the innovation pushes the frontier forward to $A_t(i)$, that will apply next period
- Step 3: The incumbent produces $y_t(i)$ with technology $A_{t-1}(i)$ and collects profits
- At time $t+1$:
- Step 4: the successful entrepreneur uses his more advanced technology $A_t(i)$ to set a lower price and to eliminate his sector's incumbent
- Step 5: A new entrepreneur invests $X_{t+1}(i)$ in $R\&D$
- $\dots$

Consequently, entrepreneurs in each sector allocate final goods to research and development, aspiring to discover novel technologies. The entrepreneur's outcome is binary, either resulting in success or failure. Upon success, the entrepreneur attains the technological frontier $A_t(i)$, which will be utilized for production in the subsequent period. At date $t+1$, the successful entrepreneur supersedes the incumbent within the sector by offering a more competitive price. The entrepreneur then remains in place, accruing monopoly profits until eventually being replaced by another entrepreneur.

Moreover, prices and wages are subject to nominal rigidities through contracts in the style of Calvo (1983). Gradual price adjustments directly influence the innovation process, as the discounted expected value of investing in R&D affects the innovation rate throughout business cycles. Monetary policy adheres to a Taylor rule, and deviations from it are considered alongside the impacts of various shocks, such as transitory technological shocks, knowledge spillover shocks, and investment shocks.

We now proceed to a comprehensive examination of the environment and challenges confronted by different agent types. Initially, we delineate the attributes of the final good producer, the employment agency, and the households' dilemmas. Largely, these aspects align with the standard framework found in contemporary dynamic stochastic general equilibrium literature. When necessary, we incorporate elements stemming from the existence of innovation-driven firms ultimately owned by households. Subsequently, we concentrate on the unique characteristics introduced by Schumpeterian considerations for innovators and intermediate good producers, particularly in the context of slow price adjustments. Lastly, we tackle aggregation issues and present the monetary authority's policy function.

2.2.1 A presentation of the common features of a New Keynesian DSGE model

2.2.1.1 The final good producer

The final consumption good is produced by a representative firm that operates in a perfectly competitive setting and that aggregates a continuum of intermediate goods $i \in (0,1)$ according to the following production function:

$$Y_t = \left(\int_0^1 Y_t(i)^{\frac{\epsilon-1}{\epsilon}} di \right)^{\frac{\epsilon}{\epsilon-1}} , \tag{2.1}$$

where Y_t is total final output, the input $Y_t(i)$ is the good produced by an intermediate level firm i, and $0 \leq \epsilon < \infty$ is the elasticity of substitution between intermediate goods.

Because of perfect competition, the final-good producer takes as given the price of its final output, P_t, and the prices of the intermediate goods, $P_t(i)$. Hence, its profit maximization problem

$$\max_{Y_t(i)} \Pi_{FG} = P_t \, Y_t - \int_0^1 P_t(i) \, Y_t(i) \; d\, i, \tag{2.2}$$

yields the demand for the i^{th} intermediate good as a negative function of its relative price, namely

$$Y_t(i) = \left(\frac{P_t(i)}{P_t}\right)^{-\epsilon} Y_t \,. \tag{2.3}$$

Since economic profits are zero under perfect competition, total nominal output is given by the sum of the nominal value of all intermediate goods i

$$P_t\, Y_t = \int_0^1 P_t(i)\, Y_t(i)\; d\,i \,, \tag{2.4}$$

which, using equation (2.3), yields the aggregate price index

$$P_t = \left[\int_0^1 P_t(i)^{1-\epsilon}\; d\,i\right]^{\frac{1}{1-\epsilon}} \,. \tag{2.5}$$

2.2.1.2 The employment agency

In our model economy, a continuum of households possesses different skills and offers specialized labour $L_{Ht}(j)$ for $j \in (0, 1)$, that gives them some degree of market power in setting wages. Since intermediate firms use a combination of specialized labour, we can think of a representative employment agency which aggregates specialized labour and turns it into the combined labour input L_t employed by the intermediate firms, namely

$$L_t = \left(\int_0^1 L_t(j)^{\frac{\gamma-1}{\gamma}}\; d\,j\right)^{\frac{\gamma}{\gamma-1}} \,, \tag{2.6}$$

where $0 \leq \gamma < \infty$ is the elasticity of substitution between each labour type.

Operating in perfect competition, the employment agency maximizes its profits with respect to $L_t(j)$ while taking as given the aggregate wage rate W_t and the prevailing labour compensation specific to each labour type j.

The solution of its optimization problem

$$\max_{L_t(j)} \Pi_{EA} = W_t\, L_t - \int_0^1 W_t(j)\, L_t(j)\; d\,j \tag{2.7}$$

yields the demand for specialized labour j as a negative function of its relative wage rate

$$L_t(j) = \left(\frac{W_t(j)}{W_t}\right)^{-\gamma} L_t\ . \tag{2.8}$$

From equation (2.8), and the competitive equilibrium for the employment agency, the aggregate wage rate is

$$W_t = \left[\int_0^1 W_t(j)^{1-\gamma}\, dj\right]^{\frac{1}{1-\gamma}}\ . \tag{2.9}$$

2.2.1.3 Households

The Budget Constraint

Each period t, a representative type j household faces the following budget constraint

$$P_t\, C_t \ +\ P_t\, I_t \ +\ P_t\, a(u_t)\widetilde{K}_t \ +\ \frac{B_t}{1+R_t} \quad \leq \quad W_t(j)\, L_t(j) \ +\ q_t\, u_t\widetilde{K}_t \ +\ B_{t-1} \ +\ D_t\ \ . \tag{2.10}$$

As of date t, the household's nominal value for its uses of funds comprises the sum of its nominal value of consumption in the final good, denoted as P_tC_t, its desired level of investment in capital goods, P_tI_t, the resources allocated to adjust the utilization rate of physical capital (if applicable), $P_ta(u_t)\widetilde{K}t$, and its end-of-period net holdings of a one-period discount bond $\frac{Bt}{1+R_t}$, where R_t represents the nominal interest rate between t and $t+1$.[1] It is assumed that the price of consumption, private investment in physical capital, and varying utilization of capital correspond to the aggregate price level P_t.

[1] The net bond holdings may be positive or negative, depending on whether the household is either a creditor or debtor. However, for this closed economy, the aggregate net bond holdings are zero in equilibrium.

Our framework accommodates a time-varying utilization of the existing stock of physical capital, $\widetilde{K}t$, less than 100%, on the condition that the household incurs a cost of varying capital utilization u_t. This real cost is captured by a convex function $a(u_t)$ that increases with u_t.[2]

Type j household's nominal sources of funds originate from its labor income, i.e., the product of its nominal wage rate $W_t(j)$ and hours worked $L_t(j)$, its nominal payments received from providing capital services to intermediate firms, from renting a portion of its existing physical capital, $u_t\,\widetilde{K_t}$, at a gross capital rental rate q_t, the nominal face value of the net discount bond holdings carried from the previous period, and the nominal dividends, D_t, obtained from its ownership of shares in the intermediate production sector operating in monopolistic competition, less the value of lump-sum taxes, net of government transfers.

Consequently, it is necessary to evaluate the dividends derived from the economic rent, partly influenced by investments in R&D. Utilizing aggregate labor and capital, a firm i, belonging to a continuum defined over $i \in (0, 1)$, produces intermediate good i in a monopolistically competitive market, thereby generating positive economic profits.

$$\Pi_{i,t} = P_t(i)\,Y_t(i) - w_t\,L_t(i) - q_t\,K_t(i)\ , \tag{2.11}$$

that are in turn paid as dividends among households. Hence, we can think of each intermediate firm as producing some good i, with a given technology discovered from past $R\&D$ that allowed it to take over sector i.

The investment in $R\&D$ has to be accounted for in each period, while being treated as a sunk cost since it is irrelevant whether or not an innovator is successful *ex post*. We will use two examples to illustrate how we account for the innovation process in the representative household's budget constraint.

Having invested $P_t\,X_t$ to reach the frontier, a failed innovator generates no profits, as depicted on the timeline of his cash flow in Figure 2.1.

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[2] It is also assumed that, in the special case where the capital stock is used at full capacity with $u_t = 1$, such that $u_t, \widetilde{K}t = K_t$, this real cost function assumes a value of zero, i.e., $a(1) = 0$.

Figure 2.1 A failed innovator's timeline for cashflows

Time period	t	t+1	t+2	t+3	t+4	t+5
Cash flow	$-P_t X_t$	0	0	0	0	0

In this case, the household's budget constraint needs only to include his initial investment. The *ex post* value of engaging in the innovation process is $-P_t X_t$.

By comparison, a successful innovator who invested $P_t\, X_t$ in $R\&D$, which turns into a successful endeavour that allows him to collect monopoly profits π_t each period, for τ periods, until it is replaced by a new innovator. Figure 2.2 illustrates the corresponding flow timeline.

Figure 2.2 A successful innovator's timeline for cashflows

Time period	t	t+1	t+2	t+3	t+τ	t+$\tau + 1$
Cash flow	$-P_t X_t$	Π_{t+1}	Π_{t+2}	Π_{t+3}	$\Pi_{t+\tau}$	0

Here, the initial investment as well as future profits should be included in their respective budget constraints. It is important, however, to highlight that the profits included in the timeline above do not exclusively result from the innovation process. Indeed, an intermediate firm is already generating profits prior to an innovator taking over. Hence, the profits generated by the intermediate firm after the takeover include both monopoly and innovation profits.

Accordingly, the overall dividends paid to households are therefore defined as:

$$D_t = \int_0^1 \left[\Pi_{i,t} - P_t\, X_t(i) \right] d\,i\,. \tag{2.12}$$

Utility maximization

Similarly to Christiano et al. (2005), household j maximizes its utility function with respect to the sequence of its budget constraints for each period, while taking into account the law of movement of capital. Its preferences for consumption embed habit formation, with an intensity parameter $h > 0$, which generates some additional intrinsic dynamics and persistence on both the demand and supply sides of the economy following various shocks. The subjective discount factor is $0 < \beta < 1$, the parameter $\theta > 0$ induces disutility of labour, and the parameter $\nu \geq 0$ implies that the Frisch elasticity of labour supply is $1/\nu$. Furthermore, we assume that the household incurs some cost of adjusting investment $S(\cdot)$, which is an increasing concave function of the growth rate in investment.

The representative household must decide how much to consume C_t, while allowing for some habit formation, how many hours $L_t(j)$ to work, how much capacity to use u_t, how much physical capital they want next period $\widetilde{K}_{t+1}$, how much to invest I_t in physical capital, and the size of their net bond holdings B_t by solving the following optimization problem:

$$\max_{C_{t+s},L_{t+s}(j),u_{t+s},I_{t+s},B_{t+s}} E_t^j \sum_{s=0}^{\infty} \beta^s \Big(ln\big(C_{t+s} - h\, C_{t+s-1}\big) - \theta \, \frac{L_{t+s}(j)^{1+\nu}}{1+\nu} \Big) \tag{2.13}$$

subject to

$$P_{t+s}\, C_{t+s} + P_{t+s}\, I_{t+s} + P_{t+s}\, a(u_t)\widetilde{K}_{t+s} + \frac{B_{t+s}}{1+i_t} \leq W_{t+s}(j)\, L_{t+s}(j) + q_{t+s}\, u_{t+s}\widetilde{K}_{t+s} + B_{t+s-1} + D_{t+s}\ , \tag{2.14}$$

$$\widetilde{K}_{t+s+1} = \mu_{_{I,t+s}} \left[1 - S\Big(\frac{I_{t+s}}{I_{t+s-1}} \Big) \right] I_{t+s} + (1-\delta)\widetilde{K}_{t+s}\ , \tag{2.15}$$

$$\ln \mu_{_{I,t+s}} = \rho_{_I}\ \ln \mu_{_{I,t+s-1}} + \epsilon_{_{I,t+s}} \tag{2.16}$$

$$K_{t+s} = u_{t+s}\widetilde{K}_{t+s}\ . \tag{2.17}$$

where E_t^j is the expectation operator conditioned of known information as of the beginning of the period t. The function $S(\ \cdot\ )$ represents a convex adjustment function cost incurred when transforming current and past investment into installed capital.[3] Moreover, an exogenous stochastic investment shock $\mu_{I,t+s}$, that affects the efficiency with which investment is transformed into capital, follows a first-order autoregressive process. Finally, we associate the following Lagrange multipliers Λ_t and Φ_t respectively, with the household's budget constraint equation (2.14), and the investment equation (2.15) at date t.

In addition, having assumed that a household j possesses some specialized skills underlying some market power over its wage rate, we also assume the existence of wage rigidities modelled with Calvo contract arrangements, with a constant proportion $1-\xi_w$ being allowed to reoptimize their wage each period. Hence, household j maximizes its expected utility weighed by the probability ξ_w of not being allowed to optimize with respect to wages subject to the labour demand function:

$$\max_{W_{t+s}(j)} E_t^j \left(\sum_{s=0}^{\infty} \xi_w^s \, \beta^s \left(-\theta \, \frac{L_{t+s}(j)^{1+\nu}}{1+\nu} \right) + \Lambda_{t+s} \, W_{t+s}(j) \, L_{t+s}(j) \right) \tag{2.18}$$

subject to

$$L_{t+s}(j) = \left(\frac{W_t(j)}{W_{t+s}} \right)^{-\gamma} L_{t+s} \, , \tag{2.19}$$

where γ is type-j labour demand elasticity to the relative wage.

Accordingly, the optimal reset wage is obtained from

$$W_t^*(j)^{-\gamma\nu-1} = \frac{\gamma-1}{\theta\,\gamma} \frac{\sum\limits_{s=0}^{\infty} \xi_w^s \beta^s \Lambda_{t+s} \, W_{t+s}^{\gamma} L_{t+s}}{\sum\limits_{s=0}^{\infty} \xi_w^s \beta^s \, W_{t+s}^{\gamma(1+\nu)} \, L_{t+s}^{1+\nu}} \, . \tag{2.20}$$

Exploiting the relevant recursions built in the summation, which, in turn, will be useful for subsequent numeric simulation, equation (2.20) can be rewritten as

[3] At the calibration stage, we assume it to be defined as $S\big(I_t/I_{t-1}\big) = (\kappa/2)\big(I_t/I_{t-1} - g_t\big)^2$. Hence, the investment adjustment cost is defined in relation with departure of physical investment growth from its steady-state trend growth, i.e. that of the technological frontier in the steady state.

$$W_t^*(j)^{-\gamma\nu-1} = \frac{\gamma-1}{\theta\gamma}\frac{Aux_{bc,t}}{Aux_{dis,t}}\ , \tag{2.21}$$

where, we define auxiliary variables associated, respectively, with the household's budget constraint in the numerator, $Aux_{bc,t}$, and the disutility of labour in the denominator, $Aux_{dis,t}$, i.e.

$$Aux_{bc,t} = \Lambda_t\, W_t^{\gamma} L_t + \xi_p\, \beta\, Aux_{bc,t+1}\ , \tag{2.22}$$

and

$$Aux_{dis,t} = \Lambda_t W_t^{\gamma(1+\nu)} L_t^{1+\nu} + \xi_p\, \beta\, Aux_{dis,t+1}\ . \tag{2.23}$$

2.2.2 The Schumpeterian add-ons in a New-Keynesian DSGE model and their implications

In academic literature and policy documents, the concept of innovation is often broad. Not all innovations amount to paradigm-shifting, as groundbreaking discoveries such as the steam engine occur infrequently. Therefore, by innovation, we refer to any development that advances the technological frontier, even incrementally. Incremental innovations can be understood as improvements in either the intermediate good itself or the production process. In our model, both types of innovations push the technological frontier forward and reduce the marginal cost of production.[4]

Building on Aghion & Howitt (1992) and Nuño (2011), our Schumpeterian growth paradigm assumes that an innovation may emerge from investing in R&D in period $t-1$ and pushes the technology level at A_{t-1}^{max} for an intermediate firm operating at date t with an endogenous probability n_{t-1}, rendering it an advanced firm. Otherwise, there is a $1-n_{t-1}$ probability that an intermediate firm is lagging, while still using an older technology level.

[4] As Kirschenbaum (2018) points out, "invention and innovation are also about revision and refinement, a gradual process of shaping, adaptation, and perhaps, elusively, perfection. Invention, in other words, is a lot like word processing itself, which allows us to continuously edit our ideas, cutting and pasting, inserting and deleting until we get what we're working on just where we want it".

The presence of price stickiness, however, introduces an additional layer of complexity absent in typical Schumpeterian growth models and recent business cycle models with Schumpeterian features without price rigidity. Specifically, since a firm's investments in R&D depend on the discounted expected profits resulting from innovating, if successful, the monopolistic rent also hinges on the expected prices that a firm will be permitted to set (with some probability that prices may be fixed and some probability that they may be adjusted).

When implementing a newly discovered innovation, we assume that an intermediate firm is allowed to set the optimal price immediately. However, at subsequent dates (quarters), barring any new innovation, the same intermediate firm is constrained by older technology, and there are probabilities that its price remains sticky for some time. Indeed, if it is not innovating, the lagging firms operate in a Calvo-type environment as in standard NK models.

Consequently, three categories of intermediate firms coexist: advanced firms that reset the optimal price, lagging firms permitted to reoptimize their respective price, and lagging firms with previously set prices.

2.2.2.1 The optimal reset price

The relationship between innovation and price adjustments is fundamental to understanding how firms navigate cost structures, competitive pressures, and inflationary environments. Innovation reduces the frictions that traditionally constrained price flexibility, such as menu costs and information lags, enabling firms to adjust prices more frequently and efficiently. This effect is particularly pronounced in industries where technology-driven pricing tools, automation, and digital platforms allow firms to react in real-time to market changes.

One key mechanism is the role of technological advancements in lowering the costs of price adjustments. Historically, firms faced substantial barriers to changing prices, including administrative expenses, customer backlash, and rigid pricing contracts. However, the development of automated pricing systems and dynamic algorithms—especially in digital markets—has minimized these barriers. In e-commerce, for instance, firms can instantly update prices based on demand fluctuations, competitor pricing, or supply chain shocks, leading to a more fluid and responsive market structure. Studies such as Zhelobodko et al. (2012) highlight how innovation in retail pricing fosters more frequent price adjustments, particularly under monopolistic competition.

Innovation also reshapes competitive dynamics, which further enhances price flexibility. When technological progress lowers entry costs, new firms can challenge incumbents, forcing existing firms to adjust prices more frequently to maintain their market position. Bils & Klenow (2004) demonstrate that markets with more competition tend to exhibit less price stickiness, as firms must constantly update their pricing strategies to remain competitive. This effect is amplified by technological improvements that facilitate faster information processing and decision-making, reducing firms' reliance on preset or rigid pricing schemes.

Additionally, innovation influences how firms respond to inflationary pressures. Sectors that incorporate advanced pricing technologies can adjust more swiftly to inflationary shocks, mitigating their impact. Gopinath & Itskhoki (2010) show that firms in technologically advanced industries exhibit higher price flexibility, allowing them to pass through cost changes more efficiently. In contrast, traditional sectors, where pricing decisions are still governed by slower, more manual processes, experience greater price inertia, delaying inflationary adjustments and potentially causing economic inefficiencies.

A striking example of how innovation reshapes price-setting behavior is seen in the rise of digital markets and online pricing mechanisms. Cavallo (2018) and Goolsbee & Klenow (2018) emphasize that e-commerce platforms have fundamentally altered how firms set prices by increasing the frequency of updates and enhancing market transparency. Online retailers continuously monitor demand, competitor prices, and inventory levels, adjusting prices multiple times a day in some cases. This dynamic pricing capability reduces inflation persistence by allowing firms to react almost instantaneously to cost changes, in contrast to traditional brick-and-mortar businesses, where price changes tend to be less frequent.

Overall, innovation acts as a catalyst for price flexibility by reducing adjustment costs, intensifying competition, and enhancing firms' ability to respond to inflation. The increasing prevalence of digital markets and algorithmic pricing further accelerates this trend, challenging traditional assumptions about price stickiness in macroeconomic models.

Innovation serves as a catalyst for reducing price rigidity by lowering the costs of price changes, intensifying market competition, and enhancing the adaptability of firms to economic fluctuations. These factors make it reasonable to assume a strong link between innovation and price adjustments, as technological progress fosters a more responsive and flexible pricing environment. This relationship not only influences individual markets but also shapes broader economic outcomes, including inflation dynamics and competitive equilibrium.

Operating in a monopolistically competitive market, intermediate firms hold market power from both their specialization and the technology used in production. In itself, it is worth noticing that the mechanics of innovation being considered also brings some support and microfoundations to the monopolistic competition *de facto* introduced in the usual New Keynesian models. Moreover, prices are fixed through Calvo contracts and set as to maximize their expected profits conditional on not being allowed to optimize.

Hence, given an initial level of technological advancement $A_t(i)$, an intermediate firm faces the following constrained minimization of their cost:

$$\min_{K_t(i),L_t(i)} W_t L_t(i) + q_t K_t(i) \tag{2.24}$$

subject to

$$Y_t(i) = \mu_{Z,t}\, A_t(i)^{1-\alpha}\, K_t(i)^{\alpha}\, L_t(i)^{1-\alpha}\ , \tag{2.25}$$

with $\alpha \in (0,1)$, where

$$\ln \mu_{Z,t} = \rho_Z\ \ln \mu_{Z,t-1} + \epsilon_{Z,t}\ , \tag{2.26}$$

so that, regardless of their individual level of technological advancement, all intermediate firms' productions are subjected to a common transitory technological shock, that follows a first-order autoregressive process.

Accordingly, the optimal capital-labour ratio (that is identical for all intermediate firms) being employed is

$$\frac{K_t}{L_t} = \frac{\alpha}{1-\alpha}\frac{W_t}{q_t}\ , \tag{2.27}$$

and the nominal marginal cost of producing an additional unit of intermediate good is given by

$$MC_t(i) = A_t(i)^{\alpha-1}\,\Omega_t \tag{2.28}$$

where Ω_t is the portion of marginal costs that is not directly dependent of the level of technology, i.e.

$$\Omega_t = \frac{q_t^{\alpha} W_t^{1-\alpha}}{\alpha^{\alpha}(1-\alpha)^{1-\alpha}} \ . \tag{2.29}$$

Hence, an improvement in technology leads to a lowering of a firm's marginal cost.

In traditional New-Keynesian models, all firms operate at the same level of technological advancement, so that, when possible, all firms set the same optimal reset price. In contrast, since the optimal reset price depends on the technology, in our set-up, with an infinite number of intermediate firms, there is an infinite number of coexisting technologies. Accordingly, there is an infinite number of reset prices because the marginal cost is a function of the technology level.

Consequently, given their respective marginal cost, intermediate firms maximize their profits with respect to their price $P_t(i)$:

$$\max_{P_t(i)} \ E_t(P_t(i) - MC_t))Y_t(i) + \left\{ \sum_{s=1}^{\infty} \xi_p^s \, \beta^s \frac{\Lambda_{t+s}}{\Lambda_t} \left[\prod_{q=1}^{s} (1 - n_{t+q-1}) \Big(P_t(i) - MC_{t+s}(i) \Big) \, Y_{t+s}(i) \right] \right\} \tag{2.30}$$

subject to

$$Y_{t+s}(i) = \left(\frac{P_t(i)}{P_{t+s}} \right)^{-\epsilon} Y_{t+s} \ , \tag{2.31}$$

To solve this problem, it must be noticed that initial technology is known and can differ between firms, while future technology levels depend as well on future investment levels in $R\&D$.

Thus, it can be shown that the optimal reset price is a function of initial technology $A_t(i)$, and of a factor F_t that is not directly dependent of the technology level:

$$P_t^*(i) = A_t(i)^{\alpha-1} F_t \ , \tag{2.32}$$

where

$$F_t = \frac{\epsilon}{\epsilon - 1} \frac{Aux_{cost,t}}{Aux_{rev,t}} \,, \tag{2.33}$$

$$Aux_{rev,t} = P_t^{\epsilon} \, Y_t + \xi_p \, \beta \, \frac{\Lambda_{t+1}}{\Lambda_t} (1 - n_t) \, Aux_{rev,t+1} \,, \tag{2.34}$$

and

$$Aux_{cost,t} = \Omega_t \, P_t^{\epsilon} \, Y_t + \xi_p \, \beta \, \frac{\Lambda_{t+1}}{\Lambda_t} (1 - n_t) \, Aux_{cost,t+1} \,. \tag{2.35}$$

2.2.2.2 The innovation process

R&D activities are conducted by entrepreneurs or innovators. If these activities lead to an innovation, the implementation of this technology by an intermediate good producer provides additional market power by producing an improved version of the intermediate good, as it reaches the new technological frontier. Consequently, the innovation process unfolds within the intermediate sector as the mechanism that drives the technological frontier outward.

Entrepreneurs or innovators invest a certain amount of final goods to increase the probability of innovating. External researchers or a new successful innovator may supplant or "leapfrog" an incumbent entrepreneur.[5] However, this prospect is uncertain, as the probability to innovate is n_t, and that of not discovering is $1-n_t$. Yet, n_t is endogenous, as it is linked to the intensity of R&D effort $\frac{X_t}{\zeta, A_t^{max}}$, where X_t is the real amount of final goods invested in R&D, A_t^{max} is the targeted technology level or frontier used in the production of date $t+1$, and $\zeta > 1$ is a scaling factor that can be derived from an innovation. Specifically, when A_t^{max} is larger, a given amount of resources X_t in R&D is associated with lower intensity, thus capturing the

[5] In our model, we abstract from step-by-step technological progress, which would imply both Schumpeterian and escape-competition effects. This can also be justified by assuming that engaging in R&D makes it prohibitively costly to develop a perfectly substitutable technology that can be used to produce a cheap and fake copy of an existing intermediate good, i.e., a knockoff.

increasing complexity of further progress. Finally, we consider the following innovation production function that exhibits diminishing marginal returns, with $\eta > 0$:

$$n_t = \left(\frac{X_t}{\zeta \, A_t^{max}} \right)^{1/(1+\eta)} \quad . \tag{2.36}$$

Furthermore, the gross growth rate of the technological frontier A_t^{max} is dictated by the probability of innovation times a spillover factor, which is subject to some first-order autoregressive stochastic component. This defines a proportional increase in productivity resulting from an innovation.[6]
Namely,

$$A_t^{max} \; = \; g_t^{max} \; A_{t-1}^{max} \; , \tag{2.37}$$

$$g_t^{max} \; = \; 1 \; + \; \sigma_t \, n_{t-1} \; , \tag{2.38}$$

where

$$\ln \sigma_t = \ln \sigma \; + \; \rho_\sigma \; \ln \sigma_{t-1} \; + \; \epsilon_{\sigma,t} \; . \tag{2.39}$$

The introduction of a stochastic spillover shock on σ_t accounts for unpredictable variations and other heterogeneities in the transmission of knowledge and/or abilities to capitalize on new innovations to push the technological frontier further. Consequently, a knowledge spillover represents the positive externality derived from an innovation, as it permanently advances the technological frontier.[7] The higher the value of

[6] While the steady state growth rate of the frontier is constant, the frontier itself could follow different paths. Indeed, a small deviation of the growth rate, caused by the stochastic nature of the model, could put the evolution of technology on different trajectories.

[7] Baldwin et al. (2005) have demonstrated, both theoretically and empirically, how knowledge spillovers generated by multinational corporations and foreign direct investments enhance growth endogenously. It is reasonable to argue that a similar effect can span across firms within an industry, as well as across industries to some extent. For example, we can consider spillovers from the diffusion of information and communication technologies. Moreover, based on industry-level data for 15 OECD countries, Saia

σ_t, the greater the extent of technology spillover, which, in turn, leads to a larger technological leap of the frontier.

In order to determine the optimal amount of final good to be invested in $R\&D$ that maximizes expected discounted profits, an entrepreneur must solve the following constrained optimization problem:

$$\max_{X_t} \beta \frac{\Lambda_{t+1}}{\Lambda_t} n_t E_t V_{t+1}(A_t^{max}) - P_t X_t \tag{2.40}$$

This problem is subject to equation (2.36). Here, $E_t V_{t+1}(A_t^{max})$ represents the expected discounted value of future profits, conditional on the entrepreneur maintaining control of the monopoly. If successful, the innovator will continue to earn monopoly profits until further innovation occurs in their sector[8].

To simplify the problem and maintain compatibility with complete markets, we assume that entrepreneurs invest in a diversified form of $R\&D$. In essence, this means that a successful entrepreneur cannot predict the specific sector in which their innovation will occur. As a result, all entrepreneurs invest the same amount of final good in $R\&D$. When an innovation takes place, the technology advances to the frontier, and the expected value of the intermediate firm becomes identical across all intermediate sectors. In accordance with equation (2.40), the optimal real investment in $R\&D$ can be expressed as

$$X_t = \beta \frac{n_t}{1+\eta} \frac{\Lambda_{t+1}}{\Lambda_t} \frac{E_t V_{t+1}(A_t^{max})}{P_t} \ . \tag{2.41}$$

To fully solve equation (2.41), we must explicitly describe the expected value of the firm for the entrepreneur. This task proves to be more challenging than initially anticipated due to the dependency on the expected path of prices.

& Albrizio (2015) recently highlighted the significance of knowledge spillovers for an economy's effectiveness in learning from the technological frontier and increasing productivity. An economy's spillovers originate from "its degree of international connectedness, ability to allocate skills efficiently, and investments in knowledge-based capital, including managerial capital and R&D."

[8] In our model, with a steady-state probability of innovation n^{ss}, the expected life span of an intermediate firm at steady-state is $1/n^{ss}$.

The first challenge involves ensuring that the evaluation of future profits follows the correct technological trajectory. For example, an innovator's technology may reach the frontier A_t^{max} in $t+1$ and remain at that level until $t+3$ when it is replaced by a more advanced firm with a newer technology and lower marginal cost of operations. In such a scenario, subsequent expected profits become irrelevant for innovation investment since the firm is no longer in business.

The second obstacle arises from price rigidities that play a crucial role in determining future profits as they influence both the profit margin and the conditional demand for a specific intermediate good. Innovative intermediate firms that advance to the frontier are automatically allowed to reoptimize their prices. In contrast, non-innovative firms may be chosen for reoptimization through Calvo contracts, with a probability ξ_p of not being permitted to optimize their prices. Given that we assume entrepreneurs/innovators diversify their investments in $R\&D$, an entrepreneur can evaluate the discounted expected profits from a potential discovery using individual prices for a range of intermediate goods, even if they do not know the specific sector in which they will innovate *ex-ante*.

To estimate the value of an innovation-implementing intermediate firm, consider an entrepreneur/innnovator who, at date t, ponders how much to invest in $R\&D$ while seeking returns from date $t+1$ onward. This evaluation must account for various possible outcomes that reflect the probability of remaining in control of the monopoly, appropriate stochastic discount factors, and the probability of price reoptimization occurring under Calvo contracts. This is why we must consider all possible contingencies that could yield returns from innovating[9].

First, let us consider the case of a new monopolist assuming control as of date $t+1$ and setting the optimal price for its intermediate good i. As the monopolist has innovated, their prevailing specific technology reaches the new technological frontier, such that $A_t(i) = A_t^{max}$. Each period, the monopolist faces a probability ξ_p of not being allowed to optimize their price. For the contingency path where price reoptimization never occurs, the expected discounted stream of profits is given by

[9] Note that if the monopolist is supplanted at a future date by a competitor's adoption of an innovation, expected profits become zero from that date forward, with no bearing on the current expected discounted flow of profits for the date t investing entrepreneur.

$$\Psi_{1t+1}(i) = \sum_{s=1}^{\infty} \xi_p^{s-1} \beta^{s-1} \frac{\Lambda_{t+s}}{\Lambda_{t+1}} \left[P_{t+1}^*(i) \pi_{p,t,t+s} - MC_{t+s}(i) \right] Y_{t+s}(i) \prod_{q=2}^{s} (1 - n_{t+q-1}) \ , \quad (2.42)$$

Here, $\prod_{q=2}^{s}(1 - n_{t+q-1})$ is the probability of not being displaced out of business at date $t + s$. The flows of revenues and costs are discounted from the perspective of date $t + 1$, as the nested sum is discounted up to the beginning of the initial cash flow pertaining to this stream, and weighted by the probability of remaining in control of the monopoly for all future periods. In particular, the date $t + 1$ cash flow has a unit probability, i.e., $\prod_{q=2}^{1}(1 - n_{t+q-1}) = 1$, as we consider a successful innovation driving the production of intermediate good i that is sold at its optimal price. Moreover, if an intermediate firm producing good i is replaced following the implementation of a new innovation in this sector at some future date T, then no additional profits will accrue from then on from the older technology.

As long as it has not been supplanted, this monopolist will be operating under technology A_t^{max}. Consequently, their marginal cost of production evolves according to:

$$MC_{t+s}(i) = A_t^{max(\alpha-1)} \, \Omega_{t+s} \ , \quad (2.43)$$

while his optimal price is set to

$$P_{t+1}^*(i) = A_t^{max(\alpha-1)} \, F_{t+1} \ , \quad (2.44)$$

and the expected demand for his good follows a path defined by

$$Y_{t+s}(i) = \left(\frac{P_{t+1}^*(i)}{P_{t+s}} \right)^{-\epsilon} Y_{t+s} \ . \quad (2.45)$$

Using equations (2.43), (2.44), and (2.45), equation (2.42) can be rewritten as

$$\Psi_{1t+1}(i) = \qquad A_t^{max(\alpha-1)(1-\epsilon)} \left\{ F_{t+1}^{(1-\epsilon)} \sum_{s=1}^{\infty} \xi_p^{s-1} \, \beta^{s-1} \, \frac{\Lambda_{t+s}}{\Lambda_{t+1}} \, P_{t+s}^{\epsilon} \, Y_{t+s} \, \prod_{q=2}^{s} (1 - n_{t+q-1}) \right.$$

$$-F_{t+1}^{-\epsilon}\sum_{s=1}^{\infty}\xi_p^{s-1}\,\beta^{s-1}\,\frac{\Lambda_{t+s}}{\Lambda_{t+1}}\,P_{t+s}^{\epsilon}\,\Omega_{t+s}Y_{t+s}\,\prod_{q=2}^{s}(1-n_{t+q-1})\Bigg\}\,. \tag{2.46}$$

Making use of equations (2.34) and (2.35), but as of $t+1$, we therefore have

$$\Psi_{1t+1}(i) = A_t^{max\,(\alpha-1)(1-\epsilon)}\left(F_{t+1}^{(1-\epsilon)}Aux_{rev,t+1} - F_{t+1}^{-\epsilon}Aux_{cost,t+1}\right), \tag{2.47}$$

with auxiliary variables associated respectively with the firm's revenues in the first term, $Aux_{rev,t}$, and costs in the second term, $Aux_{cost,t}$, i.e.

$$Aux_{rev,t+1} = P_{t+1}^{\epsilon}\,Y_{t+1} + \xi_p\,\beta\,\frac{\Lambda_{t+2}}{\Lambda_{t+1}}\,(1-n_{t+1})\,Aux_{rev,t+2}\,, \tag{2.48}$$

and

$$Aux_{cost,t+1} = \Omega_{t+1}P_{t+1}^{\epsilon}Y_{t+1} + \xi_p\,\beta\,\frac{\Lambda_{t+2}}{\Lambda_{t+1}}(1-n_{t+1})\,Aux_{cost,t+2}\,. \tag{2.49}$$

Second, let us examine all other possible cases where a new monopolist assumes control as of date $t+1$ and sets the optimal price for its intermediate good i at some future date $t+l$ with some probability $1-\xi_p$. This is followed by the contingency path where price reoptimization does not occur afterward, as there is a probability ξ_p each period of no reoptimization, even if the monopolist continues to operate. By summing over all contingent paths with the appropriate probabilistic weights, the relevant discounted stream of profits for all these contingent paths is given by

$$\Psi_{2t+1}(i) = A_t^{max\,(\alpha-1)(1-\epsilon)}\Bigg\{\sum_{l=2}^{\infty}(1-\xi_p)\beta^l\,\frac{\Lambda_{t+l}}{\Lambda_{t+1}}\prod_{q=2}^{l-1}(1-n_{t+q-1})$$

$$\Bigg[F_{t+l}^{1-\epsilon}\sum_{s=l}^{\infty}\xi_p^{s-1}\,\beta^{s-1}\,\frac{\Lambda_{t+s}}{\Lambda_{t+l}}\,P_{t+s}^{\epsilon}Y_{t+s}\prod_{q=l}^{s}(1-n_{t+q-1}) \tag{2.50}$$

$$-F_{t+l}^{-\epsilon}\sum_{s=l}^{\infty}\xi_p^{s-1}\,\beta^{s-1}\,\frac{\Lambda_{t+s}}{\Lambda_{t+l}}\,P_{t+s}^{\epsilon}\,\Omega_{t+s}Y_{t+s}\,\prod_{q=l}^{s}(1-n_{t+q-1})\Big]\Bigg\}\,.$$

Furthermore, updating equations (2.48) and (2.49) to date $t+l$, equation (2.50) can be rewritten as

$$\Psi_{2t+1}(i) = A_t^{max\,(\alpha-1)(1-\epsilon)} \tag{2.51}$$

$$\left\{\sum_{l=2}^{\infty}(1-\xi_p)\beta^l\frac{\Lambda_{t+l}}{\Lambda_{t+1}}\prod_{q=2}^{l-1}(1-n_{t+q-1})\left(F_{t+l}^{(1-\epsilon)}Aux_{rev,t+l}-F_{t+l}^{-\epsilon}Aux_{cost,t+l}\right)\right\}\,.$$

Analogously to the previous approach, utilizing the recursion embedded in the summations above and defining an auxiliary variable, $Aux_{rem,t+l}$, related to the remainder of the expected discounted profits equation (2.51), can also be presented as follows:

$$\Psi_{2t+1}(i) = A_t^{max\,(\alpha-1)(1-\epsilon)}\;Aux_{rem,t+2}\,, \tag{2.52}$$

where

$$Aux_{rem,t+2} = (1-\xi_p)\beta^2\frac{\Lambda_{t+2}}{\Lambda_{t+1}}\left(F_{t+2}^{(1-\epsilon)}Aux_{rev,t+2}-F_{t+2}^{-\epsilon}Aux_{cost,t+2}\right)+\beta\,(1-n_{t+2})\,Aux_{rem,t+3}\,. \tag{2.53}$$

Consequently, the expected value of the intermediate firm to a successful innovator is

$$E_t\,V_{t+1} = \Psi_{1t+1}(i)+\Psi_{2t+1}(i) \tag{2.54}$$

or, namely,

$$E_t\, V_{t+1} = A_t^{max(\alpha-1)(1-\epsilon)} \left(F_{t+1}^{1-\epsilon} Aux_{rev,t+1} - F_{t+1}^{-\epsilon} Aux_{cost,t+1} \,+\, Aux_{rem,t+2}\right) \,. \tag{2.55}$$

Consequently, the expected value of an intermediate firm for a successful entrepreneur/innovator is determined by the newly reached technological frontier through $A_t^{max(\alpha-1)(1-\epsilon)}$, the contribution from profits arising from being able to set the optimal price for good i as of period $t+1$ through $F_{t+1}^{1-\epsilon} Aux_{rev,t+1} - F_{t+1}^{-\epsilon} Aux_{cost,t+1}$, and the contribution to profits resulting from a later date optimal price setting with what will have become an older technology through $Aux_{rem,t+2}$.

2.2.3 The specification of monetary policy

The central bank's policy function is modelled as a Taylor-type reaction function, as it sets the nominal interest rate according to the following equation:

$$\frac{1+R_t}{1+R} = \left(\frac{1+R_{t-1}}{1+R}\right)^{\rho_R} \left[\left(\frac{\pi_t}{\pi}\right)^{\alpha_\pi} \left(\frac{Y_t}{Y_{t-1}}\, g^{-1}\right)^{\alpha_y}\right]^{1-\rho_R} \mu_{M,t} \,, \tag{2.56}$$

where

$$\ln \mu_{M,t} = \rho_M \, \ln \mu_{M,t-1} + \epsilon_{M,t} \,. \tag{2.57}$$

The parameter ρ_R represents the degree of smoothing of interest rate changes, as the monetary authority seeks to avoid excessively large fluctuations concerning its one-period-lag value, R_{t-1}, and adjusts it somewhat gradually, with weight $1-\rho_R$, in response to demand and technology shocks. The parameters α_π and α_y indicate the weights the monetary authority assigns to deviations from its inflation target, π, and its output growth target, g. The latter is defined as the growth rate of the average technology level in steady state. Finally, $\mu_{M,t}$ is an exogenous and stochastic component of monetary policy representing deviations from the Taylor-type rule, following a stationary first-order autoregressive process.

2.3 The aggregate economy

We now turn to the aggregation of the key economic variables and the equilibrium conditions.

2.3.1 The aggregate price level

From equation (2.5), the overall economy's aggregate price level can be inferred by weighting and combining each of the respective prices for the three categories of coexisting firms

$$P_t^{1-\epsilon} \;=\; \xi_p\,(1 - n_{t-1})\,(P_{t-1})^{1-\epsilon} \;+\; (1-\xi_p)\int_{n_{t-1}}^{1} P_t^*(i)^{1-\epsilon}\,d\,i \;+\; \int_0^{n_{t-1}} P_t^*(i)^{1-\epsilon}\,d\,i \;. \quad (2.58)$$

The first term above corresponds to firms operating with older technologies that are not permitted to reset their prices. The second term is associated with firms using older technologies that can reset their optimal prices. Lastly, the final group consists of new monopolists adopting the most recent technology and consequently setting their intermediate-good optimal prices.[10]

In a conventional New Keynesian model where long-term growth is exogenous, the price index comprises both prices indexed to inflation and optimal reset prices. However, in an endogenous growth environment, it is necessary to consider separately and explicitly both types of reoptimizing firms: those that have innovated and those that have not. Innovating firms all set the same optimal reset price, as they are automatically permitted to reoptimize. Lagging firms set different optimal reset prices according to their respective levels of technological advancement. The difference between these two indices, as captured by the third term on the right-hand side of equation (2.58), is related to what Aghion et al. (2019) identify as missing growth. As they emphasize, a measurement error in inflation arises from excluding the impact of innovative goods. This error has consequential implications for the effective real output growth rates. Since real output growth is obtained by subtracting inflation from nominal output growth, an overestimation of inflation results in an underestimation of output growth. This is likely to have implications for monetary policy, as well as for the design of subsidization and tax-credit policies conducive to $R\&D$, which could be considered in future work.

Exploiting the implicit recursion embedded in equation (2.58), we can show that

[10] The implied dynamics of this aggregate price level involve a corresponding New Keynesian Phillips curve that is discussed in a subsequent subsection.

$$P_t^{1-\epsilon} \;=\; \xi_p\,(1-n_{t-1})\,(P_{t-1})^{1-\epsilon} \\ +\;(1-\xi_p)(1-n_{t-1})F_t^{1-\epsilon}\,Aux_{oldtech,t-1}\;+\;n_{t-1}A_{t-1}^{max\,(\alpha-1)(\epsilon-1)}F_t^{1-\epsilon} \tag{2.59}$$

where

$$Aux_{oldtech,t-1} \;=\; n_{t-2}\,A_{t-2}^{max\,(\alpha-1)(1-\epsilon)}\;+\;(1-n_{t-2})\,Aux_{oldtech,t-2}\,. \tag{2.60}$$

The auxiliary variable, denoted by $Aux_{oldtech,t-1}$, is itself a weighted average of the older technology levels prevailing as of date $t-1$.

2.3.2 The aggregate wage rate

From the aggregate wage index, defined by equation (2.9), the overall economy's wage index can be inferred from weighting the respective wages for the workers that cannot reset their wage optimally, and those that are allowed to reset to the optimal wage, i.e.

$$W_t^{1-\gamma} \;=\; \xi_w\,(W_{t-1})^{1-\gamma}+(1-\xi_w)\,W_t^{*\,1-\gamma}\,. \tag{2.61}$$

2.3.3 Aggregate output

The aggregation of output needs to account for the fact that many intermediate firms coexist with different technology levels and, hence, specific output levels.

From equations (2.3) and (2.25), considering that the optimal capital-labour ratio is identical for all firms, and integrating over all the firms on the $[0,1]$ continuum, we can show that

$$\mu_{Z,t}\,K_t^{\alpha}L_t^{1-\alpha} = P_t^{\epsilon}Y_t\int_0^1 A_t(i)^{\alpha-1}P_t(i)^{-\epsilon}\,d\,i\,. \tag{2.62}$$

Since three classes of situations arise, the integral in equation (2.62) can be accordingly evaluated over three subintervals:

$$\mu_{Z,t}\, K_t^{\alpha} L_t^{1-\alpha} \;=\; P_t^{\epsilon} Y_t \left[\int_0^{\xi_p(1-n_{t-1})} A_t(i)^{\alpha-1} P_t(i)^{-\epsilon}\, d\,i \right.$$
$$\left. +\int_{\xi_p(1-n_{t-1})}^{1-n_{t-1}} A_t(i)^{\alpha-1} P_t^{*}(i)^{-\epsilon}\, d\,i + \int_{1-n_{t-1}}^{1} A_t(i)^{\alpha-1} P_t^{*}(i)^{-\epsilon}\, d\,i \right]. \quad (2.63)$$

The first and second intervals include old-technology-running firms that are respectively non resetting, and optimally resetting their price at $P_t^*(i)$. The last one covers the innovating firms with optimal price setting at $P_t^*(i)$. Using equation (2.32), the above equation becomes

$$\mu_{Z,t}\, K_t^{\alpha} L_t^{1-\alpha} \;=\; P_t^{\epsilon} Y_t \left[(P_{t-1})^{-\epsilon} \int_0^{\xi_p(1-n_{t-1})} A_t(i)^{\alpha-1}\, d\,i \right.$$
$$\left. +\, F_t^{-\epsilon} \int_{\xi_p(1-n_{t-1})}^{1-n_{t-1}} A_t(i)^{(1-\epsilon)(\alpha-1)}\, d\,i \,+\, F_t^{-\epsilon} \int_{1-n_{t-1}}^{1} A_t(i)^{(1-\epsilon)(\alpha-1)}\, d\,i \right]. \quad (2.64)$$

Then, since the firms involved within the first and second integrals operate older technologies, as they do not innovate, the distribution of technologies in these sectors is identical to that of the entire economy as it was one period before. Accordingly, we can define auxiliary variables that can be used to evaluate the relevant integrals by exploiting corresponding recursions. Namely, we can show that:

$$\int_0^{\xi_p(1-n_{t-1})} A_t(i)^{(\alpha-1)} di \;=\; Aux_{oldtechnonreset,t-1} \quad (2.65)$$
$$=\; \xi_p(1-n_{t-1})\, A_{t-2}^{max\,(\alpha-1)} + (1-n_{t-2}) Aux_{oldtechnonreset,t-2}\,,$$

and

$$\int_{\xi_p(1-n_{t-1})}^{1-n_{t-1}} A_t(i)^{(1-\epsilon)(\alpha-1)} di = (1-\xi_p)(1-n_{t-1}) Aux_{oldtech,t-1}\,. \quad (2.66)$$

Finally, the firms covered by the third integral have innovated and pushed the frontier, so that

$$\int_{1-n_{t-1}}^{1} A_t(i)^{(1-\epsilon)(\alpha-1)} di \;=\; A_t^{max\,(1-\epsilon)(\alpha-1)} \,. \tag{2.67}$$

Referring to equations (2.66),(2.66), and (2.67), it follows that aggregate output must satisfy

$$P_t^{\epsilon}\, Y_t \;=\; \mu_{Z,t}\, \frac{K_t^{\alpha} L_t^{1-\alpha}}{Aux_{output,t}} \,, \tag{2.68}$$

where

$$\begin{aligned} Aux_{output,t} \;=\; & \xi_p (1 - n_{t-1})(P_{t-1})^{-\epsilon} Aux_{oldtechnonreset,t-1} \\ & + (1-\xi_p)(1-n_{t-1}) F_t^{-\epsilon} Aux_{oldtech,t-1} \\ & + n_{t-1} F_t^{-\epsilon} (A_{t-1}^{max})^{(1-\epsilon)(\alpha-1)} \,. \end{aligned} \tag{2.69}$$

2.3.4 The aggregate resource constraint

Accounting for investment in $R\&D$, the aggregate resource constraint satisfies

$$C_t \,+\, I_t \,+\, a(u_t)\widetilde{K}_t \,+\, X_t \;=\; Y_t \,, \tag{2.70}$$

since the sum of private consumption, investment, resources devoted to adjust the utilization rate of capital, and investment in $R\&D$ is bounded by the production of final goods.

2.3.5 General equilibrium and key Euler equations for the model to be solved

The general equilibrium of the model requires that supply and demand be equated for the final goods market, the intermediate goods market, the credit (bond) market, the labour market, the markets for physical capital, for investment in physical capital, as well as in $R\&D$. Appendix A collects all the relevant equations that need to be satisfied.

2.3.6 Detrending and model solution

In this model, output, consumption, physical capital, investments in physical capital and investments in research and development fluctuate around a balanced growth path because of technological growth. The existence of the balanced growth path is ensured by the use of labour augmenting technology.

The variables need to be detrended before simulating the model around the steady state. Detrending can be done either by dividing aggregate variables by the technological frontier or the average technology. Since investments in $R\&D$ are a function of the technological frontier, one could argue that detrending by the technological frontier is the correct way to do it. However, because aggregate output, investment and consumption are functions of the average prevailing technology level, we can also detrend by the average technology. In the context of an exogenous growth model, this question would be irrelevant since the frontier and the average technology are one and the same. Since we are interested in the implications of endogenous Schumpeterian growth for business cycles, we prefer to perform detrending with respect to the average technology level.

It can be shown that the average technology level $\overline{A}_t$ is given by

$$\begin{aligned} \overline{A}_t &\equiv \int_0^1 A_t(i)\,d\,i \\ &= n_{t-1}A_{t-1}^{max} + (1-n_{t-1})n_{t-2}A_{t-2}^{max} + (1-n_{t-1})(1-n_{t-2})n_{t-3}A_{t-3}^{max} + \ldots \\ &= n_{t-1}A_{t-1}^{max} + (1-n_{t-1})\,\overline{A}_{t-1}\,. \end{aligned} \tag{2.71}$$

All variables with a trend, including the auxiliary variables, are made stationary, which generally requires the nominal variables to be divided by the product of P_t and some appropriate power of $\overline{A}_t$. Consequently, the detrending of many relevant variables involves the distance of a firm i's technology level relative to the average technology level prevailing in the economy, i.e. $d_t(i) \equiv A_t(i)/\overline{A}_t$.

In particular, we can show that the detrended real marginal cost for firm i is given by

$$mc_t(i) = d_t(i)^{(\alpha-1)}\,\omega_t\,, \tag{2.72}$$

where $mc_t(i) \equiv MC_t(i)/P_t$, and $\omega_t \equiv \Omega/P_t\,\overline{A}_t^{(1-\alpha)}$, while its optimal relative reset price $\phi_t^\star(i) \equiv P_t^*(i)/P_t$ is

$$\phi_t^\star(i) = d_t(i)^{(\alpha-1)}\, f_t\,, \tag{2.73}$$

where $f_t \equiv F_t/\,\overline{A}_t^{(1-\alpha)}$.

Moreover, detrended real aggregate output amounts to

$$y_t = \frac{k_t^\alpha\, L_t^{(1-\alpha)}}{\int_0^1 d_t(i)^{(\alpha-1)}di}\,, \tag{2.74}$$

while detrended real investment in $R\&D$, $x_t \equiv X_t/\overline{A}_t$ is given by

$$x_t = \beta\,\frac{n_t}{1+\eta}\,\frac{\lambda_{t+1}}{\lambda_t}\,E_t\,v_{t+1}(A_t^{max})\,, \tag{2.75}$$

with $\lambda_t \equiv \Lambda_t/(P_t\overline{A}_t)$, $E_t\,v_{t+1} \equiv E_t\,V_{t+1}(A_t^{max})/(P_{t+1}\overline{A}_{t+1})$.

Accordingly, the probability of innovating can be also written as

$$n_t = \left(\frac{x_t}{\zeta\,(\overline{A}_t/A_t^{max})^{-1}}\right)^{1/(1+\eta)}\,. \tag{2.76}$$

Once, the steady state of the detrended model is computed , the model is loglinearly-approximated around its steady state.

2.3.7 The New Keynesian Phillips curve

We now turn to the inflation dynamics in this economy with endogenous growth.[11] Given the aggregate price index derived in equation (2.59), we use the properly detrended versions of the relevant variables to find implicit dynamic equations for the inflation rate:

[11] Appendix B provides the definitions of the detrended variables and more details underlying the derivation of the New Keynesian Phillips curve.

$$\xi_p, (1-n_{t-1})\pi_{t-1}^{\epsilon-1} = 1 - f_t^{1-\epsilon}\left((1-\xi_p)(1-n_{t-1}), aux_{oldtech,t-1} + n_{t-1}d_t^{(\alpha-1)(\epsilon-1)}\right). \quad (2.77)$$

Then, referring to equation (2.33), the relevant factor in the optimal reset price is substituted into equation (2.77). Hence, the resulting equation can be rewritten as a first-order linear approximation in log-deviations from steady state, that yields the following New Keynesian Phillips curve for our model.

The New-Keynesian Phillips curve is a staple of New-Keynesian DSGE models that relates inflation with the output gap (or real marginal cost gap) and expected future inflation. In our model, the introduction of endogenous growth further complicates this relationship and shows the existence of a link between inflation and technology. Indeed, the current rate of inflation is a function of the optimal reset price (through $\hat{f}t$), the old technology still used in production ($a\hat{u}x_{oldtech,t-1}$), the growth rate of the frontier ($\hat{g}_t$) and the distance between the average technology and the frontier ($\hat{d}_t$).[12]

$$\hat{\pi}t = \Gamma 1, \hat{f}_t + \Gamma_2, aux_{old\hat{t}ech,t-1} + \Gamma_3, \hat{g}_t + \Gamma_4, \hat{d}_t \ , \quad (2.78)$$

These three additional variables also play an explicit role in a model with Schumpeterian innovation. First, as usual, a rise in the optimal reset price exerts a positive effect on inflation, with $\Gamma_1 \geq 0$. Second, the auxiliary variable reflecting the contribution from price resetting by some of the unsupplanted firms operating with prevailing older technology exerts also a positive effect on the inflation rate, since $\Gamma_2 \geq 0$. Third, growth in the technological frontier brings downward pressure on real marginal costs, which translates *ceteris paribus* in lower inflation, with $\Gamma_3 \leq 0$. Finally, $\Gamma_4 \leq 0$ can be intuitively understood from innovation pushing up the technological frontier, which hence increases the distance between A^{max} and the average technology level of firms operating in the economy, $\overline{A}_t$. This also reduces inflationary pressures. Hence, in comparison with the standard New Keynesian DGSE model with exogenous output growth and no trend inflation, the price dynamics in our model involves an extended-like New Keynesian Phillips curve.

[12] As defined in Appendix B, $d_t \equiv A_t^{max}/\bar{A}_t$

2.4 Parameter calibration and characteristics of the shocks

2.4.1 Taking the model to the data

Typically, a calibration exercise involves constraining the values of the model parameters to effectively target a specified set of stylized facts or moments. Ensuring that the simulated data from the model are directly comparable with the empirical data is a crucial, yet often overlooked, step in the process. In the case of an exogenous growth setup, overlooking this aspect might not have significant consequences. However, with Schumpeterian endogenous growth, this oversight could lead to erroneous comparisons.

Traditionally, simulated data around the model's steady state are compared with detrended empirical data. This oversight might be benign in the case of an exogenous growth model, as growth is deterministic and lacks a built-in cyclical component. However, in the context of endogenous growth, the economy's growth rate is influenced by cyclical components at medium frequencies. Therefore, a necessary first step is to reconstruct the trending simulated data series by adding back the trend, as follows:

$$ln(Y_t^{trending,simulated}) = ln(Y_t^{cyclical,simulated}) + ln(A_{t-1}) \tag{2.79}$$

By constructing a new set of trending simulated variables, we ensure comparability between the model and empirical data. An HP filter is applied to both sets of data before generating the moments and co-movements.

While some parameter values were set based on standard values found in the literature, other parameters in the calibration were derived from the US data available for the 1960Q1 to 2018Q2 sample.

We employ Fernald (2017)'s data series on utilization-adjusted total factor productivity (TFP) to construct a cumulative TFP series compounded at quarterly rates. Civilian non-institutional population data are obtained from the U.S. Bureau of Labor and Statistics to compute the *per capita* variables. The Implicit Price Deflator serves as a proxy for the price level, and all macroeconomic aggregates are sourced from the U.S. Bureau of Economic Analysis. We utilize series on Real Gross Domestic Product, Research and Development, Real Gross Private Domestic Investment, and Real Personal Consumption Expenditures.

2.4.2 Calibration associated with the features akin to that a typical New Keynesian DSGE model

The calibration of most parameters related to the New Keynesian aspects of the model follows standard practice. Table 2.1 displays the values used for calibrating the key parameters of the model. The share of capital in the production function (α), the discount rate (β), the depreciation rate of physical capital (δ), and the monetary policy parameters adhere to conventional values found in the literature. The steady-state gross trend inflation (π) is set at 1, meaning that the inflation rate is zero in the steady-state.

Additionally, we assume full capacity utilization in the steady-state, i.e., $u = 1$. Market power in labor supply and the provision of intermediate goods result in widely accepted wage and price markups of approximately 20%, which is the benchmark established in Christiano et al. (2005). This value corresponds to an elasticity of substitution of 6 between intermediate goods and labor types. The disutility of labor (θ) is set so that steady-state labor constitutes approximately 0.33 of the hours endowment per period.

Lastly, price changes in our model are semi-endogenous since, in any given period, two types of firms can reset prices: innovative firms introducing a new product and incumbent firms that have been selected to do so in their ongoing operations.

2.4.3 Calibration associated with the Schumpeterian features of the model

Calibrating Schumpeterian models is a relatively new endeavour in economic research. We will base our calibration on the statistical moments of the variables related to research and development and technological advancement.

As shown in Figure B.1, from 1960Q1 and 2018Q2, the share of GDP dedicated to $R\&D$ investment has varied between 2.22% ans 2.96%, averaging a 2.68%-share of GDP over the period. We use this sample average as the steady state target for investment in $R\&D$ by calibrating the production function parameters at $\zeta = 10^8$ and $\eta = 11$.

Given the relationship between the frontier growth rate, the spillover and the innovation probability, there are additional degrees of freedom left as the variation along one dimension can be compensated by a change in another. This additional degree of freedom will prove particularly interesting when studying the dynamics of the model. For instance, different restrictions will yield significantly different impulse response

Table 2.1 Calibration of key parameters

Parameter	Value	Description
α	1/3	Share of capital in intermediate-good production
β	.99	Discount rate
δ	.025	Depreciation rate of physical capital
π	1	Steady-state gross inflation rate
u	1	Steady-state capacity utilization rate
ϵ	6	Elasticity of substitution of intermediate-good demand
γ	6	Elasticity of substitution of labour demand
ξ_p	$1-\frac{(1-.66)}{(1-n_{ss})}$	Calvo parameter for prices
ξ_w	.66	Calvo parameter for wages
θ	5	Weight on the disutility of labour
ν	1	Utility parameter that determines
		the Frisch elasticity of labour supply, $(\frac{1}{\nu})$
h	.5	Consumption habit formation
κ	1	Investment adjustment-cost parameter
σ_{ss}	$\frac{(g_{ss}-1)}{n_{ss}}$	Steady-state value for the extent of knowledge-spillover
ζ	10^8	Scaling parameter in the innovation production function
η	11	Diminishing-return parameter in the innovation production function

Note: As explained further below, the joint determination of n_{ss} and σ_{ss} is made to replicate the long-run value of the growth rate of the technological frontier g_{ss}, that is set at the historical average of real GDP *per capita* from 1960Q1 to 2018Q2.

functions.

2.4.4 Persistence and variance of the shocks

Investment and monetary policy shocks parameters, shown in Table 2.2, are derived from New Keynesian literature and find their sources in Justiniano et al. (2010).

The spillover and transitory technology shocks are two components of TFP. The transitory technology shock

Table 2.2 Calibration of the monetary and investment shocks

Parameter	Value	Description
ρ_m	0.4	Monetary policy shock persistence
σ_m	0.001	Monetary policy shock variance
ρ_μ	0.2	Investment shock persistence
σ_μ	0.005	Investment shock variance

can be seen as the cyclical component of TFP. We, therefore, calibrate both persistence and volatility to match the cyclical component of TFP. We, then, perform a grid search to minimize the Euclidian distance between the volatilities of observed and simulated TFP. The results from this calibration are reported in Table 2.3.

Table 2.3 Calibration of the technology and knowledge-spillover shocks

Parameter	Value	Description
ρ_z	0	Technology shock persistence
σ_z	0.005	Technology shock variance
ρ_σ	0.62	Knowledge-spillover shock persistence
σ_σ	0.018	Knowledge-spillover shock variance

In the next section, we show that our model exhibits relative volatilities, correlations and autocorrelations consistent with the observed data.

2.5 Consistency of the theoretical model with the data

Although our calibration approach does not aim to precisely replicate the sample statistical moments observed in the data, comparing key moments of the HP cyclical components of simulated and empirical data enables us to assess the consistency of the mechanisms incorporated in the model. Tables 2.4, 2.5, and 2.6 present relative volatilities, contemporaneous correlations, and autocorrelations for various macroeconomic variables.[13]

Table 2.4 demonstrates that the simulated relative volatilities compared to output are reasonably consistent

[13] Given the computing power at our disposal and using a grid search approach, we have made a compromise between processing time and accuracy at this time. Further refinements could be pursued in the future.

Table 2.4 Relative volatilities in the cyclical component of the variables with respect to cyclical output according to the endogenous growth model

	Output	Consumption	Investment	$R\&D$ Investment	Price Level	Inflation
Empirical	1	0.39	2.17	1.09	0.37	0.13
Simulated	1	0.59	2.99	0.96	0.23	0.21

with their sample counterparts. Notably, the model closely matches the volatility of $R\&D$. As anticipated, both investment in physical capital and investment in $R\&D$ exhibit greater volatility than output, with relative volatilities of 2.17 and 0.96, respectively. Cyclical consumption is somewhat more volatile in the model than in the data, and investment in physical capital is also somewhat too volatile compared to the data.[14] Furthermore, the price level and inflation rate in the model exhibit slightly less and slightly more volatility, respectively, than the observed data.

Table 2.5 Contemporaneous correlations in the cyclical component for both empirical and simulated data

		Output	Consumption	Investment	$R\&D$ Investment	Price Level	Inflation
Consumption	Empirical	0.80	1				
	Simulated	0.95					
Investment	Empirical	0.92	0.74	1			
	Simulated	0.97	0.79				
$R\&D$ Investment	Empirical	0.40	0.41	0.35	1		
	Simulated	0.68	0.58	0.72			
Price Level	Empirical	-0.53	-0.46	-0.51	-0.18	1	
	Simulated	-0.56	-0.45	-0.58	-0.49		
Inflation	Empirical	0.18	0.04	0.16	0.14	-0.18	1
	Simulated	-0.12	-0.16	-0.06	0.13	0.23	

The signs and magnitudes of various contemporaneous correlations between the HP cyclical components of the macroeconomic variables displayed in Table 2.5 are fairly similar in both empirical and simulated data,

[14] By utilizing a smaller increment in the grid-search procedure and adjusting the parameters for the intensity of habit persistence in consumption and the convex adjustment cost of investment in physical capital, it is likely that the moments for the simulated data will more closely align with the empirical moments, though this is not our primary objective here.

although they often appear somewhat larger for the simulated series. For example, simulated correlations between $R\&D$ investment and output, and $R\&D$ and consumption are 0.68 and 0.58, compared to 0.40 and 0.41 in the data. It is particularly noteworthy that the values of contemporaneous correlation between the price level and various macroeconomic variables are negative and closely resemble their empirical counterparts. Simultaneously, the values of the correlation between the inflation rate and other variables are relatively weak, ranging between -0.18 and 0.23, suggesting that the observed discrepancy may not be very significant.

Lastly, Table 2.6 presents the autocorrelation coefficients at orders 1 to 4 for the cyclical components of the macroeconomic variables. The model tends to reasonably replicate the empirical autocorrelations of the variables, especially for consumption. The model exhibits slightly more persistence in output, investments in physical capital, and inflation, and slightly less persistence in $R\&D$ investment and the price level.

Table 2.6 Autocorrelation functions of the cyclical component for both empirical and simulated data

	Lag	-1	-2	-3	-4
Output	Empirical	0.84	0.62	0.38	0.16
	Simulated	0.92	0.78	0.59	0.37
Consumption	Empirical	0.86	0.69	0.50	0.28
	Simulated	0.86	0.68	0.47	0.25
Investment	Empirical	0.90	0.72	0.50	0.29
	Simulated	0.95	0.83	0.66	0.45
$R\&D$ Investment	Empirical	0.89	0.70	0.45	0.24
	Simulated	0.77	0.55	0.35	0.17
Price Level	Empirical	0.93	0.82	0.66	0.48
	Simulated	0.90	0.67	0.41	0.16
Inflation	Empirical	0.42	0.30	0.20	0.18
	Simulated	0.59	0.19	-0.07	-0.21

With a reasonable calibration of the model, it is thus possible to obtain key moments and comovements for essential macroeconomic variables that are consistent with their observed counterparts.

2.6 The business cycle implications of varying the steady state probabilities of innovating

In our model, the balanced growth rates of the technological frontier and of the trend growth rate of real output are determined by the product of both the steady-state probability of innovating, n_{ss}, and the steady-state extent of technology spillover, σ_{ss}, as the latter induces a larger technological jump of the frontier. Hence,

$$g_{ss} = 1 + \sigma_{ss}\, n_{ss}\ . \tag{2.80}$$

Specifically, it is possible to consider different combinations of n_{ss} and σ_{ss} that generate the same value of g_{ss}. However, existing evidence in the literature does not provide much guidance on calibrating σ_{ss} versus n_{ss}.[15]

Nevertheless, at this stage, it is insightful to examine how the impulse response functions of the macroeconomic variables to each shock differ when comparing two economies. In the first one, we set $n_{ss} = 0.05$ and $\sigma_{ss} = 0.097$, while in the second one, these values are respectively assigned as $n_{ss} = 0.35$ and $\sigma_{ss} = 0.0139$. Consequently, both economies are consistent with a steady-state quarterly gross growth rate of $g_{ss} = 1.00485$, or approximately 2% *per annum*. The former economy is associated with a lower probability of innovating, typically connected with a more substantial "jump," while the latter is characterized by more frequent, albeit smaller, discoveries that are more easily spread and do not necessitate as large a value for σ_{ss}.

Although we maintain a constant steady-state real growth rate of output, a different steady-state probability of innovation has a distinct impact on the dynamics of the economy, which was absent in the conventional model with exogenous growth. Indeed, in our setup, a larger value of n_{ss} implies not only that a greater proportion of intermediate firms will be innovating in the steady state, but also that the degree of effective price flexibility is higher as the proportion of firms reoptimizing intermediate good prices has increased. This is because innovating firms are allowed to set the optimal markup related to the innovating intermediate

[15] Furthermore, in the real world, variation across industries and different time periods may imply that this decomposition could differ over time. This would introduce an additional level of intriguing complexity over an extended data range, which is beyond the scope of the current paper.

product.

We now discuss how the impulse response functions of the cyclical macroeconomic variables following different shocks are altered when considering an economy with a "low" steady-state probability of innovation, say 5%, *versus* an economy with a relatively "high" steady-state probability of innovation, say 35%.

2.6.1 The impulse response functions for the cyclical components of the variables following a transitory technology shock

As can be seen in Figures 2.3 and 2.4, when changing n_{ss} from 5% to 35%, a positive transitory technology shock does not lead to a very significantly different cyclical dynamics on output, consumption, investment in physical capital, hours worked, capacity utilization, physical capital stock and the optimal reset price. However, this does not mean that it is without consequences. Initially, a cyclical wealth effect dominates on hours worked, while it builds up on consumption. Then, the cyclical intratemporal and intertemporal substitution effects on hours worked takes the lead. Also, as expected the rise in the marginal product of physical capital first leads to a positive cyclical response of capacity utilization, while the increased incentive to invest in physical capital peaks up and start to rise its stock.

With $n_{ss} = 0.35$, there is a sharper immediate negative response of inflation and drop in nominal interest rates lasting a few quarters. Also, the induced rise in the marginal productivities of labour and capital translates into transitory positive cyclical impacts on the wage index, the optimal reset wage and the return on physical capital. Furthermore, this positive transitory positive technology shock has a transitory, albeit persisting beyond 25 quarters on investments into the value of an intermediate firm, $R\&D$ investments, the innovation probability, and finally average real growth.

2.6.2 The impulse response functions for the cyclical components of the variables following an efficiency of investment shock

After a temporary positive improvement on the efficiency of investment in physical capital, a quick temporary rise in the return on physical capital follows, that quickly brings up temporarily cyclical capacity utilization and private investment. The increase in optimal reset wage and return on capital both push the marginal cost up through the dynamic complementarity of inputs. The cyclical impulse response functions in Figures 2.5 are not much different whether $n_{ss} = 0.05$ or $n_{ss} = 0.35$.

As shown in the lower panel of Figure 2.6, when $n_{ss} = 0.35$, a positive shock on the efficiency of investment leads to a persistent cyclical increase in the probability of innovating, in the discounted expected value of the intermediate firms, and in $R\&D$ investments. Here too, the average growth rate of real output also exhibits a persistent cyclical increase.

2.6.3 The impulse response functions for the cyclical components of the variables following a monetary policy shock

We now turn to the cyclical impact of a restrictive monetary policy, brought about by a “discretionary” rise in the nominal interest above the Taylor-type reaction function. The net effect on the nominal interest rate therefore is the combination of the monetary shock and the immediate reaction to inflation- and output-growth gaps. As illustrated in Figure 2.7, the nominal interest rate initially goes up, while the inflation rate goes down. This is accompanied by a quick cyclical downturn in real output, in consumption, in hours worked, in capacity utilization and in investment in physical capital, along with a cyclical decrease in the real return on capital. A higher steady-state innovation-probability value of $n_{ss} = 0.35$, compared with $n_{ss} = 0.05$, generally leads to somewhat weaker cyclical fluctuations in the above variables, except for inflation that is brought down more quickly.

At the same time, there is a smaller negative cyclical impact on marginal cost, while the wage index temporarily goes up, even though the optimal reset wage is cyclically lower.

Meanwhile, this restrictive policy shock causes a fairly more important cyclical downturn in the probability of innovating that lowers a little more the value of intermediate firms and invesments in $R\&D$. However, despite these consequences, the cyclical impact of average real growth rate does not differ as much with either $n_{ss} = 0.35$ or $n_{ss} = 0.05$. The cumulative impact of lower compounded growth is not necessary negligible however. This still needs to be assessed.

2.6.4 The impulse response functions for the cyclical components of the variables following a knowledge-spillover shock

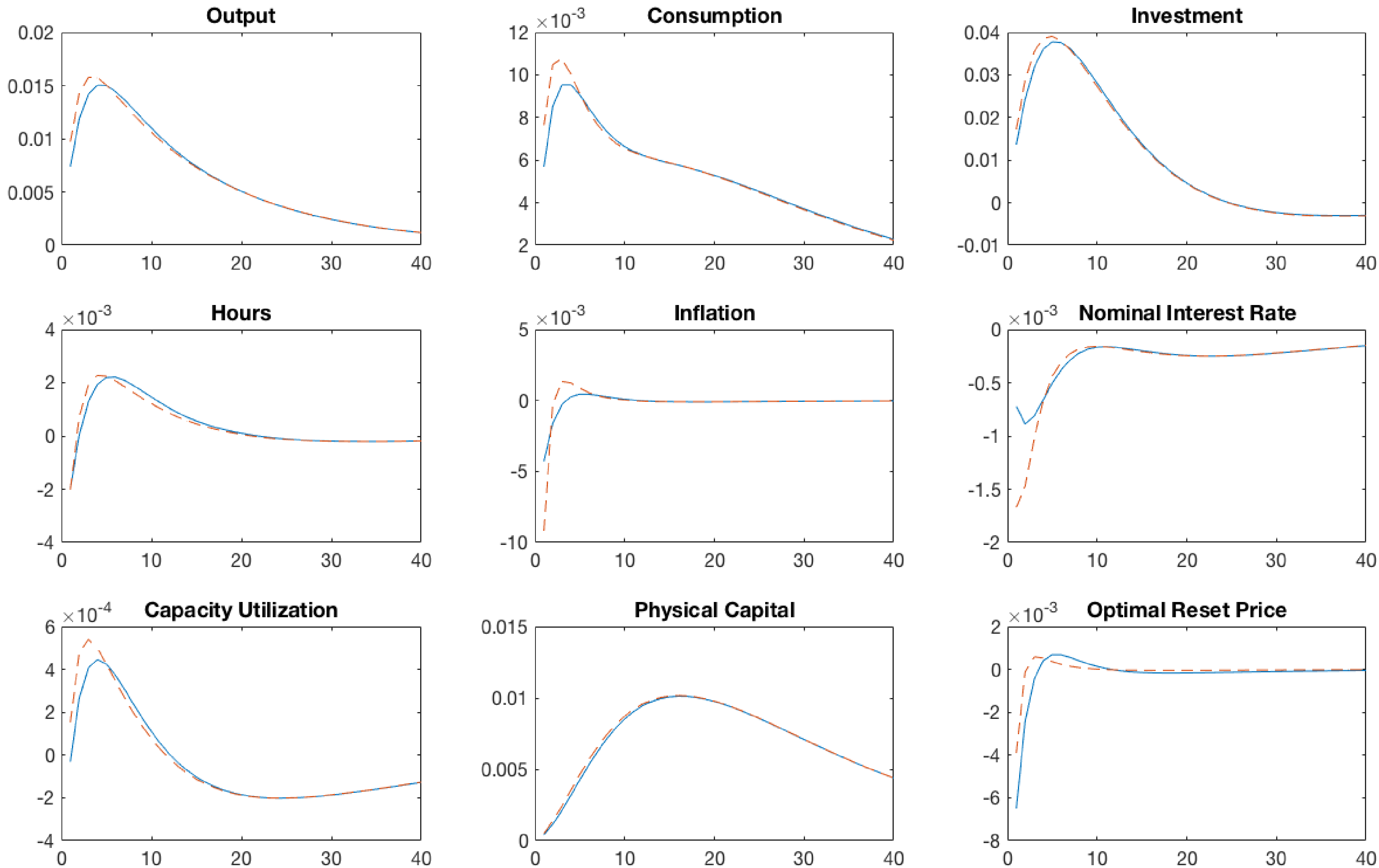
The last shock to be considered is specific to a model with endogenous growth and has no analogue in exogenous growth business cycle models.

As it is apparent in Figures 2.9 and 2.10, the cyclical dynamic responses to a transitory positive knowledge-spillover shock is significantly sensitive to the steady-state probability of innovation. Focussing first our attention to Figure 2.10, an increased extent of knowledge-spillover cyclically lowers for some quarters both the value of intermediate firms' discounted profits and investments in $R\&D$, while cyclically lowering the probability of innovation. However, the cyclical impacts on the innovation probability and the average real output growth rate are fairly subdued when $n_{ss} = 0.05$. This is not so for $n_{ss} = 0.35$. Instead, the positive cyclical response oft the average growth rate is much higher and more persistent. Moreover, in this case, for about 5 quarters, there initially is a bigger decrease in each of the investment in $R\&D$, the value of the discounted firm and the probability of innovation, then followed by a positive 4- to 5-quarter rise in these variables, prior to a sustained but slight negative impact.

Now considering the cyclical impulse responses of the other macroeconomics variables, an appreciably higher value of n_{ss}, such as 35%, leads to significantly more volatile cyclical effects of a given positive knowledge-spillover shock, as there are, in part, stronger intertemporal substitution effects that are engineered by this shock. To some extent, as seen in Figure 2.9, it is as if the cyclical impulse response functions are somewhat squeezed for many graphics for a higher value of n_{ss}.

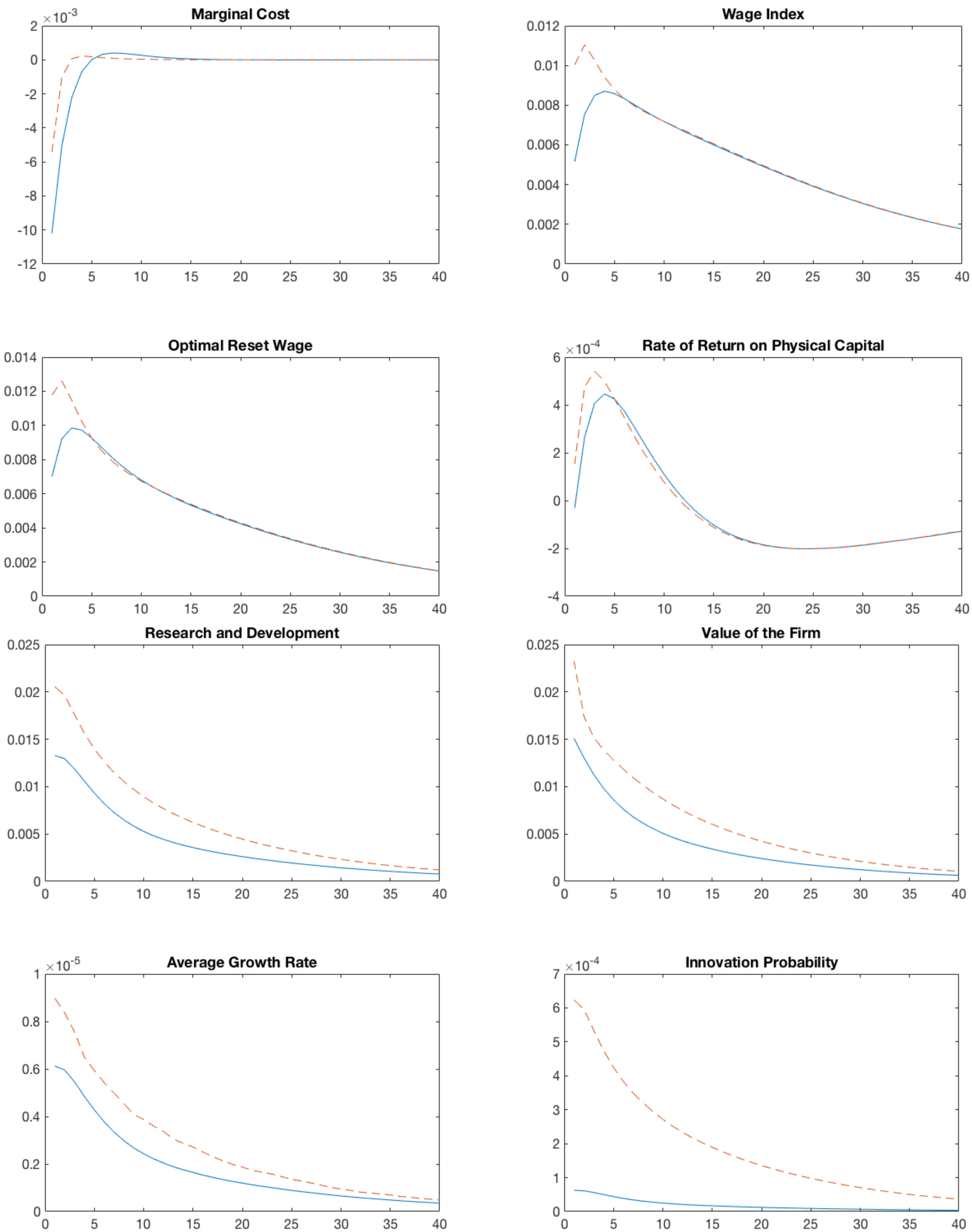
There are two main conclusions we can draw from the impulse response function to the spillover shock. The spillover shock could be the main source of cyclical fluctuation of the growth rate of the economy. An estimated model would allow to compute the variance decomposition and confirm the intuition gathered from the impulse response functions. A larger innovation probability at the steady state quickens the model's reaction to a spillover shock. Most variables such as the nominal interest rate and the optimal reset price return to their steady state much quicker when $n_{ss} = 35\%$. We will now turn to an assessment of the welfare implications of various (σ_{ss}, n_{ss}) pairs. It can be expected that two effects will be in play. On one hand, if a higher value of n_{ss} brings about more volatility of cyclical consumption, this will lower welfare. On the other hand, if it raises trend permanent income, and therefore trend consumption, it will be welfare-raising.

Figure 2.3 Impulse response functions of the HP cyclical components of macroeconomic variables following a transitory technology shock



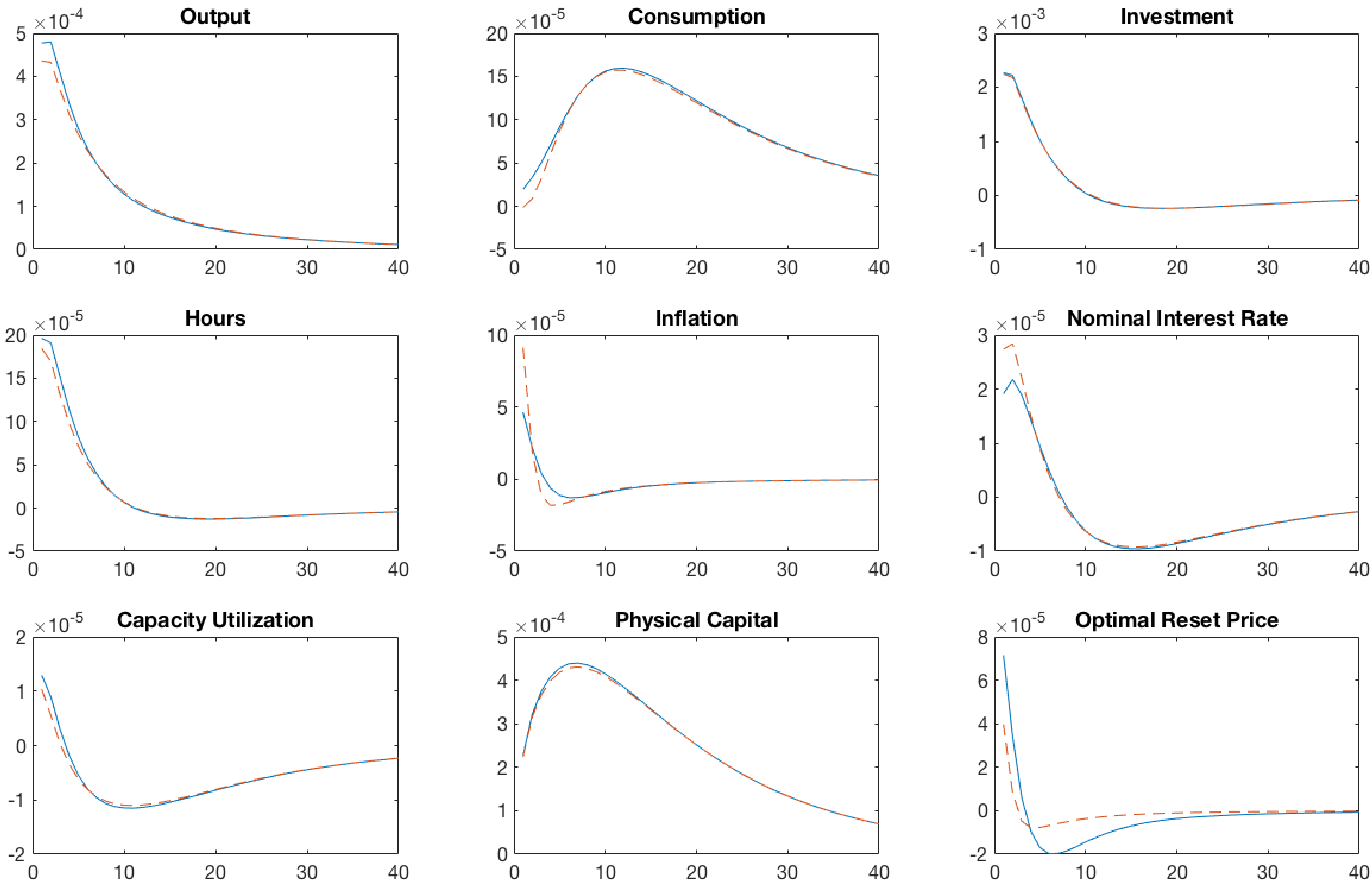
Solid line : n_{ss} = 5%; dashed line: n_{ss} = 35%

Figure 2.4 Impulse response functions of the HP cyclical components of macroeconomic variables following a transitory technology shock



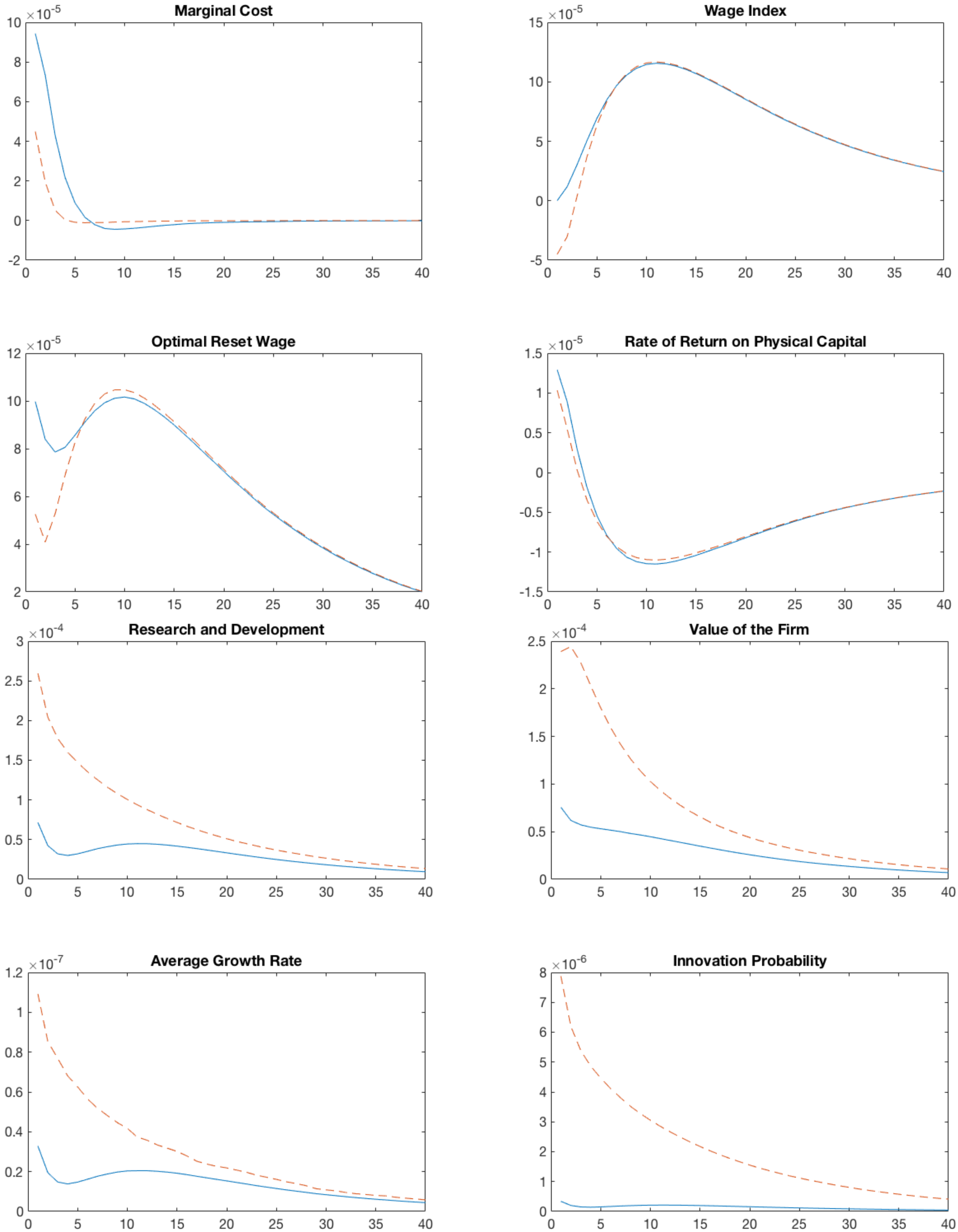
Solid line : n_{ss} = 5%; dashed line: n_{ss} = 35%

Figure 2.5 Impulse response functions of the HP cyclical components of macroeconomic variables following an efficiency-investment shock



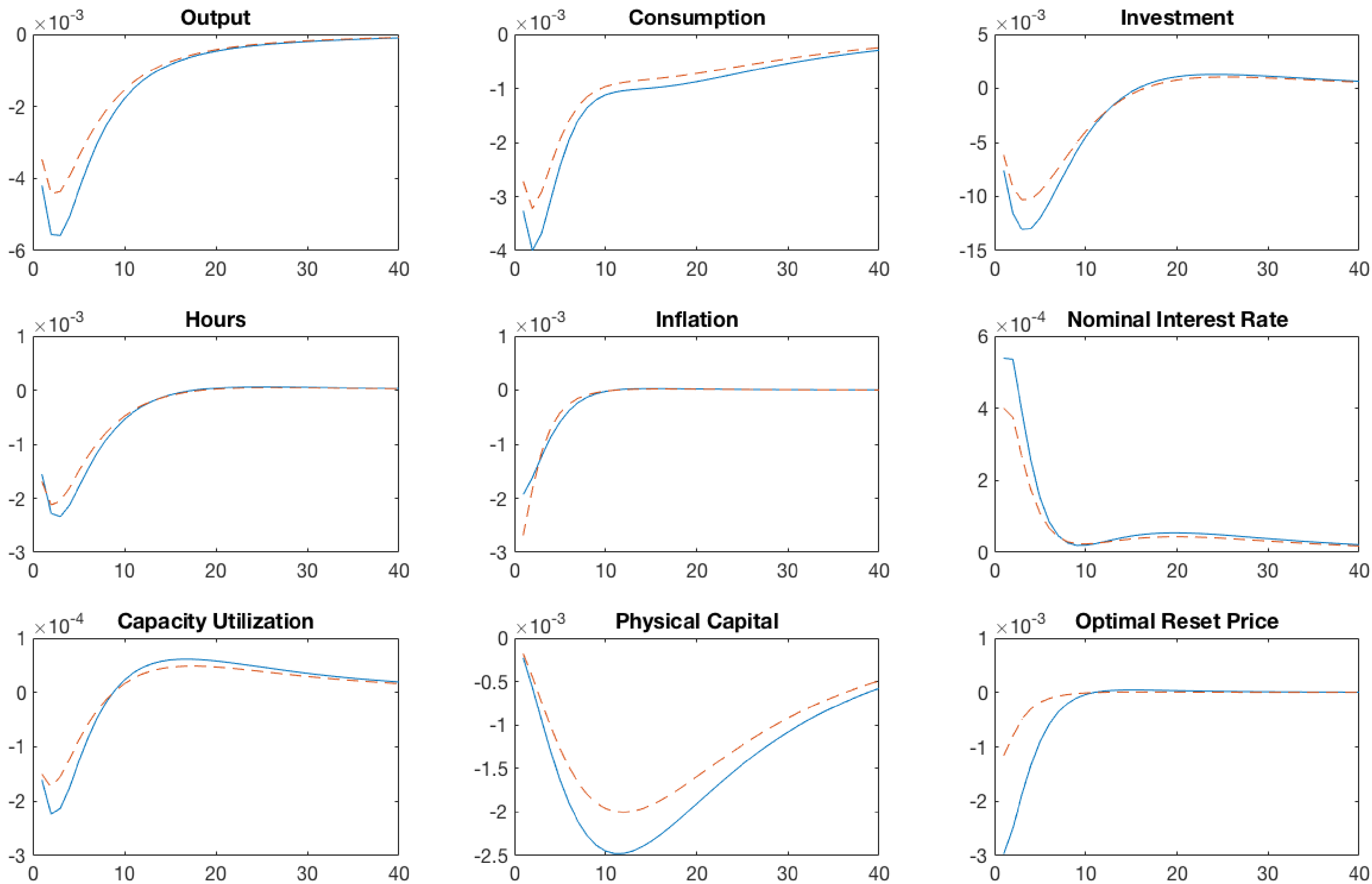
Solid line : n_{ss} = 5%; dashed line: n_{ss} = 35%

Figure 2.6 Impulse response functions of the HP cyclical components of macroeconomic variables following an efficiency-investment shock



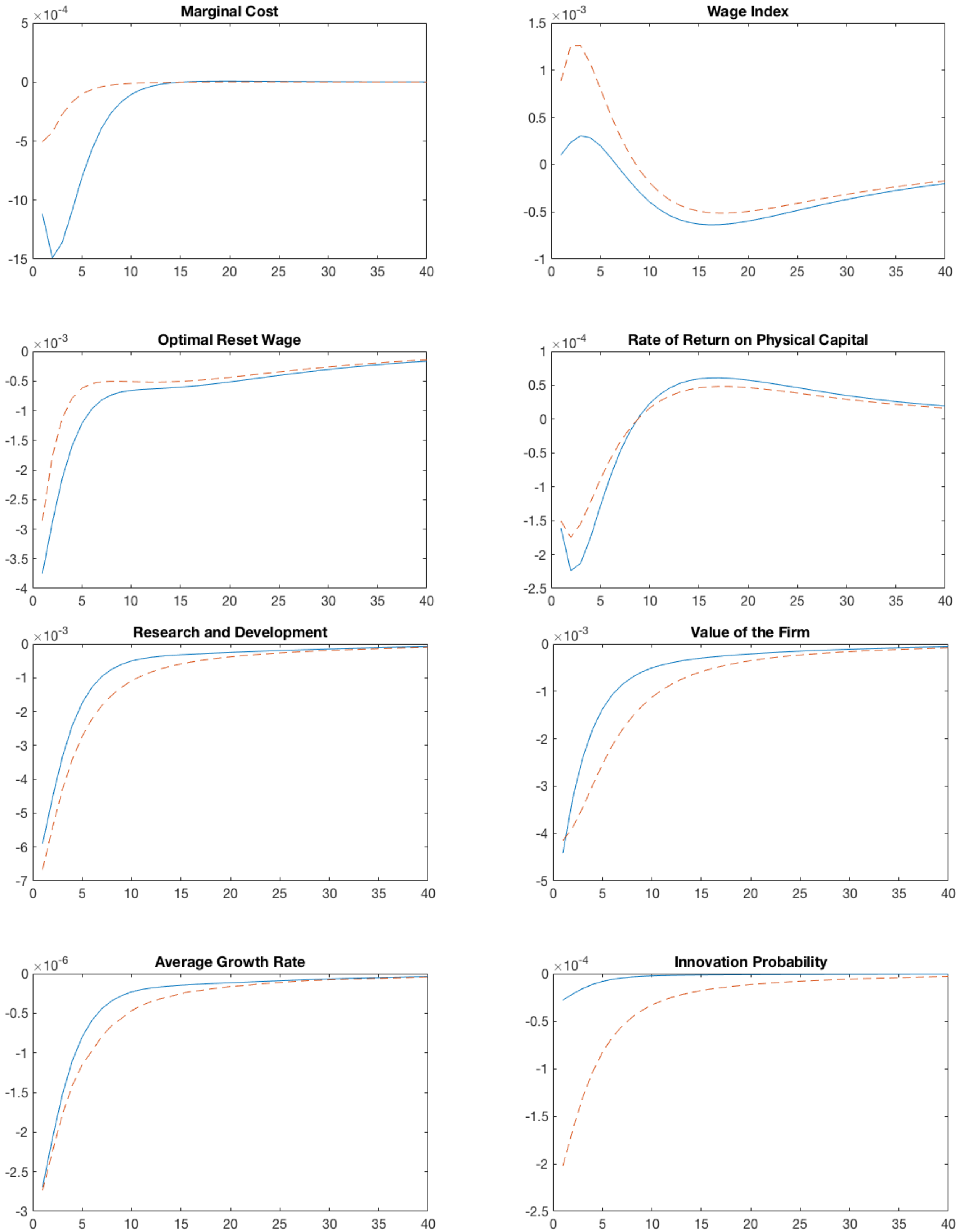
Solid line : n_{ss} = 5%; dashed line: n_{ss} = 35%

Figure 2.7 Impulse response functions of the HP cyclical components of macroeconomic variables following a monetary policy shock



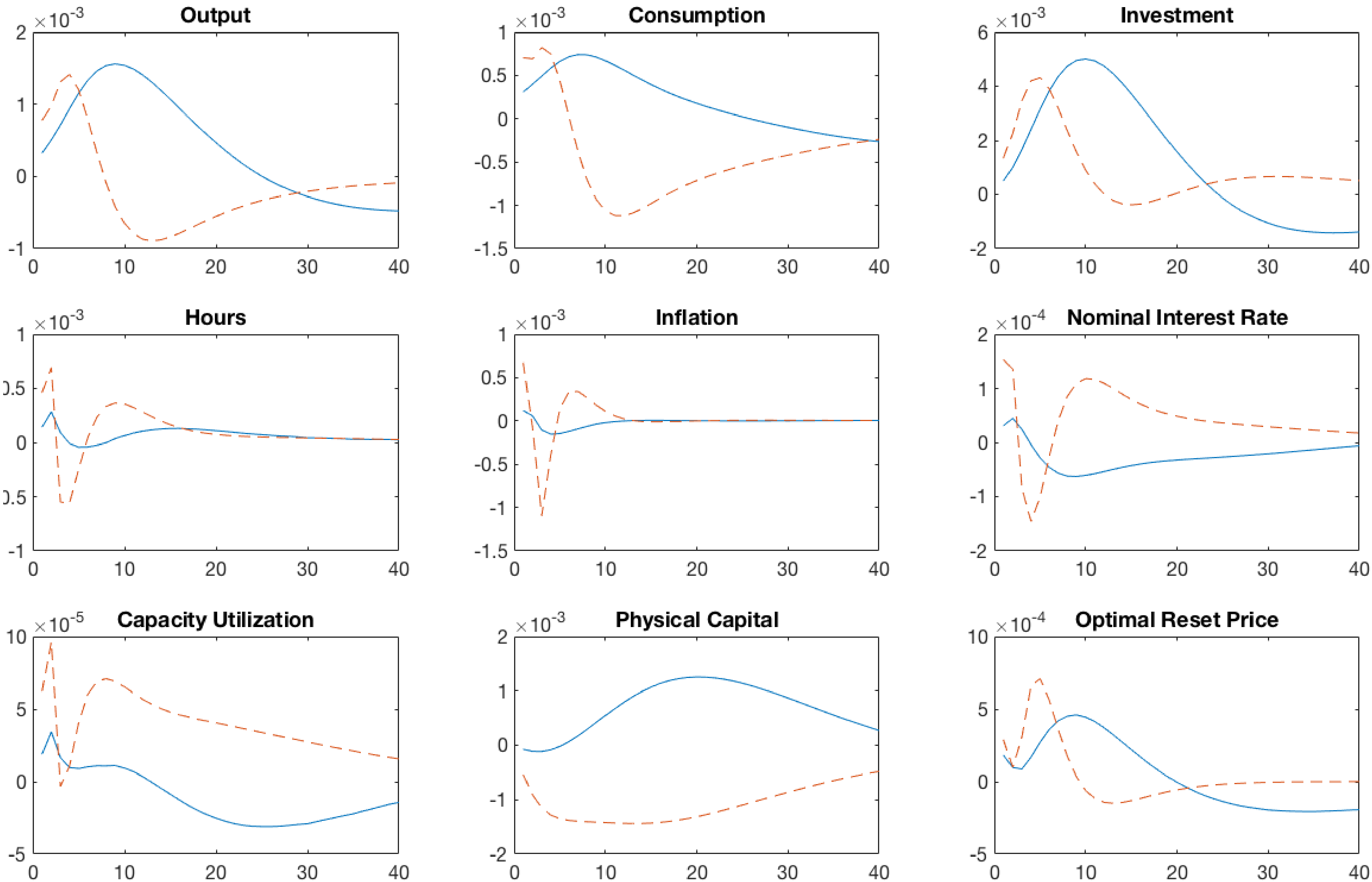
Solid line : n_{ss} = 5%; dashed line: n_{ss} = 35%

Figure 2.8 Impulse response functions of the HP cyclical components of macroeconomic variables following a monetary policy shock



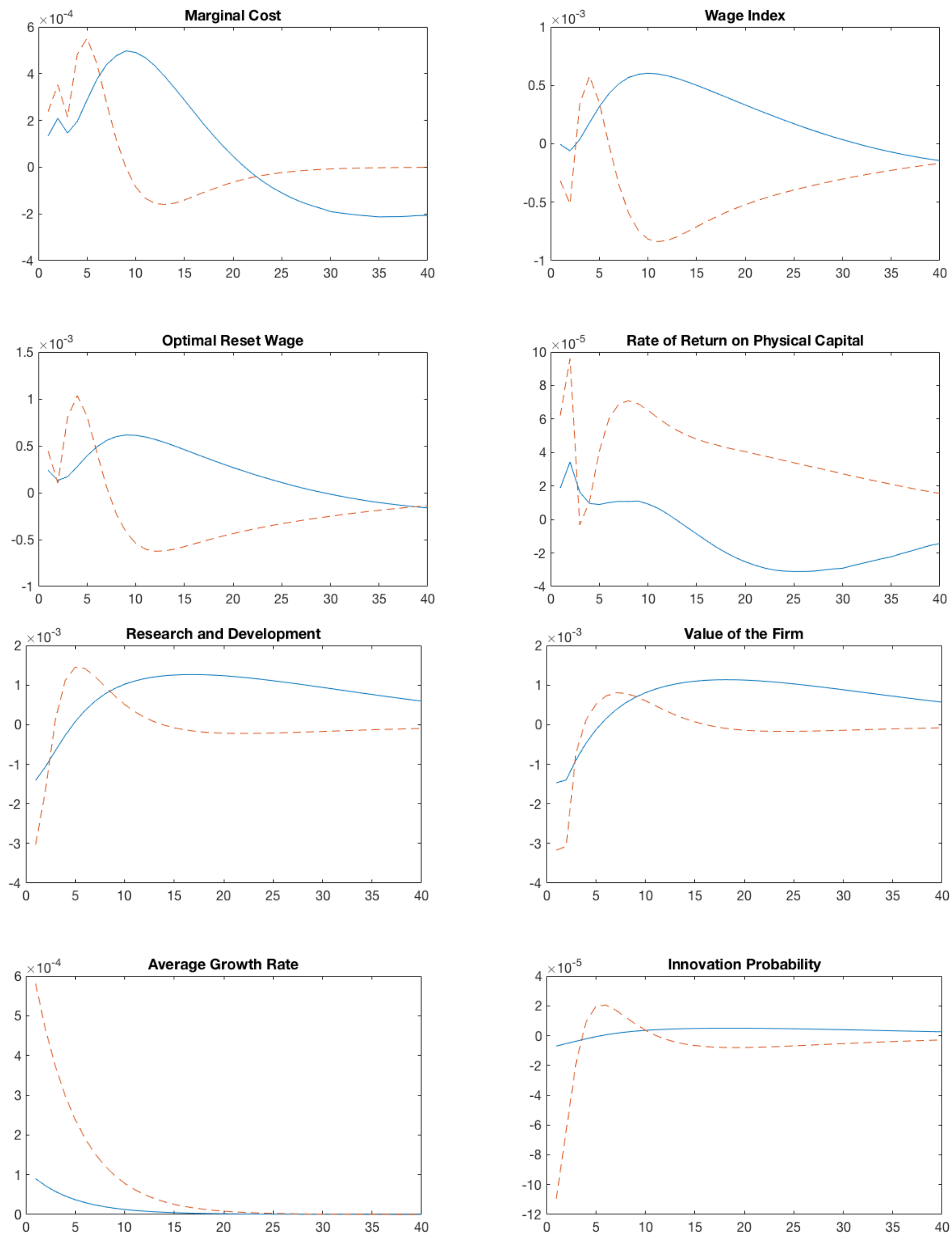
Solid line : n_{ss} = 5%; dashed line: n_{ss} = 35%

Figure 2.9 Impulse response functions of the HP cyclical components of macroeconomic variables following a knowledge-spillover shock



Solid line : n_{ss} = 5%; dashed line: n_{ss} = 35%

Figure 2.10 Impulse response functions of the HP cyclical components of macroeconomic variables following a knowledge-spillover shock



Solid line : n_{ss} = 5%; dashed line: n_{ss} = 35%

2.7 The welfare implications of different steady-state probability of innovation

In our model, the innovation probability is the odds that, for a given amount of investments in $R\&D$, the process is successful in introducing a new technology. For the firm in monopolistic position in its sector, it also corresponds to the probability of being supplanted by an innovator. As such, this can be seen as the proportion of firms that enters or exits the unit mass intermediate sector.

Having looked at the impulse response functions of a New Keynesian model with endogenous growth, we now turn to the welfare implications of the interaction between business cycles and endogenous growth. Our aim is to assess the welfare cost of different calibrations of the Schumpeterian growth mechanisms, while keeping constant the steady-state growth rate of the technological frontier. Since $g_{ss} = 1 + \sigma_{ss} n_{ss}$, both n_{ss} and σ_{ss} can be varied while keeping g_{ss} invariant, in order to study the welfare implications of different combinations of steady-state probability of innovation and spillover. For instance, a high degree of knowledge-spillover can compensate for a lower innovation probability.

The infinite-horizon sum of the present discounted value of flow utilities across households define their welfare, which can be recursively expressed as follows:

$$W_t \;=\; U(C_t, L_t) \;+\; \beta\, W_{t-1}\,. \tag{2.81}$$

Accordingly, applying the methodology developed by Schmitt-Grohé & Uribe (2004), it is possible to make welfare comparisons between a benchmark scenario for the (n_{ss}, σ_{ss})-pair, indexed by subscript B, and alternative pairs, generically indexed by subscript A. We choose to contrast various scenarios relative to a benchmark using consumption-equivalent differences as a more tangible measure of welfare. Namely, it is thus expressed as a percentage of steady state consumption that would make a household indifferent between two scenarios.

A first measure, evaluated at the non-stochastic steady states, reckons the fraction of consumption that would have to be given up each period in the alternative scenario to reach the same welfare level as in the benchmark scenario. It is defined as:

$$C_{ss} = 1 - exp\Big((1-\beta)\cdot[W_{B,ss} - W_{A,ss}]\Big) \tag{2.82}$$

where $W_{B,ss}$ is the benchmark and $W_{A,ss}$ is the alternative. Accordingly, when $W_{B,ss} < W_{A,ss}$, then $C_{ss} > 0$.

A second measure can also be evaluated at stochastic means of the value functions obtained from the model simulations. It hence takes into account the interaction between the alternative (n_{ss}, σ_{ss})-pair interacts with the various random shocks built-in the model. In this case, the corresponding consumption-equivalent welfare loss is given by:

$$C_m = 1 - exp\Big((1-\beta)\cdot\big[E(W_{B,t}) \ - \ E\big(W_{A,t}\big)\big]\Big) \tag{2.83}$$

with $C_m > 0$ when $E(W_{B,t}) < E(W_{A,t})$.

As previously mentioned, different combinations of (n, σ), that yield a yearly steady-state gross growth rate of $g_{ss} = 1.01954$ *per annum*, are used to compute and to compare the non-stochastic steady-state and stochastic-means consumption-equivalent welfare measures. While the former (C_{ss}) is obtained directly, the latter first requires to compute the stochastic means of the variables of interest prior to calculating C_m.

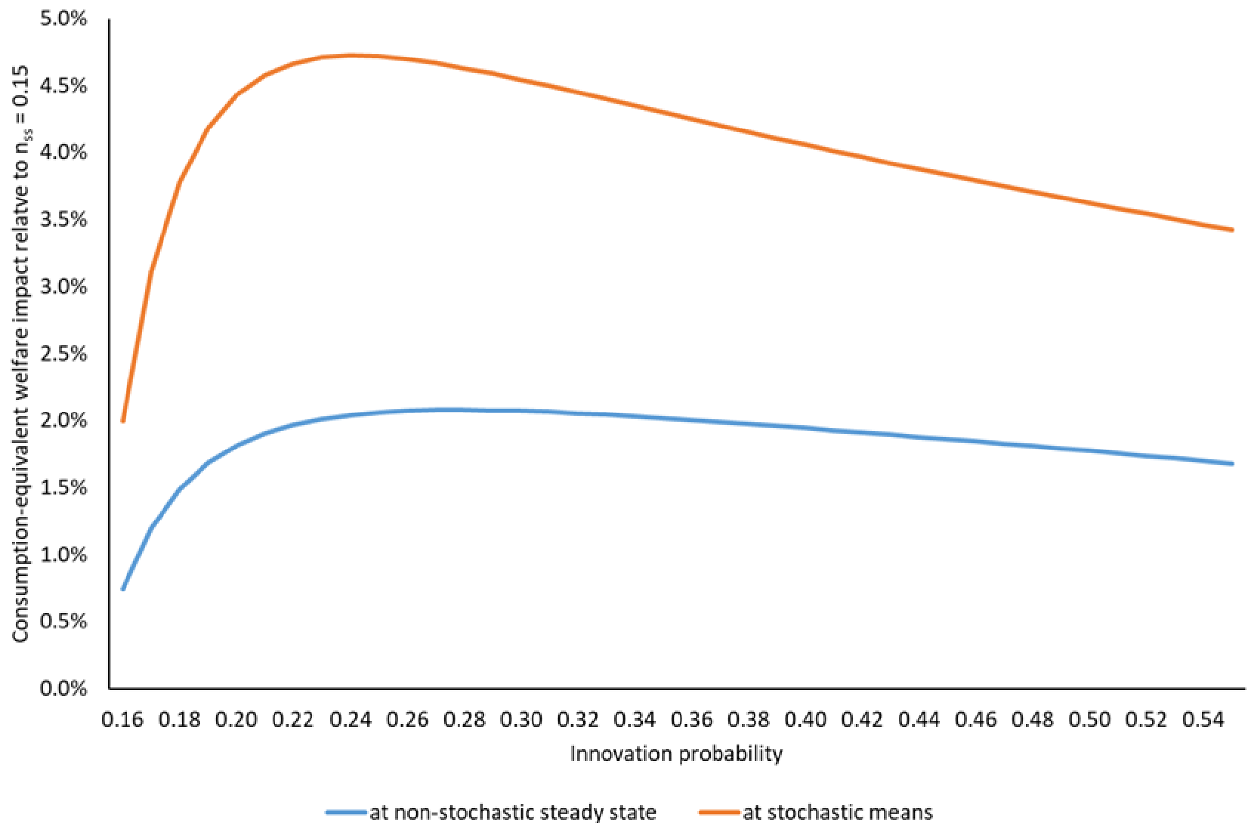
Since $\sigma_{ss} = \frac{g_{ss}-1}{n_{ss}}$, for the calibrated value of $g_{ss} = 1.00485$ per quarter, we set the benchmark calibration at a quarterly steady-state innovation probability of $n_{ss} = 0.15$, along with $\sigma_{ss} = 0.3233$. For the alternative scenarios, the innovation probability n are varied from 0.16 to 0.54. The corresponding spillover parameter σ_{ss} is changed accordingly from 8.9815×10^{-3} to 3.0313×10^{-2}. Figure 2.11 shows the results from these simulations.

There are two main conclusions. First, the welfare impact of varying the steady-state innovation probability, even for a constant value of g_{ss} is non negligible. Indeed, to be indifferent between an innovation probability given by $n_{ss} = 0.15$ instead of $n_{ss} = 0.23$, a household would have to be compensated with a higher steady state consumption of 2.0% according to the non-stochastic the steady-state measure, and 4.7% according to the stochastic-means measure. Second, there is significant non linearity when comparing the welfare impact of different (n_{ss}, σ_{ss}) environments. Indeed, the consumption-equivalent welfare-comparison curve is positively sloped and steeper between $n_{ss} = 0.16$ and $n_{ss} = 0.2$. It reaches a maximum at $n_{ss} = 0.28$, before slowly decreasing.

Two opposing forces are simultaneously at work in determining welfare. On the one hand, an increase in the steady-state innovation probability leads to a greater turnover in firms who set prices more often. As it is even more strongly revealed by the C_m measure, a higher value for n_{ss} allows the model economy to be closer to an economy without any price rigidities. Indeed, the reduction in the price wedge is welfare improving. On the other hand, when the innovator decides how much to invest in R&D, only private benefits are taken into account. Since an increase in the innovation probability amounts also to a lower probability for an incumbent to remain in operation, this reduces the discounted value of the innovation, with its consequential reduction in welfare.

Hence, there is also a positive externality from innovating that is not considered by the private innovator, while the society benefits from additional permanent push of the technological frontier. From a normative angle, The larger the gap between the private and social values of innovating, the more investment in R&D is socially suboptimal. This suggests some issues worth considering in future research on the appropriate length of intellectual property right policy.

Figure 2.11 Consumption-equivalent welfare impact of raising the steady-state probability of innovating from $n_{ss} = 0.15$ and beyond, with a given steady-state technological frontier gross growth rate set at $g_{ss} = 1.0195$ per annum or $1.0195^{0.25} = 1.00485$ per quarter.



2.8 Conclusion

Despite the development of modern endogenous growth models, most DSGE business cycle models still rely on exogenous neoclassical long-term growth, focusing solely on fluctuations around the trend growth. By incorporating Schumpeterian innovation into the intermediate production sector within a New Keynesian framework with nominal wage and price stickiness, we demonstrate that endogenous decisions to invest in $R\&D$ not only affect the likelihood of innovation and technological frontier advancement but also introduce a significant transmission channel for common shocks (such as aggregate technological, investment efficiency, and monetary shocks) as well as for spillover shocks, which were absent in exogenous growth models. In fact, the interaction between innovation and optimal price-setting in the intermediate sector reveals how the technological frontier advances while demonstrating that increased innovation leads to greater price flexibility in a world with nominal rigidities.

From a theoretical standpoint, our hybrid model highlights and addresses new challenges in modeling and simulation. Firstly, investment decisions in $R\&D$ must consider the interactions between endogenous growth features and New Keynesian characteristics of the model. Indeed, the expected discounted profits resulting from an innovation are influenced by both the probability of being replaced by a new innovator and, if not replaced, the probability of optimally resetting the price in subsequent periods after an innovation. This has implications for the aggregation of the intermediate production sector. Secondly, the introduction of Schumpeterian innovation, which determines the expected growth rate of the economy, allows for the consideration of knowledge spillover shocks, adding a new dimension not found in traditional business cycle models. This new shock represents the unpredictable variations and heterogeneities in knowledge transmission and the ability to capitalize on new innovations to further advance the technological frontier.

From a methodological perspective, we demonstrate that the computation, presentation, and interpretation of impulse response functions must be approached differently when accounting for both endogenous growth and business cycles. Specifically, the cyclical IRFs of endogenous and exogenous growth models are not directly comparable, as exogenous growth remains entirely acyclical while endogenous growth rates fluctuate cyclically around a steady state.

In terms of the model's ability to replicate key stylized business cycle characteristics, we find that a reasonable calibration in line with standard practices, given the current trade-off between accuracy and processing times, allows the model to produce moments and comovements relating to output, consumption, investment in physical capital, and investment in $R\&D$ that are consistent with empirical observations.

When comparing models with endogenous and exogenous growth, we find that the aggregate business cycle responses to common shocks, such as disturbances in aggregate productivity, investment efficiency, or monetary policy, remain relatively similar in magnitude when the steady-state probability of innovating in a quarter is relatively low (e.g., 5%). However, with endogenous Schumpeterian growth, the introduction of a new knowledge spillover shock, while sensitive to calibration, offers additional insight into the average growth rate of the economy at various frequencies, even beyond those typically considered in the literature. This suggests promising directions for future research.

Furthermore, we have shown that the dynamic responses of macroeconomic variables can differ significantly in terms of amplitude and shape when altering the steady-state probability of innovation while maintaining a constant steady-state or trend growth rate of real output. This occurs because a higher steady-state probability of innovation, typically associated with a "smaller" leap in the technological frontier, corresponds to a smaller steady-state extent of technology spillover, implying that smaller discoveries spread more easily. However, a crucial new mechanism is revealed: a higher probability of innovation reduces the extent of nominal price rigidities since incoming innovative firms are allowed to set optimal prices.

Therefore, there are significant welfare implications for different combinations of steady-state innovation probability and knowledge-spillover extent, even when maintaining the same 1.95% *per annum* value for the steady-state growth of the technological frontier. Specifically, an economy with a 23% steady-state quarterly probability of innovation rather than 15%, considering dynamic interactions, results in a consumption-equivalent welfare impact of a 4.7% higher level.

In conclusion, our model raises several new questions that warrant future research and could not be addressed within standard DSGE New Keynesian frameworks. Possible extensions to our model may explore the following topics: the importance of inflation and effective real growth mismeasurements due to the growth and business cycle implications of innovation; the modeling of optimal monetary policy in an environment accounting for innovation; whether the monetary authority's reaction should differ based on the source of output and inflation fluctuations; the business cycle and growth implications of fiscal policy in this context, including an assessment of various government policies intended to encourage innovation; and the business cycle, growth, and welfare implications of transitioning from a higher to a lower steady-state growth rate of the technological frontier.

CHAPTER 3

BAYESIAN ESTIMATION INSIGHTS INTO ENDOGENOUS GROWTH AND BUSINESS CYCLES

ABSTRACT

This paper extends the Mahroug and Paquet (2023) model by integrating critical elements such as positive trend inflation and price and wage indexation into a New Keynesian Dynamic Stochastic General Equilibrium (NK DSGE) framework. The enhancements are aimed at capturing more realistic macroeconomic dynamics, particularly in the context of contemporary monetary policies and labor market behaviors.

The incorporation of positive trend inflation reflects the persistent, low-level inflation observed in modern economies, especially pertinent in the post-Great Recession era, where central banks have targeted positive inflation rates. This addition allows the model to more accurately portray the impact of inflation expectations on economic decisions and policy-making. Similarly, the inclusion of price and wage indexation acknowledges the real-world scenario of delayed adjustments in prices and wages to economic changes and indexation clauses imbedded in some contracts. This aspect is crucial in explaining the observed stickiness in prices and wages in response to economic shifts.

Using Bayesian estimation techniques and data spanning from 1954 Q3 to 2018 Q3, we find that the model's estimated parameters for New Keynesian components align with existing literature, lending credibility to its foundational assumptions. A notable result is the significant spillover effect from innovation, highlighting the importance of R&D in driving long-term growth. However, the model also reveals high persistence in technological and spillover shocks, suggesting an external dependency in generating economic persistence that might overshadow the model's endogenous growth dynamics.

This paper opens avenues for further research, including extending the R&D investment horizon to better capture endogenous growth dynamics, exploring the mechanisms of price and wage stickiness, and examining policy implications in more depth. The paper concludes that while the model successfully integrates important aspects of modern economies and offers valuable insights, continued research is essential to enhance its accuracy and applicability in economic analysis and policy formulation.

3.1 Introduction

To further our understanding of the interactions of long-term growth with the business cycle, we need to move to the estimation stage. The added value of an estimation is twofold. First, we need to evaluate whether the New Keynesian components of the model produce estimates that are consistent with the existing literature amongst others, Smets & Wouters (2007) and Justiniano et al. (2011). If they are, the endogenous growth component is a seamless addition to the New Keynesian theoretical framework. Otherwise embedding Schumpeterian growth has implications on previously agreed upon estimates. Second, we aim at providing estimates of parameters that are not traditionally estimated. Mahroug and Paquet (2023) calibrated Schumpeterian growth parameters based on steady state ratios and relative volatilities. One of the challenges faced by their calibration was the trade-off between the innovation probability (the probability that someone who invests in research and development is successful and makes a discovery) and the spillover effect (the speed at which new ideas propagate into the economy). They are left with an additional degree of freedom that is difficult to anchor. This paper aims, therefore, at providing that additional anchor that will allow further research based on the estimation stemming from this exercise.

In the wake of the Great Financial Crisis one of the criticisms levelled at DSGE models was the lack of endogenous growth either in the medium or the long-term. Comin & Gertler (2006), Comin & Hobijn (2009), Amano et al. (2012), and Comin et al. (2014) build upon Romer (1990) seminal work by using expanding variety in a dynamic stochastic general equilibrium model.Annicchiarico et al. (2011) consider a NK model with Calvo staggered prices and wages with endogenous growth operating through non-rival access to knowledge. Annicchiarico & Pelloni (2014) examine how nominal rigidities affect uncertainty on long-term growth, when prices and wages are preset with a one-period lag, in a model with labour, as single input, is divided between producing output or $R\&D$. Cozzi et al. (2017) show the implication of financial conditions for innovation dynamics in a NK model with Schumpeterian growth with price and wage rigidities arising from specific adjustment costs. Finally, to evaluate the sources of the productivity slowdown following the Great Recession, Anzoategui et al. (2017) estimate a DSGE model with staggered Calvo sluggish adjustments of wages and final-good prices, featuring endogenous growth via an expanding variety of intermediate goods resulting from public learning-by-doing in the $R\&D$ process and an endogenous pace of technology adoption.

In section 2, we present sthe New-Keynesian structural model with built-in Schumpeterisn features in an economy with positive steady-state inflation and both wage and price indexation mechanisms. Section 3 presents the data used in the Bayesian estimation, the measurement equations and the prior densities. In section 4, we proceed to discuss the results of the Bayesian estimation and suggest two key extensions which may improve the overall fit of the model and allow for a better framework in term of policy analysis.

3.2 The model

This section details the augmented model's structure and its theoretical underpinnings. Building upon Mahroug and Paquet (2023), the model developed in this paper,is a New Keynesian DSGE model augmented with Schumpeterian growth in the intermediate good sector along with Calvo nominal rigidities on prices and wages. This model allows for dynamic interactions between growth and the business cycle. We incorporate price and wage indexation, drawing on seminal works such as Galı & Gertler (1999) and Gali et al. (2005). For positive trend inflation, we refer to the insights of Ascari & Sbordone (2014), which underscore the impact of trend inflation on macroeconomic dynamics. These additions, along with a set of additional shocks to accommodate more observables in the Bayesian estimation process, to achieve better data alignment.

3.2.1 The final good producer

The final consumption good is produced by a representative firm that operates in a perfectly competitive setting and that aggregates a continuum of intermediate goods $i \in (0,1)$ according to the following production function:

$$Y_t = \left(\int_0^1 Y_t(i)^{\frac{\epsilon_t - 1}{\epsilon_t}} di \right)^{\frac{\epsilon_t}{\epsilon_t - 1}} , \tag{3.1}$$

where Y_t is total final output, the input $Y_t(i)$ is the good produced by an intermediate level firm i, and $0 \leq \epsilon_t < \infty$ is the elasticity of substitution between intermediate goods and is assumed to evolve according to:

$$ln\, \epsilon_t = (1 - \rho_\epsilon)\, ln\, \epsilon + \rho_\epsilon ln\, \epsilon_{t-1} + \varepsilon_{\epsilon,t} \tag{3.2}$$

with $\varepsilon_{\epsilon,t}$ an i.i.d. process centered around 0.

This type of law of motion, and that according to the conventional practice, it is assumed that parameters such as those linked to elasticities are assumed to be characterized by their logarithms move possibly smoothly around their respective long-run mean.

Because of perfect competition, the final good producer takes as given the price of its final output, P_t, and the prices of the intermediate goods, $P_t(i)$. Hence, its profit maximization problem

$$\max_{Y_t(i)} \Pi_{FG} = P_t\, Y_t - \int_0^1 P_t(i)\, Y_t(i)\; d\, i \tag{3.3}$$

yields the demand for the i^{th} intermediate good as a negative function of its relative price, namely

$$Y_t(i) = \left(\frac{P_t(i)}{P_t}\right)^{-\epsilon_t} Y_t\,. \tag{3.4}$$

Since economic profits are zero under perfect competition, total nominal output is given by the sum of the nominal value of all intermediate goods i

$$P_t\, Y_t = \int_0^1 P_t(i)\, Y_t(i)\; d\, i\,, \tag{3.5}$$

3.2.2 The employment agency

In our model economy, a continuum of households possesses different skills and offers specialized labour $L_t(j)$ for $j \in (0,1)$, that gives them some degree of market power in setting wages. Since intermediate firms use a combination of specialized labour, we can think of a representative employment agency which aggregates specialized labour and turns it into the combined labour input L_t employed by the intermediate firms, namely

$$L_t = \left(\int_0^1 L_t(j)^{\frac{\gamma_t - 1}{\gamma_t}}\, d\, j\right)^{\frac{\gamma_t}{\gamma_t - 1}}\,, \tag{3.6}$$

where

$$ln\,\gamma_t = (1-\rho_\gamma)\,ln\,\gamma + \rho_\gamma ln\,\gamma_{t-1} + \epsilon_{\gamma,t} \tag{3.7}$$

is the log-elasticity of substitution between each labour type.

Operating in perfect competition, the employment agency maximizes its profits with respect to $L_t(j)$ while taking as given the aggregate wage rate W_t and the prevailing labour compensation specific to each labour type j.

The solution of its optimization problem

$$\max_{L_t(j)} \Pi_{EA} = W_t\,L_t - \int_0^1 W_t(j)\,L_t(j)\;d\,j \tag{3.8}$$

yields the demand for specialized labour j as a negative function of its relative wage rate

$$L_t(j) = \left(\frac{W_t(j)}{W_t}\right)^{-\gamma_t} L_t\;. \tag{3.9}$$

From equation (3.9), and the competitive equilibrium for the employment agency, the aggregate wage rate is given by

$$W_t = \left[\int_0^1 W_t(j)^{1-\gamma_t}\,dj\right]^{\frac{1}{1-\gamma_t}}\;. \tag{3.10}$$

3.2.3 The household

The representative household must decide how much to consume C_t, while allowing for some habit formation, how many hours $L_t(j)$ to work, how much capacity to use u_t, how much physical capital they want next period $\widetilde{K}_{t+1}$, how much to invest I_t in physical capital, and the size of their net bond holdings B_t by solving the following optimization problem:

$$\max_{C_{t+s},L_{t+s}(j),u_{t+s},\tilde{K}_{t+s+1},B_{t+s}} E_t^j \sum_{s=0}^{\infty} \beta^s \left(ln\big(C_{t+s} - h\, C_{t+s-1}\big) - \theta\, \frac{L_{t+s}(j)^{1+\nu}}{1+\nu} \right) \tag{3.11}$$

subject to the sequences of budget constraints:

$$\begin{aligned} P_{t+s}\, C_{t+s} + P_{t+s}\, I_{t+s} + P_{t+s}\, a(u_t)\widetilde{K}_{t+s} + \frac{B_{t+s}}{1+r_{t+s}} \leq W_{t+s}(j)\, L_{t+s}(j) + q_{t+s}\, u_{t+s}\widetilde{K}_{t+s} \\ + B_{t+s-1} + D_{t+s} + T_{t+s}\ , \end{aligned} \tag{3.12}$$

$$\widetilde{K}_{t+s+1} = \mu_{I,t+s} \left[1 - S\left(\frac{I_{t+s}}{I_{t+s-1}} \right) \right] I_{t+s} + (1-\delta)\widetilde{K}_{t+s}\ , \tag{3.13}$$

$$\ln \mu_{I,t+s} = \rho_I\ \ln \mu_{I,t+s-1} + \epsilon_{I,t+s} \tag{3.14}$$

$$K_{t+s} = u_{t+s}\widetilde{K}_{t+s}\ . \tag{3.15}$$

where E_t^j is the expectation operator conditioned of known information as of the beginning of period t, D_t are the dividends the household receives from the intermediate sector and T_t is a lump sum transfer from the government. The function $S(\,\cdot\,)$ represents a convex adjustment function cost incurred when transforming current and past investment into installed capital. We assume it to be defined as $S\big(I_t/I_{t-1}\big) = (\kappa/2)\big(I_t/I_{t-1} - g_t\big)^2$. Hence, the investment adjustment cost is defined in relation to the departure of physical investment growth from its steady-state trend growth, i.e., that of the technological frontier in the steady state. We include a habit formation parameter h to mimic the persistence of consumption over time. Function $a(u_t)$ represents the cost of varying the utilization of capital. Moreover, an exogenous stochastic investment shock $\mu_{I,t+s}$, which affects the efficiency with which investment is transformed into capital, follows a first-order autoregressive process.

The government transfers a fixed share of output to the households such as:

$$T_t = (1 - \frac{1}{tx_t})Y_t, \tag{3.16}$$

where tx_t is the inverse of the average taxation rate and follows an autoregressive process centered around its steady state:

$$ln\, tx_t = (1 - \rho_{tx})\, ln\, tx + \rho_{tx} ln\, tx_{t-1} + \epsilon_{tx,t}. \tag{3.17}$$

In addition, along with a household j possesses some specialized skills underlying market power over its wage rate, we also assume the existence of wage rigidities modelled with Calvo contract arrangements, with a constant proportion $1 - \xi_w$ being allowed to reoptimize their wage each period. When the household is not allowed to reoptimize, its wages are indexed to realized inflation. Hence, household j maximizes its expected utility weighed by the probability ξ_w of not being allowed to optimize with respect to wages subject to the labour demand function. Algebraically that is

$$\max_{W_t(j)} E_t^j \left(\sum_{s=0}^{\infty} \xi_w^s\, \beta^s \Big(-\theta\, \frac{L_{t+s}(j)^{1+\nu}}{1+\nu} \Big) + \Lambda_{t+s}\, W_t(j) \pi_{w,t,t+s}\, L_{t+s}(j) \right) \tag{3.18}$$

subject to

$$L_{t+s}(j) = \left(\frac{W_t(j)\pi_{w,t,t+s}}{W_{t+s}} \right)^{\gamma_t} L_{t+s}\ , \tag{3.19}$$

and

$$\pi_{w,t,t+s} = \prod_{k=1}^{s} \pi_{t+k-1}^{\iota_w} \pi^{1-\iota_w}. \tag{3.20}$$

Accordingly, the optimal reset wage is obtained from

$$W_t^*(j)^{-\gamma_t \nu - 1} = \frac{\gamma_t - 1}{\theta\, \gamma_t} \frac{\sum_{s=0}^{\infty} \xi_w^s \beta^s \Lambda_{t+s} \left(W_t \pi_{w,t,t+s}\right)^{\gamma_t} L_{t+s}}{\sum_{s=0}^{\infty} \xi_w^s \beta^s \left(W_t \pi_{w,t,t+s}\right)^{\gamma_t (1+\nu)} L_{t+s}^{1+\nu}} . \tag{3.21}$$

where Λ_t is the Lagrange multiplier associated with the household's budget constraint equation (3.12).

3.2.4 The intermediate good producers

Operating in a monopolistically competitive market, intermediate firms hold market power from both their diversification and the technology used in production. In itself, it is worth noticing that the mechanics of innovation being considered also bring some support and microfoundations to the monopolistic competition *de facto* introduced in the usual New Keynesian models. Moreover, prices are fixed through Calvo contracts and set as to maximize their expected profits conditional on not being allowed to optimize.

Hence, given an initial level of technological advancement $A_t(i)$, an intermediate firm faces the following constrained minimization of their cost:

$$\min_{K_t(i), L_t(i)} W_t L_t(i) + q_t K_t(i) \tag{3.22}$$

subject to its production function

$$Y_t(i) = \mu_{Z,t}\, A_t(i)^{1-\alpha}\, K_t(i)^{\alpha}\, L_t(i)^{1-\alpha} , \tag{3.23}$$

with $\alpha \in (0,1)$, where

$$\ln \mu_{Z,t} = \rho_Z \, \ln \mu_{Z,t-1} + \epsilon_{Z,t} \, , \tag{3.24}$$

so that, regardless of their individual level of technological advancement, all intermediate firms' productions are subjected to a common transitory technological shock, which follows a first-order autoregressive process.

Accordingly, the optimal capital-labour ratio (that is identical for all intermediate firms) being employed is

$$\frac{K_t}{L_t} = \frac{\alpha}{1-\alpha} \frac{W_t}{q_t} \, , \tag{3.25}$$

and the nominal marginal cost of producing an additional unit of intermediate good is given by

$$MC_t(i) = A_t(i)^{\alpha-1} \, \Omega_t \tag{3.26}$$

where Ω_t is the portion of marginal costs that is not directly dependent of the level of technology, i.e.

$$\Omega_t = \frac{q_t^{\alpha} W_t^{1-\alpha}}{\alpha^{\alpha}(1-\alpha)^{1-\alpha}} \, . \tag{3.27}$$

Hence, an improvement in technology leads to a lowering of a firm's marginal cost.

In traditional New-Keynesian models, all firms operate at the same level of technological advancement, so that, when possible, all firms set the same optimal reset price. In contrast, since the optimal reset price depends on the technology, in our set-up, with an infinite number of intermediate firms, there is an infinite number of coexisting technologies. Accordingly, there is an infinite number of reset prices because the marginal cost is a function of the technology level.

Consequently, given their respective marginal cost, intermediate firms maximize their profits with respect to their price $P_t(i)$:

$$\max_{P_t(i)} \; E_t \Bigg\{ \sum_{s=0}^{\infty} \xi_p^s \, \beta^s \frac{\Lambda_{t+s}}{\Lambda_t} \Bigg[n_{t+s-1} \Bigg(P_t(i) \pi_{p\,t,t+s} - MC_{t+s}(i) \Bigg) Y_{t+s}(i) \Bigg] \Bigg\} \tag{3.28}$$

subject to the demand for good i

$$Y_{t+s}(i) = \left(\frac{P_t(i)}{P_t} \right)^{-\epsilon} Y_{t+s}. \tag{3.29}$$

The fraction of intermediate firms that are not allowed to reoptimize has their prices indexed to inflation

$$P_t(i) = P_{t-1}(i) \pi_{t-1}^{\iota_p} \pi^{1-\iota_p}, \tag{3.30}$$

and define the cumulative indexation between t and $t+s$ is given by

$$\pi_{p,t,t+s} = \prod_{k=1}^{s} \pi_{t+k-1}^{\iota_p} \pi^{1-\iota_p}. \tag{3.31}$$

Thus, it can be shown that the optimal reset price is a function of initial technology $A_t(i)$, and of a factor F_t that is not directly dependent on the technology level:

$$P_t^*(i) = A_t(i)^{\alpha-1} F_t \; , \tag{3.32}$$

where

$$F_t = \frac{\epsilon_t}{\epsilon_t - 1} \frac{\sum_{s=0}^{\infty} \xi_p^s \, \beta^s \frac{\Lambda_{t+s}}{\Lambda_t} (\pi_{p,t,t+s})^{-\epsilon_t} \Omega_{t+s} P_t^{\epsilon}{}_t Y_{t+s}}{\sum_{s=0}^{\infty} \xi_p^s \, \beta^s \frac{\Lambda_{t+s}}{\Lambda_t} (\pi_{p,t,t+s})^{1-\epsilon_t} P_t^{1-\epsilon_t} Y_{t+s}} \; . \tag{3.33}$$

3.2.5 The innovation process

$R\&D$ activities are conducted by entrepreneurs/innovators. If these lead to an innovation, the implementation of this technology by an intermediate good producer conveys some additional market power from producing an improved version of the intermediate good, as it reaches the new technological frontier. Hence, the innovation process unfolds within the intermediate sector as the mechanism that pushes the technological frontier outward.

An entrepreneur/innovator invests some amount of final goods to raise the probability of innovating. Outside researchers or a new successful innovator supplants or "leapfrogs" an incumbent entrepreneur.[1] This prospect is, however, uncertain, as the probability of innovating is n_t, and that of not discovering is $1 - n_t$. Yet, n_t is endogenous, as it is linked to the intensity of $R\&D$ effort $\frac{X_t}{\zeta A_t^{max}}$, where X_t is the real amount of final goods invested in $R\&D$, A_t^{max} is the targeted technology level, or frontier, that will be used in date t+1 production, and $\zeta > 1$ is a scaling factor applied to the investment in $R\&D$. For a larger value of when A_t^{max} a given amount of resources X_t devoted to $R\&D$ is associated with a lower level of research intensity, thus capturing the increasing complexity of further progress. Finally, the innovation production function that exhibits diminishing marginal returns, with $\eta > 0$:

$$n_t = \left(\frac{X_t}{\zeta A_t^{max}} \right)^{1/(1+\eta)} . \tag{3.34}$$

Furthermore, the gross growth rate of the technological frontier A_t^{max} is dictated by the probability of innovation times a spillover factor, which is subject to some first-order autoregressive stochastic component. This defines a proportional increase in productivity resulting from an innovation.[2]

Namely,

[1] In our model, we therefore abstract from step-by-step technological progress, that would imply both Schumpeterian and escape-competition effects. This can also be justified by assuming that, from engaging in $R\&D$, it is prohibitively costly to develop a perfectly substitutable technology that can be used to produce a cheap and faked copy of an existing intermediate good, i.e. a knockoff.

[2] While the steady state growth rate of the frontier is constant, the frontier itself could follow different paths. Indeed, a small deviation of the growth rate, caused by the stochastic nature of the model, could put the evolution of technology on different trajectories.

$$A_t^{max} \; = \; g_t^{max} \; A_{t-1}^{max} \; , \tag{3.35}$$

$$g_t^{max} \; = \; 1 \; + \; \sigma_t \, n_{t-1} \; , \tag{3.36}$$

where

$$\ln \sigma_t = (1 - \rho_\sigma) \ln \sigma \; + \; \rho_\sigma \; \ln \sigma_{t-1} \; + \; \epsilon_{\sigma,t} \; . \tag{3.37}$$

To choose the amount of final good to be invested in $R\&D$ that maximizes expected discounted profits, the entrepreneur faces the following constrained optimization problem

$$\max_{X_t} \beta \, \frac{\Lambda_{t+1}}{\Lambda_t} \, n_t \, E_t \, V_{t+1}(A_t^{max}) - P_t \, X_t \tag{3.38}$$

subject to equation (3.34), where $E_t V_{t+1}(A_t^{max})$ is the expected discounted value of future profits contingent on the entrepreneur remaining at the helm of the monopoly. If successful, the innovator will collect monopoly profits as long as no further innovation occurs in its sector.[3]

For convenience and in a way that is compatible with complete markets, we assume that entrepreneurs invest in a diversified form of $R\&D$. Strictly speaking, this is as if an entrepreneur, provided one is successful, does not know in which sector one may end up. This implies that all entrepreneurs will invest the same amount of final good in $R\&D$. Consequently, if an innovation occurs, the technology jumps to the frontier and the expected value of the intermediate firm will be the same regardless of the intermediate sector. In accordance with the problem in equation (3.38), the optimal real investment in $R\&D$ is given by

[3] In our model, with a steady-state probability of innovation n^{ss}, the expected life span of an intermediate firm at steady-state is $1/n^{ss}$.

$$X_t = \beta \, \frac{n_t}{1+\eta} \, \frac{\Lambda_{t+1}}{\Lambda_t} \, \frac{E_t V_{t+1}(A_t^{max})}{P_t} \, . \tag{3.39}$$

To complete the solution to equation (3.39), we still need to write explicitly the expected value of the firm to the entrepreneur. This happens to be more challenging than it may look at first, as it relies on the path that prices are expected to follow.

To calculate the value of an innovation-implementing intermediate firm, let us consider an entrepreneur/innovator who, at date t, ponders how much to invest in $R\&D$ while seeking some returns from date $t+1$ onward. This evaluation must take into account different possible outcomes that reflect the probability of remaining at the helm of the monopoly, the appropriate stochastic discount factors, as well as the probability that price reoptimization occurs in a Calvo setting. This is why all possible contingencies that could deliver some return from innovating are cibsudered.[4]

First, let us think about the case of a new monopolist taking over as of date $t+1$ and setting the optimal price for its intermediate good i. As he has innovated, his prevailing specific technology reaches the new technological frontier, so that $A_t(i) = A_t^{max}$. He faces each period the probability ξ_p of not being allowed to optimize its price afterwards, so that, for the contingency path that price reoptimization never occurs, his expected discounted stream of profits would be given by

$$\Psi_{1t+1}(i) = \sum_{s=1}^{\infty} \xi_p^{s-1} \, \beta^{s-1} \, \frac{\Lambda_{t+s}}{\Lambda_{t+1}} \left[P_{t+1}^{*}(i) \pi_{p,t+1,t+s} - MC_{t+s}(i) \right] Y_{t+s}(i) \, \prod_{q=2}^{s} (1 - n_{t+q-1}) \tag{3.40}$$

where $\prod_{q=2}^{s} (1 - n_{t+q-1})$, is the probability of not being displaced out of business at date $t+s$. The flows of revenues and costs are discounted from the perspective of date $t+1$, as the nested sum is discounted up to the beginning of the initial cash flow pertaining to this stream, and weighted by the probability of remaining at the helm of the monopoly for all periods in the future. In particular, date $t+1$ cash flow has a

4 Notice that if the monopolist is supplanted at a future date by a competitor's adoption of an innovation, expected profits are to become zero from that date forward, with no bearing on the current expected discounted flow of profits for date t investing entrepreneur.

unit probability, i.e. $\prod_{q=2}^{1}(1-n_{t+q-1})=1$, as we consider a successful innovation driving the production of intermediate good i that is sold at its optimal price. Moreover, if an intermediate firm producing good i is replaced following the implementation of a new innovation in its sector at some future date T, then no additional profits will accrue from then on from the older technology.

As long as it has not been supplanted, this monopolist will be operating under technology A_t^{max}. Hence, his marginal cost of production evolves according to

$$MC_{t+s}(i) = A_t^{max(\alpha-1)}\,\Omega_{t+s}\;, \tag{3.41}$$

while his optimal price is set to

$$P_{t+1}^{*}(i) = A_t^{max(\alpha-1)}\,F_{t+1}\;, \tag{3.42}$$

and the expected demand for his good follows a path defined by

$$Y_{t+s}(i) = \left(\frac{P_{t+1}^{*}(i)\pi_{p,t+1,t+s}}{P_{t+s}}\right)^{-\epsilon_t} Y_{t+s}\;. \tag{3.43}$$

Using equations (3.41), (3.42), and (3.43), equation (3.40) can be rewritten as

$$\Psi_{1t+1}(i) = \qquad A_t^{max(\alpha-1)(1-\epsilon_t)}\left\{F_{t+1}^{(1-\epsilon_t)}\sum_{s=1}^{\infty}\xi_p^{s-1}\,\beta^{s-1}\,\frac{\Lambda_{t+s}}{\Lambda_{t+1}}\,P_{t+s}^{\epsilon_t}\,Y_{t+s}\,\prod_{q=2}^{s}(1-n_{t+q-1})\right.$$

$$\left.-F_{t+1}^{-\epsilon_t}\sum_{s=1}^{\infty}\xi_p^{s-1}\,\beta^{s-1}\,\frac{\Lambda_{t+s}}{\Lambda_{t+1}}\,P_{t+s}^{\epsilon_t}\,\Omega_{t+s}Y_{t+s}\,\prod_{q=2}^{s}(1-n_{t+q-1})\right\}. \tag{3.44}$$

Making use of equations (2.34) and (2.35), but as of $t+1$, we therefore have

$$\Psi_{1t+1}(i) = A_t^{max(\alpha-1)(1-\epsilon)} \left(F_{t+1}^{(1-\epsilon)} Aux_{rev,t+1} - F_{t+1}^{-\epsilon} Aux_{cost,t+1} \right), \quad (3.45)$$

with auxiliary variables associated respectively with the firm's revenues in the first term, $Aux_{rev,t}$, and costs in the second term, $Aux_{cost,t}$, i.e.

$$Aux_{rev,t+1} = P_{t+1}^{\epsilon} Y_{t+1} + \xi_p \beta \frac{\Lambda_{t+2}}{\Lambda_{t+1}} (1 - n_{t+1}) Aux_{rev,t+2}, \quad (3.46)$$

and

$$Aux_{cost,t+1} = \Omega_{t+1} P_{t+1}^{\epsilon} Y_{t+1} + \xi_p \beta \frac{\Lambda_{t+2}}{\Lambda_{t+1}} (1 - n_{t+1}) Aux_{cost,t+2}. \quad (3.47)$$

Second, let us turn to all the other possible cases of a new monopolist taking over as of date $t+1$ and setting the optimal price for its intermediate good i at some future date $t+l$ with some probability $1-\xi_p$, yet followed by the contingency path that price reoptimization does not occur afterwards, as there is a probability ξ_p each period of no reoptimization, even if the monopolist remains in operation. Summing over all contingent paths, with the proper probabilistic weights, the relevant discounted stream of profits for all these contingent paths is given by

$$\Psi_{2t+1}(i) = A_t^{max(\alpha-1)(1-\epsilon_t)} \left\{ \sum_{l=2}^{\infty} (1-\xi_p)\beta^l \frac{\Lambda_{t+l}}{\Lambda_{t+1}} \prod_{q=2}^{l-1} (1 - n_{t+q-1}) \right.$$

$$\left[F_{t+l}^{1-\epsilon_t} \sum_{s=l}^{\infty} \xi_p^{s-1} \beta^{s-1} \frac{\Lambda_{t+s}}{\Lambda_{t+l}} P_{t+st}^{\epsilon} Y_{t+s} \prod_{q=l}^{s} (1 - n_{t+q-1}) \right. \quad (3.48)$$

$$\left. \left. - F_{t+l}^{-\epsilon_t} \sum_{s=l}^{\infty} \xi_p^{s-1} \beta^{s-1} \frac{\Lambda_{t+s}}{\Lambda_{t+l}} P_{t+s}^{\epsilon_t} \Omega_{t+s} Y_{t+s} \prod_{q=l}^{s} (1 - n_{t+q-1}) \right] \right\}.$$

Furthermore, updating equations (3.46) and (3.47) to date $t+l$, equation (3.48) can be rewritten as

$$\Psi_{2t+1}(i) = A_t^{max(\alpha-1)(1-\epsilon_t)} \left\{ \sum_{l=2}^{\infty} (1-\xi_p)\beta^l \frac{\Lambda_{t+l}}{\Lambda_{t+1}} \prod_{q=2}^{l-1} (1-n_{t+q-1}) \left(F_{t+l}^{(1-\epsilon_t)} Aux_{rev,t+l} - F_{t+l}^{-\epsilon_t} Aux_{cost,t+l} \right) \right\} \tag{3.49}$$

In an analogous manner as before, making use of the recursion built in the summation above and defining an auxiliary variable, $Aux_{rem,t+l}$, associated with the remainder of the expected discounted profits equation (3.49) can also be displayed as

$$\Psi_{2t+1}(i) = A_t^{max(\alpha-1)(1-\epsilon_t)} \; Aux_{rem,t+2} \; , \tag{3.50}$$

where

$$Aux_{rem,t+2} = (1-\xi_p)\beta^2 \frac{\Lambda_{t+2}}{\Lambda_{t+1}} \left(F_{t+2}^{(1-\epsilon_t)} Aux_{rev,t+2} - F_{t+2}^{-\epsilon_t} Aux_{cost,t+2} \right) + \beta \left(1-n_{t+2}\right) Aux_{rem,t+3} \; . \tag{3.51}$$

Consequently, the expected value of the intermediate firm to a successful innovator is

$$E_t \, V_{t+1} = \Psi_{1t+1}(i) + \Psi_{2t+1}(i) \tag{3.52}$$

or, namely,

$$E_t \, V_{t+1} = A_t^{max(\alpha-1)(1-\epsilon_t)} \; \left(F_{t+1}^{1-\epsilon_t} Aux_{rev,t+1} - F_{t+1}^{-\epsilon_t} Aux_{cost,t+1} \; + \; Aux_{rem,t+2} \right) \; . \tag{3.53}$$

Hence, the expected value of an intermediate firm for a successful entrepreneur/innovator is determined by the newly reached technological frontier through $A_t^{max(\alpha-1)(1-\epsilon_t)}$, the contribution from profits arising from being able to set the optimal price for good i as of period $t+1$ through $F_{t+1}^{1-\epsilon_t} Aux_{rev,t+1} -$

$F_{t+1}^{-\epsilon_t} Aux_{cost,t+1}$, and the contribution to profits resulting from a later date optimal price setting with what will have become an older technology through $Aux_{rem,t+2}$.

3.2.6 The specification of monetary policy

The central bank's policy function is modelled as a Taylor-type reaction function, as it sets the nominal interest rate according to the following equation:

$$\frac{1+R_t}{1+R} = \left(\frac{1+R_{t-1}}{1+R}\right)^{\rho_R} \left[\left(\frac{\pi_t}{\pi}\right)^{\alpha_\pi} \left(\frac{Y_t}{Y_{t-1}}\, g^{-1}\right)^{\alpha_y}\right]^{1-\rho_R} \mu_{M,t} \ , \tag{3.54}$$

where

$$\ln \mu_{M,t} = \rho_M \, \ln \mu_{M,t-1} + \epsilon_{t_{M,t}} \ . \tag{3.55}$$

The ρ_R parameter represents the degree of smoothing of interest rate changes, as the monetary authority seeks to avoid too-large changes with respect to its one-period-lag value, R_{t-1}, and adjusts it somewhat gradually, with weight $1-\rho_R$, following demand and technology shocks. The parameters α_π and α_y are the monetary authority's weights attached to deviations from its inflation target, π, and its output growth target, g. The latter is defined as the growth rate of the average technology level in steady state. Finally, $\mu_{M,t}$ is an exogenous and stochastic component of monetary policy representing deviations from the Taylor-type rule, that follows a stationary first-order autoregressive process.

3.3 Bayesian estimation

3.3.1 The elements needed for a Bayesian estimation

We seek the posterior distribution of a set of parameters which combines a likelihood function with prior information. In this section, we discuss the data used for estimation, how we map said data to the model and the prior densities of each estimated parameter.

We have introduced a set of shocks that may not necessarily be structural to the model to increase the number of observables used in the estimation. In Bayesian estimation of DSGE models, the necessity for having as many shocks as observable variables is driven by two fundamental requirements: model identification and capturing the dynamics of the data.

Capturing the dynamics of the data is another crucial aspect of model estimation. In real economies, various types of shocks can influence different aspects of the economic data. These shocks might include technology advancements, policy changes, demand fluctuations, and other external disturbances. A DSGE model with a diverse set of shocks can more accurately reflect the complex dynamics of the real economy. If there are fewer shocks than observable variables, the model might fail to capture all the sources of fluctuations observed in the data, leading to an incomplete or misleading understanding of economic dynamics. Having a sufficient number of shocks allows the model to explain the variability in each observable variable comprehensively. This is particularly important for understanding how different economic policies or external events might influence various aspects of the economy.

Model identification is a critical aspect of any econometric estimation, including Bayesian approaches. For a model to be identified, each observable variable must have a unique mapping to the model's elements, such as parameters and shocks. If the number of shocks is less than the number of observable variables, the model may suffer from under-identification. This situation arises when multiple sets of model parameters can explain the observed data equally well, making it impossible to uniquely determine the model's parameters. Essentially, under-identification leads to a scenario where the model cannot distinguish between the effects of different economic forces on the observable variables. By matching the number of shocks with the number of observables, each observable is uniquely associated with specific model dynamics, ensuring that the model is properly identified and that the parameter estimates are meaningful.

Appendix D includes a discussion a more detailed discussion on some other considerations related to the Bayesian estimation of DSGE models.

3.3.2 The data

The national accounts data is extracted from the Federal Reserve Bank of St. Louis' FRED Database at a quarterly frequency for a sample spanning 1954 Q3 to 2018 Q3. Nominal consumption (PCEC) and investments (FPI) are deflated, with the implicit price deflator (GDPDEF) and calculated on a per capita basis by dividing them by the civilian noninstitutional population (LNS10000000) obtained from the Bureau of Labor and Statistics (BLS). Output (GDPC1) is already in real terms and needs only to be computed on a per capita basis. To compute the number of hours worked, we take the average number of hours worked in the non-farm business sector (PRS85006023) multiply it by the employment level to get the total number of hours worked and is expressed on a per capita basis. Inflation is the quarterly rate of growth of the implicit price deflator.

The inclusion of endogenous growth in the model necessitates adding two observables to the one used by Smets & Wouters (2007). We use Fernald's measure of Total Factor Productivity (TFP) and the aggregate $R\&D$ (Y694RC) from the Bureau of Economic Analysis. We apply to $R\&D$ the same transformations we applied to investment and consumption.

We use the shadow interest rate from Wu & Xia (2016) instead of the Federal Funds rate for a couple of reasons. First, the shadow rate effectively captures the stance of monetary policy in periods where traditional policy rates are constrained by the ZLB. This becomes particularly relevant in empirical analysis, as it provides a more accurate reflection of monetary conditions during periods of binding constraints on nominal interest rates. Additionally, the shadow rate can serve as a proxy for unconventional monetary policy measures, offering a continuous metric that extends the conventional policy rate's interpretation beyond the ZLB. This approach allows for a more nuanced analysis of monetary policy impacts in various economic environments, including those not directly constrained by the ZLB. By incorporating the Wu-Xia shadow rate, the model gains an enhanced ability to reflect real-world policy dynamics, thereby improving the robustness and relevance of the estimation, especially in scenarios where conventional policy rates do not fully capture the monetary policy stance due to the presence of the ZLB. This methodological choice thus enriches the analysis by bridging the gap between theoretical models and the complex realities of monetary policy implementation.

3.3.3 Measurement equations

To perform a Bayesian estimation, it is necessary to specify measurement equations which map observed variables from empirical data to variables used in the model. In contrast with the usual application in standard exogenous NK DSGE models in the literature, this model has an explicitly specified growth trend, which must be accounted for when mapping empirical data to the model. We will use a first difference in both empirical and simulated data pertaining to macroeconomic aggregates, the number of hours worked and the federal funds rate:

$$dln(TFP_t^{obs}) = ln\left(\frac{Z_t}{Z_{t-1}}\right) + (1-\alpha)ln\left(\frac{A_t}{A_{t-1}}\right) \tag{3.56}$$

$$dln(i_t^{obs}) = ln\left(\frac{i_t}{i_{t-1}}\right) + ln\left(\frac{A_t}{A_{t-1}}\right) \tag{3.57}$$

$$dln(c_t^{obs}) = ln\left(\frac{c_t}{c_{t-1}}\right) + ln\left(\frac{A_t}{A_{t-1}}\right) \tag{3.58}$$

$$dln(w_t^{obs}) = ln\left(\frac{w_t}{w_{t-1}}\right) + (1-\alpha)ln\left(\frac{A_t}{A_{t-1}}\right) \tag{3.59}$$

$$dln(y_t^{obs}) = ln\left(\frac{y_t}{y_{t-1}}\right) + ln\left(\frac{A_t}{A_{t-1}}\right) \tag{3.60}$$

$$dln(L_t^{obs}) = ln\left(\frac{L_t}{L_{t-1}}\right) \tag{3.61}$$

$$dln(x_t^{obs}) = ln\left(\frac{x_t}{x_{t-1}}\right) + ln\left(\frac{A_t}{A_{t-1}}\right) \tag{3.62}$$

$$d(r_t^{obs}) = r_t - r_{t-1} \tag{3.63}$$

The inclusion of trend inflation allows for the inclusion of the level of inflation as a measurement equation:

$$\pi_t^{obs} = \pi_t \tag{3.64}$$

Our vector of observables is :

$$\left[dln(TFP_t^{obs}),\ dln(i_t^{obs}),\ dln(c_t^{obs}),\ dln(w_t^{obs}),\ dln(y_t^{obs}),\ dln(L_t^{obs}),\ dln(x_t^{obs}),\ d(r_t^{obs})\right] \tag{3.65}$$

Table 3.1 Prior Distribution of New Keynesian Parameters

	Description	Prior Density	Mean	Standard Deviation
α	Capital share	Normal	0.3	0.05
γ	Elasticty of substitution between labor	Normal	6	1
ϵ	Elasticty of substitution between intermediate goods	Normal	6	1
h	Habit formation	Normal	0.5	1
ν	Inverse Frisch elasticity	Normal	2	0.75
α_y	Taylor rule output growth	Normal	0.125	0.05
α_π	Taylor rule inflation	Normal	1.7	0.3
ρ_r	Taylor rule smoothing	Beta	0.6	0.2
ξ_p	Calvo prices	Beta	0.5	0.1
ξ_w	Calvo wages	Beta	0.5	0.1
ι_p	Price indexation	Beta	0.5	0.15
ι_w	Wage indexation	Beta	0.5	0.15
π	Trend inflation	Normal	1.005	0.001

3.3.4 Discussion of prior distributions

We have taken a parsimonious approach to setting priors. We fix some parameters to values agreed upon in the literature. For instance, we set the depreciation rate of capital at capacity δ, the discount rate β and the steady state government ratio to output to 22%. In table 3.1, we summarize the priors of the parameters traditionally found in the New Keynesian literature, which are in line with those used in Smets & Wouters (2007) and Justiniano et al. (2011).

These parameters are estimated as a way of ensuring the internal consistency of the model. We need to study whether the inclusion of an endogenous growth framework in a New Keynesian model creates dynamics which would lead us to estimates that deviate from the literature. Estimating the Schumpeterian growth components of the model is a relatively new endeavour, as discussed in previous sections. We have therefore decided to set uniform priors for those parameters. In Bayesian estimation, the utilization of a uniform prior distribution is predominantly guided by the desire to reflect a state of uncertainty or the absence of prior knowledge regarding a parameter's value. Characterized as non-informative or weakly informative, the uniform prior assigns equal probability to all values within a specified range, demonstrating a stance of neutrality and allowing for an objective analysis. This approach is particularly advantageous when maintaining objectivity is crucial, as it minimizes the influence of subjective beliefs and prevents the introduction of potential biases into the estimates. From a computational standpoint, uniform priors offer simplicity, contributing to the efficiency and ease of implementation in complex models. Additionally, they serve a practical purpose by bounding the parameter space, which is beneficial in situations where certain parameter values are implausible or outside the realm of theoretical or practical possibility. By restricting parameters to a feasible range, uniform priors help in avoiding extreme or unrealistic estimates that may arise due to anomalies in the data. Overall, the choice of a uniform prior in Bayesian estimation represents a strategic decision to allow the data to predominantly inform the posterior distribution, ensuring a more data-driven and less assumption-laden analytical process.

Table 3.2 Endogenous Growth Parameters

	Description	Prior Density	Mean	Minimum	Maximum
σ	Spillover effect	Uniform	0.03	0.001	0.06
ζ	Innovation scalaing parameter	Uniform	200000	100000	300000
η	Inverse marginal diminishing returns of investing in $R\&D$	Uniform	10	1	20

We set identical priors to both the persistence and standard deviations of all exogenous shocks. The prior distribution of the shocks' persistence is a beta with a mean of 0.5 and a standard deviation of 0.2. The standard deviation's prior density is an inverse gamma with a 0.2 mean and a standard deviation of 2. The shocks are independent and therefore, their covariances are equal to zero.

3.4 Results

Table 3.3 Results of the Bayesian estimation

	Prior Mean	Posterio rMean	5th Percentile	95th Percentile
ξ_p	0.5	0.649	0.646	0.653
ξ_w	0.5	0.611	0.606	0.618
ι_p	0.5	0.850	0.835	0.865
ι_w	0.5	0.712	0.703	0.722
α_π	1.5	1.349	1.344	1.356
ρ_r	0.75	0.404	0.398	0.410
α_y	0.125	0.276	0.271	0.282
ν	1	1.092	1.072	1.110
h	0.7	0.873	0.868	0.877
ϵ	6	7.188	7.101	7.264
γ	6	5.633	5.585	5.689
α	0.33	0.296	0.288	0.304
π	1.005	1.003	1.003	1.003
tx	1.28	1.238	1.234	1.242

The parameters which relate to the New Keynesian components of the model are in line with what is observed in the literature. The Calvo parameters of both the price and wage rigidity are both at approximately 0.6, meaning that contracts will last on average 2.5 quarters, which is consistent with the findings of Bils & Klenow (2004) and Smets & Wouters (2007). The estimation yields a high degree of price and wage indexation, which are respectively at 0.85 and 0.71. The estimated price elasticity of demand ϵ shows that the markup over the perfectly competitive price is $\frac{\epsilon}{\epsilon-1} = 16\%$ while the wage markup is 21%.

Inflation is estimated to be somewhat lower than the previously anticipated 2%, which aligns with the introduction of the Schumpeterian component in the model. As firms innovate, they introduce new products that reduce the marginal cost of production over time. This, in turn, places downward pressure on price levels and decreases trend inflation. This dynamic is consistent with the findings of Aghion et al. (2019), who examine the relationship between creative destruction and "missing growth." According to their argument, the process of creative destruction—where new, superior products replace outdated ones—complicates the accurate measurement of inflation. Specifically, the Consumer Price Index (CPI) relies on a fixed basket of goods, which fails to adequately account for the introduction of these new, improved, and often cheaper goods. As a result, the CPI tends to overstate inflation, since it does not fully incorporate the value and price reductions associated with the innovation. Consequently, this overstatement of inflation leads to an underestimation of actual economic growth, as the true cost of living is lower than what the CPI suggests. Thus, creative destruction presents a significant challenge in accurately measuring inflation and growth, highlighting the need for more refined approaches to capture the effects of innovation in economic metrics.

Monetary policy exhibits a high degree of reaction to output growth which can be in part explained by the use of the Wu & Xia (2016) shadow rate. Instead of being stuck at the lower bound, the shadow rate goes below 0 as unconventional monetary policy is employed to stimulate the economy during the global financial crisis of 2008. The corollary to this point is that the degree of smoothing of monetary policy ρ_r is lower than its prior. The prior mean for this parameter is 0.75 while the posterior mean is 0.4. The inverse of the Frisch elasticity ν is close to 1. The habit formation parameter h is similarly close to its prior.

Table 3.4 Posterior distribution of the endogenous growth parameters

	Prior Mean	Mean	5th Percentile	95th Percentile
σ_{ss}	0.02	0.039	0.039	0.040
ζ	20000	24539	24528	24552
η	11	6.075	5.915	6.232

The major contribution of this paper to the literature is the estimation of the parameters related to endogenous growth. The growth rate of the frontier, which in our model is also the growth rate of the economy is 2.4%. The concavity of the production function is determined by the parameter η whose mean is estimated to be 6.075, implying that the exponent of the production function is $\frac{1}{1+\eta} = 0.14$. The innovation production function exhibits a high degree marginal diminishing returns. The spillover effect is larger than anticipated at 0.039. While the interpretation of this number is difficult, it can be translated into an equivalent innovation probability, as there is a linear relationship between growth, spillover and innovation probability. According to our estimation, each quarter, an innovation occurs in 15.4% of the intermediate sectors.

Furthermore, it must be kept in mind that the nature of innovation represented in the model remains somewhat broad and does not require it necessarily to be pathbreaking at each displacement of the technological frontier. In the real world, even some organizational innovation or milder production innovation that reduces the marginal cost counts as an innovation in our stylized model economy. Hence, the estimates we find might be deemed as a summary statistics representing broadly characteristics of the extent of innovation activities for the aggregate economy.

Table 3.5 Posterior distribution of the shock parameters

	Prior Mean	Mean	5th Percentile	95th Percentile
ρ_I	0.5	0.404	0.388	0.417
ρ_m	0.5	0.669	0.658	0.681
ρ_z	0.5	0.988	0.980	0.995
ρ_β	0.5	0.571	0.556	0.587
ρ_σ	0.5	0.939	0.932	0.947
ρ_ϵ	0.5	0.596	0.583	0.608
ρ_γ	0.5	0.686	0.681	0.693
ρ_x	0.5	0.733	0.724	0.742
ρ_{tx}	0.5	0.972	0.963	0.980
σ_z	0.1	0.01	0.0093	0.0108
σ_{tx}	0.1	0.0078	0.0073	0.0084
σ_μ	0.1	0.094	0.086	0.1015
σ_σ	0.1	0.2185	0.2006	0.2353
σ_β	0.1	0.0241	0.0219	0.0262
σ_γ	0.1	0.1532	0.1342	0.168
σ_ϵ	0.1	0.0599	0.0552	0.0647
σ_x	0.1	0.6212	0.5538	0.687
σ_m	0.1	0.01	0.0084	0.0131

Finally we now turn to the persistence of the exogenous shocks. While it is understandable that fiscal shocks are persistent, the high degree of persistence of the neutral technology shock and the spillover shocks may be somewhat surprising. Indeed, part of the motivation for including endogenous growth in a New Keynesian model is to be able to endogenously generate persistence. Our estimation points pertaining to the persistence of the shocks seem to remain strongly responsible for generating persistence. One conjecture that may underlie the weaker than expected contribution of $R\&D$ to the inherent persistence in the economy may depend on the particular implementation of creative destruction. One avenue that may be worth considering in future research would be to extend the time-to-build period needed to increase the probability of innovating. A three month investment horizon limits the ability of the model to create additional persistence. Hence a possibility would be to have a multiperiod investment in $R\&D$. Shocks from several quarters back would have longer-lasting effects without the need for high degrees of persistence of the shocks.

3.5 Conclusion

This paper builds on the Mahroug & Paquet (2021) model by introducing elements such as positive trend inflation and price and wage indexation into the New Keynesian Dynamic Stochastic General Equilibrium (NK DSGE) framework. These technical additions are crucial for capturing more realistic macroeconomic dynamics, particularly in the context of contemporary monetary policy and labor market behaviors.

Integrating positive trend inflation into the model is essential for mirroring the persistent, low-level inflation observed in modern economies. This aspect has been especially pertinent since the 1980s, where central banks have actively targeted a positive inflation rate, whether officially or unofficially. By incorporating this feature, the model more accurately reflects the impact of inflation expectations on various economic decisions, particularly monetary policy.

The inclusion of price and wage indexation is another significant enhancement. This addition recognizes the reality that not all prices and wages adjust instantaneously to economic changes. It introduces a degree of nominal rigidity, a key aspect of New Keynesian economics, and helps explain the observed stickiness in prices and wages in response to economic fluctuations.

The Bayesian estimation results of this enhanced model provide several key insights. Firstly, the estimated parameters are consistent with established New Keynesian models, which validates the foundational assumptions of the framework. A significant finding is the large spillover effect from innovation, underscoring the pivotal role of $R\&D$ in driving long-term economic growth. However, the model also indicates high persistence in technological and spillover shocks, suggesting a reliance on external factors for generating economic persistence and potentially overshadowing the model's endogenous growth mechanisms.

While the model successfully integrates key aspects of modern economies and offers valuable insights, it also highlights the need for ongoing research to refine its accuracy and applicability in economic analysis and policy formulation.

There is a need to refine the model to include a longer time to build horizon in $R\&D$ investments, allowing for a more nuanced understanding of how such investments impact economic growth over time. Exploring the mechanisms and impacts of price and wage stickiness could also provide deeper insights, especially in various inflationary environments. Moreover, the policy implications of these findings, particularly in optimizing monetary and fiscal policies to support innovation-led growth, could be worth exploring. Conducting robustness checks under different economic scenarios and empirical validation in diverse economies may enhance as well the model's applicability and reliability in policy-making contexts.

CONCLUSION

Cette thèse s'attaque à la compréhension des liens complexes entre la croissance endogène schumpétérienne et les fluctuations économiques causées par des chocs réels et monétaires, tout en tenant compte des rigidités néokeynésiennes de prix et de salaires dans l'économie. D'une part, nous avons élaboré quelques variantes de modèles macroéconomiques intégrés de la croissance endogène schumpétérienne et des cycles économiques à l'aide de l'approche d'équilibre général dynamique stochastique pour étudier les réponses dynamiques de variables clés à différentes sources de perturbation. Nous avons également montré que différents niveaux de probabilité d'innovation à l'état stationnaire ont vraisemblablement un impact non négligeable sur la flexibilité des prix et sur le bien-être des agents économiques. D'autre part, une estimation bayésienne des paramètres clés de la version la plus générale du modèle a permis de vérifier la plausibilité et la pertinence du modèle d'innovation par destruction créatrice pour le cycle économique.

Le chapitre 1 a présenté le fondement d'un premier modèle hybride intégrant à la fois les principes de la croissance endogène schumpétérienne et les rigidités nominales nouvelles keynésiennes, et leur interaction. Ce modèle comprenait une structure unique de brevets qui lui permettait de modéliser deux niveaux technologiques distincts dans le secteur de la production intermédiaire. Cette première approche met en évidence l'importance de la RD et de ses effets d'entraînement (de retombées ou de spillovers), posant ainsi les bases pour une analyse plus poussée de leur impact sur les investissements en RD et les effets de la politique monétaire dans une économie avec innovation. La décomposition de la variance a notamment révélé que le choc sur les retombées (ou spillover) de l'innovation joue un rôle significatif dans la fluctuation des investissements en RD et en capital physique.

Le chapitre 2 a élargi l'étude de l'interaction entre la croissance schumpétérienne et l'impact des rigidités des prix et des salaires sur les cycles économiques en le libérant du système de brevets plus contraignant qu'il avait adopté. Cette extension du modèle a ouvert la voie à une interaction plus complexe et diversifiée entre la technologie et la rigidité des prix et a permis une analyse plus riche et plus détaillée, en explorant comment les variations dans la probabilité d'innovation à l'état stationnaire influencent significativement les réponses macroéconomiques et le bien-être économique. Le modèle et sa calibration reproduisent efficacement les éléments clés du cycle économique. Ils révèlent l'impact significatif des variations de la probabilité d'innovation de l'état stationnaire sur les réponses macroéconomiques. Une probabilité d'innovation plus élevée entraîne des retombées technologiques étendues et réduit les rigidités des prix nominaux. Conséquemment, nous observons également qu'une différence dans les probabilités d'innovation à l'état stationnaire a des conséquences substantielles sur le bien-être. Par exemple, une économie avec une probabilité d'innovation plus élevée à l'état stationnaire présente un niveau de bien-être nettement supérieur. Cela illustre à quel point l'innovation et ses retombées ont un impact économique non négligeable.

Finalement, le chapitre 3 s'est intéressé à l'estimation bayésienne des paramètres d'une version plus générale du modèle du cycle avec rigidités et innovations schumpétériennes. Notamment, en comparaison avec les chapitres précédents, il introduit un taux d'inflation tendancielle à l'état stationnaire strictement positif, ainsi que des mécanismes d'indexation des prix et des salaires. Cette première estimation, à notre connaissance, d'un tel modèle du cycle permet de valider la plausibilité et la pertinence du modèle élaboré et de renforcer la compréhension de la dynamique complexe qui lie l'innovation à la croissance et aux cycles économiques. L'approche empirique nous permet d'obtenir une représentation plus nuancée et plus précise de la dynamique économique et souligne le rôle crucial de la RD et de ses effets de retombées.

Les résultats obtenus témoignent de l'importance de la rigidité des prix et des salaires, l'impact de la destruction créatrice ainsi que l'influence de la politique monétaire sur la croissance à la fréquence des cycles économique. L'estimation du modèle confirme la robustesse de plusieurs paramètres types des modèles standards du cycle. De plus, la prise en compte originale d'éléments propres à l'innovation schumpétérienne permet d'estimer des paramètres qui lui sont associés. Malgré les avancées documentées, l'estimation du modèle révèle également des limites de la spécification actuelle et pointe vers des améliorations à envisager. En particulier, nous trouvons que la persistance du modèle repose en grande partie sur la persistance des chocs. Nos constats s'avèrent utiles pour formuler des propositions afin d'affiner et d'élargir davantage notre cadre empirique dans les recherches futures.

Plusieurs angles constituent des avenues particulièrement intéressantes et prometteuses autour des trois thématiques suivantes.

Premièrement, il pourrait fort bien s'avérer pertinent d'étendre la période de gestation d'un investissement en RD conduisant à une hausse significative de la probabilité d'innovation schumpétérienne, à plus d'une période, comme c'est typiquement supposé dans la littérature. La modélisation des investissements en RD sur plusieurs périodes correspondrait vraisemblablement davantage à la réalité du processus d'innovation, en tenant compte du temps nécessaire à la recherche, au développement, et à la commercialisation des innovations. En enrichissant l'impact cumulatif, des investissements courants et subséquents en RD ainsi que leur persévérance sur les résultats en termes d'innovations permettraient possiblement de mieux reproduire la dynamique entre l'investissement initial en RD et ses retombées économiques. De plus, cela pourrait fournir une base plus solide pour la formulation des politiques économiques et l'analyse de la croissance à long terme. De plus, cette amélioration pourrait augmenter la persistance observée des effets de différents chocs sur les variables du modèle, sans avoir recours à des coefficients d'autocorrélation tendant vers l'unité dans les lois de mouvement des chocs exogènes.

Deuxièmement, une étude consacrée à la modélisation de la politique monétaire optimale dans des environnements riches en innovations et avancées technologiques pourrait éclairer la conduite de cette politique. Typiquement dans la littérature, tout comme dans cette thèse, la politique monétaire est représentée par une fonction de réaction de Taylor avec une composante stochastique, afin d'examiner ensuite la réponse dynamique des variables macroéconomiques d'intérêt, notamment à un choc monétaire. Or, les analyses ont tendance à occulter la contribution de la partie réactive de la politique monétaire, bien qu'elle ait sa propre contribution au cycle économique. Ainsi, des variations dans la formulation et la modélisation de la politique monétaire et de ses effets mêmes sur l'innovation ne sont vraisemblablement pas triviales.

Une avenue prometteuse de recherche explorerait comment les banques centrales pourraient adapter leurs stratégies pour répondre efficacement aux fluctuations causées par les mécanismes de croissance endogène. Par exemple, Annicchiarico & Pelloni (2021) ont récemment étudié la question de la politique monétaire optimale au sein d'un cadre nouveau keynésien intégrant des innovations horizontales qui motivait l'adoption d'une cible d'inflation positive et montrant que la réactivité de la politique monétaire devrait tendre à s'atténuer en contexte de croissance endogène. À notre connaissance, la question de la réaction dynamique optimale de la politique monétaire n'a pas été considérée dans un contexte de croissance endogène schumpétérienne.

Finalement, il serait intéressant d'examiner les conséquences de changements dans le taux de croissance tendancielle de la frontière technologique, comme le suggèrent récemment des études sur le ralentissement de la croissance de la productivité dans les pays développés. Fernald et al. (2023) ont voulu savoir si le ralentissement de la croissance de la productivité observé en Europe et aux États-Unis depuis les années 2000 était dû à des chocs externes ou à un changement à long terme. L'article suggère que le ralentissement de la croissance de la productivité totale des facteurs est la principale cause de ce phénomène. Cela ouvre de nouvelles pistes quant à la modélisation.

Afin de refléter cette dynamique, il faudrait permettre un changement dans le taux de croissance réel de la production agrégée à l'état stationnaire. Deux mécanismes sont possibles. Un premier consiste à analyser les transitions entre deux états stationnaires différents. Un second, plus complexe, mais peut-être plus intéressant, endogénéiserait complètement le taux de croissance à l'état stationnaire. Ceci ajouterait ainsi une autre source de non-stationnarité et permettrait à la croissance à long terme de fluctuer en fonction des conditions économiques.

En somme, nous croyons que les avancées développées dans cette thèse et les réflexions additionnelles qu'elles suscitent sont un encouragement à traiter simultanément la croissance schumpétérienne et les cycles économiques dans la modélisation macroéconomique.

APPENDIX A

CHAPTER 1 - DETAILED VARIANCE DECOMPOSITION

Table A.1 Variance Decomposition t=5 quarters

	Technology Shock	**Monetary Shock**	**Investment Shock**	**Spillover Shock**
Output	2%	12%	85%	0%
Consumption	3%	0%	97%	0%
Investment	1%	37%	61%	1%
R&D	60%	5%	32%	4%
Physical Capital	1%	56%	31%	12%
Capacity Utilization	21%	4%	75%	0%
Wages	79%	1%	18%	2%
Labor	21%	6%	73%	0%
Rate of return on capital	21%	4%	75%	0%
Interest Rate	13%	12%	74%	1%

Note: The table shows the variance decomposition of different economic indicators at t=5 quarters.

Table A.2 Variance Decomposition t=10 quarters

	Technology Shock	**Monetary Shock**	**Investment Shock**	**Spillover Shock**
Output	2%	15%	81%	2%
Consumption	2%	5%	90%	3%
Investment	1%	35%	62%	2%
R&D	46%	15%	30%	9%
Physical Capital	1%	46%	36%	17%
Capacity Utilization	20%	5%	75%	1%
Wages	55%	18%	16%	11%
Labor	19%	7%	75%	0%
Rate of return on capital	20%	5%	75%	1%
Interest Rate	13%	13%	71%	2%

Note: The table shows the variance decomposition of different economic indicators at t=10 quarters.

Table A.3 Variance Decomposition t=20 quarters

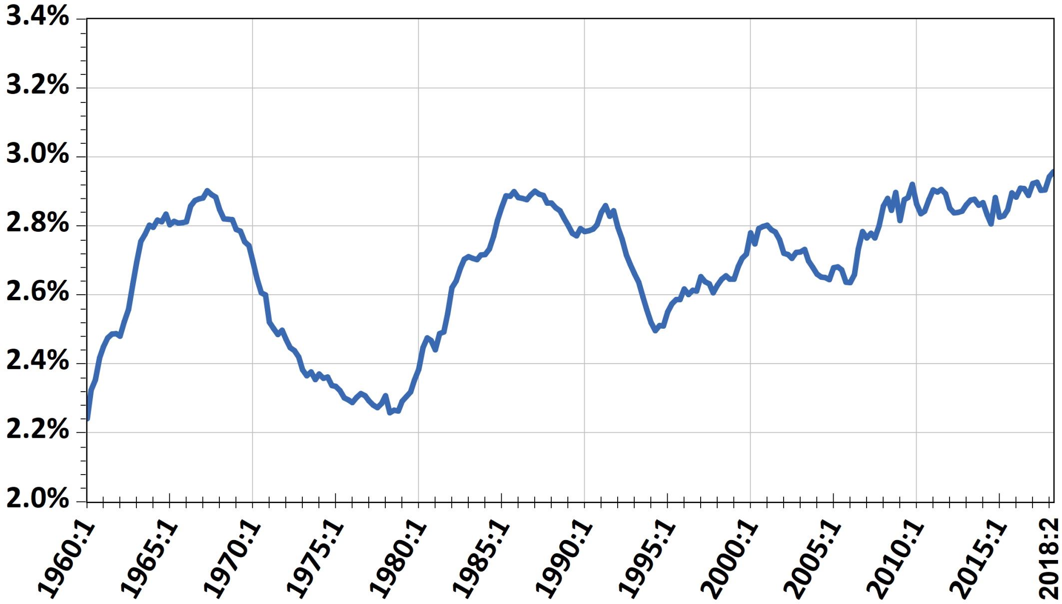
	Technology Shock	**Monetary Shock**	**Investment Shock**	**Spillover Shock**
Output	2%	16%	75%	7%
Consumption	2%	14%	72%	12%
Investment	1%	34%	62%	3%
R&D	31%	19%	30%	20%
Physical Capital	0%	36%	34%	30%
Capacity Utilization	18%	8%	70%	4%
Wages	20%	29%	24%	27%
Labor	18%	7%	75%	0%
Rate of return on capital	18%	8%	70%	4%
Interest Rate	12%	14%	69%	5%

Note: The table shows the variance decomposition of different economic indicators at t=20 quarters.

APPENDIX B

CHAPTER 2: IMPULSE RESPONSE FUNCTIONS AND WELFARE ANALYSIS

Figure B.1 *R&D* investment-to-GDP ratio in the United States (1960Q1-2018Q2)



APPENDIX C

CHAPTER 2: EQUILIBRIUM AND OPTIMALITY EQUATIONS

C.1 Equilibrium equations

In the equations below, Λ_t and Φ_t are the Lagrange multipliers associated respectively with the household's budget constraint equation (3.12), and the investment equation (3.13) at date t.

$$\frac{1}{C_t - hC_{t-1}} - \Lambda_t P_t - \beta \frac{h}{C_{t+1} - hC_t} = 0 \tag{C.1}$$

$$-\frac{\Lambda_t}{1+r_t} + \beta \Lambda_{t+1} = 0 \tag{C.2}$$

$$q_t - P_t a'(u_t) = 0 \tag{C.3}$$

$$-\Lambda_t P_t + \Phi_t \mu_t \left[\frac{I_t}{I_{t-1}} S'\left(\frac{I_t}{I_{t-1}}\right) + 1 - S\left(\frac{I_t}{I_{t-1}}\right) \right] + \beta \Phi_{t+1} \mu_{t+1} \frac{I_{t+1}^2}{I_t^2} S'\left(\frac{I_{t+1}}{I_t}\right) \tag{C.4}$$

$$-\Phi_t + \beta \Lambda_{t+1}\big(q_{t+1} u_{t+1} - P_{t+1} a(u_{t+1})\big) + \beta \Phi_{t+1}(1-\delta) = 0 \tag{C.5}$$

$$K_t = u_t \widetilde{K}_t \; . \tag{C.6}$$

$$W_t^*(j)^{-\gamma\nu - 1} = \frac{\gamma - 1}{\theta\gamma} \frac{Aux_{bc,t}}{Aux_{dis,t}} \tag{C.7}$$

$$\frac{K_t}{L_t} = \frac{\alpha}{1-\alpha}\frac{W_t}{q_t} \tag{C.8}$$

$$F_t = \frac{\epsilon}{\epsilon - 1}\,\frac{Aux_{cost,t}}{Aux_{rev,t}} \tag{C.9}$$

$$n_t = \left(\frac{X_t}{\zeta\, A_t^{max}}\right)^{1/(1+\eta)} \tag{C.10}$$

$$A_t^{max} \;=\; g_t^{max}\;\, A_{t-1}^{max} \tag{C.11}$$

$$g_t^{max} \;=\; 1 \;+\; \sigma_t\, n_{t-1} \tag{C.12}$$

$$X_t = \beta\,\frac{n_t}{1+\eta}\,\frac{\Lambda_{t+1}}{\Lambda_t}\;\frac{E_t V_{t+1}(A_t^{max})}{P_t}\;. \tag{C.13}$$

$$E_t\, V_{t+1} = A_t^{max\,(\alpha-1)(1-\epsilon)}\;\big(F_{t+1}^{1-\epsilon} Aux_{rev,t+1} - F_{t+1}^{-\epsilon} Aux_{cost,t+1} \;+\; Aux_{rem,t+2}\big) \tag{C.14}$$

$$\begin{aligned} P_t^{1-\epsilon} \;&=\; \xi_p(1-n_{t-1})(P_{t-1})^{1-\epsilon} \\ &\quad+\;(1-\xi_p)(1-n_{t-1})F_t^{1-\epsilon}\,Aux_{oldtech,t-1} \;+\; n_{t-1}A_{t-1}^{max\,(\alpha-1)(\epsilon-1)}F_t^{1-\epsilon} \end{aligned} \tag{C.15}$$

$$W_t^{1-\gamma} \;=\; \xi_w\,(W_{t-1})^{1-\gamma} + (1-\xi_w)\,W_t^{*\,1-\gamma} \tag{C.16}$$

$$P_t^{\epsilon}\,Y_t \;=\; \mu_{Z,t}\,\frac{K_t^{\alpha}\,L_t^{1-\alpha}}{Aux_{output,t}} \tag{C.17}$$

$$C_t \;+\; I_t \;+\; a(u_t)\widetilde{K}_t \;+\; X_t \;=\; Y_t\,, \tag{C.18}$$

$$\frac{1+R_t}{1+R} = \left(\frac{1+R_{t-1}}{1+R}\right)^{\rho_R}\left[\left(\frac{\pi_t}{\pi}\right)^{\alpha_\pi}\left(\frac{Y_t}{Y_{t-1}}\,g^{-1}\right)^{\alpha_y}\right]^{1-\rho_R}\;\mu_{M,t} \tag{C.19}$$

Auxiliary variables:

$$Aux_{bc,t} = \Lambda_t\,W_t^{\gamma}\,L_t + \xi_p\,\beta\,Aux_{bc,t+1}\,, \tag{C.20}$$

$$Aux_{dis,t} = \Lambda_t\,W_t^{\gamma(1+\nu)}L_t^{1+\nu} + \xi_p\,\beta\,Aux_{dis,t+1}\,. \tag{C.21}$$

$$Aux_{rev,t} = P_t^{\epsilon}Y_t + \xi_p\,\beta\,\frac{\Lambda_{t+1}}{\Lambda_t}(1-n_t)\,Aux_{rev,t+1}\,, \tag{C.22}$$

$$Aux_{cost,t} = \Omega_t\,P_t^{\epsilon}\,Y_t + \xi_p\,\beta\,\frac{\Lambda_{t+1}}{\Lambda_t}(1-n_t)\,Aux_{cost,t+1}\,. \tag{C.23}$$

$$Aux_{rem,t+2} = (1-\xi_p)\,\beta^2\,\frac{\Lambda_{t+2}}{\Lambda_{t+1}}\left(F_{t+2}^{(1-\epsilon)}Aux_{rev,t+2} - F_{t+2}^{-\epsilon}Aux_{cost,t+2}\right) \quad \text{(C.24)}$$

$$Aux_{oldtech,t-1} \;=\; n_{t-2}\,A_{t-2}^{max\,(\alpha-1)(1-\epsilon)} \;+\; (1-n_{t-2})\,Aux_{oldtech,t-2} \quad \text{(C.25)}$$

$$\begin{aligned} Aux_{output,t} \;=\; & \xi_p(1-n_{t-1})(P_{t-1}\pi_{p\,t-1,t})^{-\epsilon}Aux_{oldtechnonreset,t-1} \\ & + (1-\xi_p)(1-n_{t-1})F_t^{-\epsilon}Aux_{oldtechreset,t-1} \\ & + n_{t-1}F_t^{-\epsilon}(A_{t-1}^{max})^{(1-\epsilon)(\alpha-1)} \end{aligned} \quad \text{(C.26)}$$

$$Aux_{oldtechnonreset,t} \;=\; n_{t-1}A_{t-1}^{max\,\alpha-1} + (1-n_{t-1})Aux_{oldtechnonreset,t-1} \quad \text{(C.27)}$$

Laws of motions for the various stochastic shocks:

$$\ln\mu_{I,t+1} = \rho_I\,\ln\mu_{I,t} + \epsilon_{I,t} \quad \text{(C.28)}$$

$$\ln\mu_{Z,t} = \rho_Z\,\ln\mu_{Z,t-1} + \epsilon_{Z,t} \quad \text{(C.29)}$$

$$\ln\sigma_t = \ln\sigma \;+\; \rho_\sigma\,\ln\sigma_{t-1} \;+\; \epsilon_{\sigma,t} \quad \text{(C.30)}$$

$$\ln\mu_{M,t} = \rho_M\,\ln\mu_{M,t-1} + \epsilon_{M,t} \quad \text{(C.31)}$$

C.2 The New Keynesian Phillips curve

C.2.1 Detrended variables

The first step to derive the Phillips curve is to detrend the relevant variables of the model:

$$y_t \equiv \frac{Y_t}{\overline{A_t}^{1-\alpha}} \tag{C.32}$$

$$f_t \equiv \frac{F_t}{\overline{A_t}^{1-\alpha} P_t} \tag{C.33}$$

$$aux_{cost,t} \equiv \frac{Aux_{cost,t}}{\overline{A_t}^{2-\alpha} P_t^{1+\epsilon}} \tag{C.34}$$

$$aux_{rev,t} \equiv \frac{Aux_{rev,t}}{\overline{A_t} P_t^{\epsilon}} \tag{C.35}$$

$$\omega_t \equiv \frac{\Omega_t}{\overline{A_t}^{1-\alpha}} \tag{C.36}$$

$$aux_{oldtech,t} \equiv \frac{Aux_{oldtech,t}}{\overline{A_t}^{(\alpha-1)(1-\epsilon)}} \tag{C.37}$$

$$\lambda_t \equiv \Lambda_t A_t^{max} P_t \tag{C.38}$$

C.2.2 Detrended optimal reset price and marginal cost

Then, detrending the optimal reset price and approximating it around the steady state implies:

$$f_t = \frac{\epsilon}{\epsilon - 1}\, \frac{aux_{costt}}{aux_{revt}} \tag{C.39}$$

$$aux_{costt} = \omega_t\, y_t + \frac{\xi_p\, \beta\, \lambda_{t+1}}{\lambda_t}\, {g_t}^{1-\alpha}\, {\pi_{t+1}}^{\epsilon}\, \left(1 - n_t\right)\, aux_{costt+1} \tag{C.40}$$

$$aux_{revt} = y_t + \left(1 - n_t\right)\, \frac{\xi_p\, \beta\, \lambda_{t+1}}{\lambda_t}\, {\pi_{t+1}}^{\epsilon - 1}\, aux_{revt+1} \tag{C.41}$$

C.2.2.1 Auxiliary variable for cost

Using the following steady state values

$$aux_{cost} = \omega\, y + \xi_p\, \beta\, g^{1-\alpha}\, \pi^{\epsilon}\, \left(1 - n\right)\, aux_{cost}\ , \tag{C.42}$$

$$\left(1 - \xi_p\, \beta\, g^{1-\alpha}\, \pi^{\epsilon}\, \left(1 - n\right)\right) aux_{cost} = \omega\, y\ , \tag{C.43}$$

$$aux_{cost} = \frac{\omega\, y}{1 - \xi_p\, \beta\, g^{1-\alpha}\, \pi^{\epsilon}\, \left(1 - n\right)}\ , \tag{C.44}$$

we find:

$$\begin{aligned} \widehat{aux}_{cost,t} = {} & \frac{\omega y}{aux_{cost}} \left(\widehat{\omega}_t + \widehat{y}_t\right) \\ & - \frac{aux_{cost} - \omega y}{aux_{cost}} \left[\frac{n}{1-n}\widehat{n}_t + \widehat{aux}_{cost,t+1} - \widehat{\lambda}_t + \widehat{\lambda}_{t+1} + \epsilon\, \widehat{\pi}_{t+1} + (1-\alpha)\widehat{g}_{t+1}\right] \end{aligned} \tag{C.45}$$

C.2.2.2 Auxiliary variable for revenue

Using the following steady state values

$$aux_{rev} = y + (1-n)\;\xi_p\,\beta\,\pi^{\epsilon-1}\,aux_{rev}\;, \tag{C.46}$$

$$(1-(1-n)\;\xi_p\,\beta\,\pi^{\epsilon-1}\,)\,aux_{rev} = y\;, \tag{C.47}$$

$$aux_{rev} = \frac{y}{1-(1-n)\;\xi_p\,\beta\,\pi^{\epsilon-1}}\;, \tag{C.48}$$

we find:

$$\widehat{aux}_{rev,t} = \frac{y}{aux_{rev}}\widehat{y}_t - \frac{aux_{rev}-y}{aux_{rev}}\left[\frac{n}{1-n}\widehat{n}_t + \widehat{aux}_{cost,t+1} - \widehat{\lambda}_t + \widehat{\lambda}_{t+1} + (\epsilon-1)\widehat{\pi}_{t+1}\right] \tag{C.49}$$

C.2.3 From the price index to the inflation rate equation

Using the price index, we derive a log-linearized approximation of inflation around the steady state:

The price index equation:

$$P_t^{1-\epsilon} = \xi_p(1-n_{t-1})P_{t-1}^{1-\epsilon} + (1-\xi_p)\int_{n_{t-1}}^{1} {P^\star}_t^{1-\epsilon}(i)di + \int_0^{n_{t-1}} {P^\star}_t(i)^{1-\epsilon} \tag{C.50}$$

$$P_t^{1-\epsilon} = \xi_p(1-n_{t-1})P_{t-1}^{1-\epsilon} + (1-\xi_p)(1-n_{t-1})F_t^{1-\epsilon}Aux_{oldtech,t-1} + n_{t-1}F_t^{1-\epsilon}A_{t-1}^{max\,(\alpha-1)(\epsilon-1)}\;. \tag{C.51}$$

Corresponding inflation equation (after detrending):

$$\xi_p(1-n_{t-1})\pi_{t-1}^{\epsilon-1} = 1 - f_t^{1-\epsilon}\left((1-\xi_p)(1-n_{t-1})aux_{oldtech,t-1} + n_{t-1}d_t^{(\alpha-1)(\epsilon-1)}\right) \tag{C.52}$$

or, alternatively,

$$\xi_p \pi_{t-1}^{\epsilon-1} = (1-n_{t-1})^{-1} - f_t^{1-\epsilon}\left((1-\xi_p)aux_{oldtech,t-1} + \frac{n_{t-1}}{1-n_{t-1}} d_t^{(\alpha-1)(\epsilon-1)}\right) . \tag{C.53}$$

The distance between the average technology and the frontier:

$$\bar{A}_t = n_{t-1}A_t^{max} + (1-n_{t-1})\bar{A}_{t-1} \tag{C.54}$$

$$\frac{\bar{A}_t}{A_t^{max}} = n_{t-1} + (1-n_{t-1})\frac{\bar{A}_{t-1}}{A_t^{max}} \tag{C.55}$$

$$d_t \equiv \frac{A_t^{max}}{\bar{A}_t} \tag{C.56}$$

$$d_t^{-1} = n_{t-1} + (1-n_{t-1})d_{t-1}^{-1}g_t^{-1} . \tag{C.57}$$

The gross growth rate of the technological frontier:

$$g_t = 1 + \sigma_t n_{t-1} \tag{C.58}$$

$$n_{t-1} = \frac{g_t - 1}{\sigma_t} . \tag{C.59}$$

The inflation rate equation can be rewritten as a follows:

$$
\begin{aligned}
&\xi_p\,(1-\frac{g_t-1}{\sigma_t})\,\pi_{t-1}^{\epsilon-1} \qquad \text{(C.60)}\\
&\quad= 1-f_t^{1-\epsilon}\left((1-\xi_p)(1-\frac{g_t-1}{\sigma_t})\,aux_{oldtech,t-1}+\frac{g_t-1}{\sigma_t}\left(\frac{g_t-1}{\sigma_t}\;+\;(1-\frac{g_t-1}{\sigma_t})d_{t-1}^{-1}g_t^{-1}\right)^{(1-\alpha)(\epsilon-1)}\right)\,.
\end{aligned}
$$

Finally, the linear approximation around the steady state brings leads to the following version of the New Keynesian Phillips curve:

$$
\begin{aligned}
\hat{\pi}_t = &-\frac{\left(aux_{old}\left(\frac{g-1}{\sigma}-1\right)\,(\xi-1)+\frac{g-1}{\sigma\left(\frac{g-1}{\sigma}-\frac{\frac{g-1}{\sigma}-1}{d\,g}\right)^{(\alpha-1)\,(\epsilon-1)}}\right)\,(\epsilon-1)}{f^{\epsilon}\,\left(f^{1-\epsilon}\,\left(aux_{old}\left(\frac{g-1}{\sigma}-1\right)\,(\xi-1)+\frac{g-1}{\sigma\left(\frac{g-1}{\sigma}-\frac{\frac{g-1}{\sigma}-1}{d\,g}\right)^{(\alpha-1)\,(\epsilon-1)}}\right)-1\right)}\,\hat{f}_t\\
&+\frac{f^{1-\epsilon}\,\left(\frac{g-1}{\sigma}-1\right)\,(\xi-1)}{f^{1-\epsilon}\,\left(aux_{old}\left(\frac{g-1}{\sigma}-1\right)\,(\xi-1)+\frac{g-1}{\sigma\left(\frac{g-1}{\sigma}-\frac{\frac{g-1}{\sigma}-1}{d\,g}\right)^{(\alpha-1)\,(\epsilon-1)}}\right)-1}\,\widehat{aux}_{oldtech,t-1} \qquad \text{(C.61)}\\
&+\frac{\xi\,\left(\frac{g-1}{\sigma}-1\right)\,\left(\frac{f^{1-\epsilon}\left(\frac{1}{\sigma\left(\frac{g-1}{\sigma}-\frac{\frac{g-1}{\sigma}-1}{d\,g}\right)^{(\alpha-1)\,(\epsilon-1)}}-\frac{(\alpha-1)\,(\epsilon-1)\,(g-1)\left(\frac{1}{\sigma}-\frac{1}{\sigma\,d\,g}+\frac{\frac{g-1}{\sigma}-1}{d\,g^2}\right)}{\sigma\left(\frac{g-1}{\sigma}-\frac{\frac{g-1}{\sigma}-1}{d\,g}\right)^{(\alpha-1)\,(\epsilon-1)+1}}\right)}{\xi\left(\frac{g-1}{\sigma}-1\right)}-\frac{\frac{f^{1-\epsilon}\,(g-1)}{\sigma\left(\frac{g-1}{\sigma}-\frac{\frac{g-1}{\sigma}-1}{d\,g}\right)^{(\alpha-1)\,(\epsilon-1)}}-1}{\sigma\,\xi\left(\frac{g-1}{\sigma}-1\right)^2}\right)}{f^{1-\epsilon}\,\left(aux_{old}\left(\frac{g-1}{\sigma}-1\right)\,(\xi-1)+\frac{g-1}{\sigma\left(\frac{g-1}{\sigma}-\frac{\frac{g-1}{\sigma}-1}{d\,g}\right)^{(\alpha-1)\,(\epsilon-1)}}\right)-1}\,\hat{g}_t
\end{aligned}
$$

$$-\frac{f^{1-\epsilon}\,\left(\frac{g-1}{\sigma}-1\right)\,\left(\alpha-1\right)\,\left(\epsilon-1\right)\,\left(g-1\right)}{\sigma\,d^2\,g\,\left(f^{1-\epsilon}\,\left(aux_{old}\left(\frac{g-1}{\sigma}-1\right)\,\left(\xi-1\right)+\frac{g-1}{\sigma\left(\frac{g-1}{\sigma}-\frac{\frac{g-1}{\sigma}-1}{d\,g}\right)^{\left(\alpha-1\right)\,\left(\epsilon-1\right)}}\right)-1\right)\,\left(\frac{g-1}{\sigma}-\frac{\frac{g-1}{\sigma}-1}{d\,g}\right)^{\left(\alpha-1\right)\,\left(\epsilon-1\right)+1}}\,\hat{d}_t\,.$$

That is

$$\hat{\pi}_t=\Gamma_1\,\hat{f}_t+\Gamma_2\,a\hat{u}x_{oldtech,t-1}+\Gamma_3\,\hat{g}_t+\Gamma_4\,\hat{d}_t\;, \tag{C.62}$$

with $\Gamma_1\geq 0$, $\Gamma_2\geq 0$, $\Gamma_3\leq 0$ and $\Gamma_4\leq 0$.

APPENDIX D

CHAPTER 3: APPENDIX ON BAYESIAN ESTIMATION OF DSGE MODELS

D.1 Why Bayesian estimation and some key considerations

The preference for Bayesian estimation over maximum likelihood estimation (MLE) in DSGE modeling is attributed to several compelling advantages. Bayesian methods adeptly integrate prior theoretical knowledge and empirical findings into the estimation process, enhancing parameter identification, especially in cases of limited data. Unlike MLE, which offers only point estimates, Bayesian estimation elucidates the full probability distribution of parameters, providing a comprehensive assessment of uncertainty and parameter variability. This is crucial in macroeconomic contexts, where understanding the range of plausible parameter values is as important as the estimates themselves. Furthermore, Bayesian approaches streamline the comparison and evaluation of different models using probabilistic tools like the Bayes Factor. These methods also exhibit greater robustness in managing complex models with numerous parameters or latent variables, where MLE may struggle with issues of convergence or parameter identification. Thus, Bayesian estimation is a more versatile and informative tool in contemporary macroeconomic modeling.

Here are some consideration to take into account

- Bayesian Estimation in DSGE Models: The Bayesian framework in DSGE modeling involves updating priors with observed data using Bayes' theorem. This process refines initial parameter beliefs (priors), integrating them with the likelihood of observing the data, resulting in the posterior distribution. The theorem's formulation ensures that parameter estimates are informed both by prior theoretical or empirical knowledge and by the observed data.
- Prior Distribution Specification: Priors in Bayesian analysis are critical. They encapsulate existing knowledge or assumptions about parameters. For instance, normal priors might be used for parameters expected to hover around a mean value, while gamma priors might be suitable for variance parameters. The choice of priors can significantly influence posterior estimates, especially in cases of limited data.
- Likelihood Function and Model Solution: The likelihood function is the probability of the data given the parameters. Constructing this function involves solving the DSGE model under different parame-

ter configurations and comparing the model's predictions with actual economic data. This comparison is pivotal for assessing the model's fit and guiding the parameter estimation process.

- Metropolis-Hastings Algorithm: The Metropolis-Hastings algorithm facilitates sampling from complex posterior distributions. This algorithm proposes new parameter values based on a proposal distribution and decides on their acceptance based on an acceptance ratio. The algorithm's efficiency is crucial for ensuring a representative sample from the posterior distribution.

- Model Calibration and Bayesian Inference: The calibration and estimation process in Bayesian DSGE modeling involves a blend of theory-driven parameter setting and data-driven parameter adjustment. Calibration sets some parameters based on theoretical or empirical benchmarks, while Bayesian inference tunes the remaining parameters to best fit the observed data.

D.2 Technical summary

D.2.1 Bayesian estimation framework

Bayesian estimation in the context of DSGE models is a methodological approach that combines prior beliefs about economic parameters with empirical data to refine these beliefs. The fundamental equation guiding this process is Bayes' theorem:

$$P(\theta|y) = \frac{P(y|\theta)P(\theta)}{P(y)} \tag{D.1}$$

where:

- θ represents the vector of model parameters.
- y denotes the observed data.
- $P(\theta|y)$ is the posterior distribution of the parameters given the data.
- $P(y|\theta)$ is the likelihood function.
- $P(\theta)$ is the prior distribution of the parameters.
- $P(y)$ is the marginal likelihood of the observed data.

D.2.2 Prior distributions

The choice of prior distributions $P(\theta)$ is critical in Bayesian estimation. Priors can be categorized into:

- Informative Priors: Derived from previous empirical research, expert knowledge, or theoretical models. They are used when substantial information is available about the parameters.
- Non-informative or Weakly Informative Priors: Applied when less is known about the parameters. These priors are intentionally vague, allowing the data to primarily influence the posterior distribution.

In DSGE modeling, priors need to be chosen carefully to ensure they are consistent with economic theory while remaining flexible enough to learn from the data.

D.2.3 Likelihood function

The likelihood function $P(y|\theta)$ in DSGE models is often complex due to the nonlinear nature of these models. Key considerations include:

- Model Solution: Before computing the likelihood, the DSGE model is typically linearized or log-linearized around a steady state.
- State-Space Representation: Many DSGE models are formulated in a state-space framework to facilitate the use of the Kalman filter, which is instrumental in evaluating the likelihood function for time series data.

D.2.4 Posterior distribution

The posterior distribution $P(\theta|y)$ is the updated belief about the model parameters after considering the data. It is central to Bayesian inference and is used for:

- Parameter Estimation: The posterior distribution provides a range of plausible values for each parameter.

- Uncertainty Quantification: It allows for the calculation of credible intervals, offering a probabilistic interpretation of parameter uncertainty.
- Predictive Analysis: The posterior can be used for forecasting and policy analysis, as it encapsulates both prior information and data-driven insights.

D.2.5 Computational considerations

Given the complexity of the posterior distribution in DSGE models, direct analytical solutions are often unfeasible. This necessitates the use of numerical methods like MCMC to sample from the posterior. The convergence and efficiency of these algorithms are crucial for reliable inference.

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